

Handout 0: Background & Acknowledgements

This lecture series on the magnetic field evolution before recombination consists of 5 lectures:

1. Numerical approaches to magnetogenesis
2. Numerical approaches to relic gravitational waves
3. Decaying MHD turbulence
4. Dynamos
5. Magnetic fields during recombination

To enhance our intuition about these topics, it is useful to combine theory with practical examples using computer simulations. Here we use the PENCIL CODE (Pencil Code Collaboration, 2021), **but you are welcome to use other tools to address questions of interest**.

1 Background

My main background is originally in dynamo theory, with applications to solar dynamos on the one hand and galactic dynamos on the other. In both cases, we observe (and want to explain) *large-scale magnetic fields*, in addition to *small-scale magnetic fields* that are also always present. The solar magnetic field is broadly antisymmetric about the equator and varies cyclicly with a period of about 22-year, whereas the magnetic fields of many spiral galaxies is symmetric about the equatorial plane (Parker, 1971).

2 Acknowledgements

Much of the science behind the topics of the 5 lectures above I learned gradually by working with other people who have taught me a lot of new things. In some fields, **my knowledge is still quite incomplete, so please *don't hesitate in correcting me!***

I might not have entered to topic of primordial magnetic fields without the opportunity provided by Nordita of interacting with people from very different fields. When I was an assistant professor at Nordita during 1995 (before moving to Newcastle upon Tyne in 1996), Kari Enqvist and Poul Olesen told me that a big problem with primordial magnetic fields are the small length scales. This is where my knowledge about inverse turbulent cascades (Frisch et al., 1975; Pouquet et al., 1976) became useful. This led to my first paper in this field (Brandenburg et al., 1996).

Since 2010, Tina Kahniashvili introduced me to many new aspects; see Kahniashvili et al. (2010) for our first paper. This culminated in work on relic gravitation waves with Alberto Roper Pol; see (Roper Pol et al., 2020). During the Nordita program on the *Origin, Evolution, and Signatures of Cosmological Magnetic Fields* in 2015 (<https://indico.fysik.su.se/event/3988/>), organized by Tina Kahniashvili, Tanmay Vachaspati, and myself, I learned about the *chiral magnetic effect* from Alexey Boyarsky and Oleg Ruchayskiy, which led to joint work with my long-term collaborator Igor Rogachevskii (Rogachevskii et al., 2017) and our Nordita postdoc Jennifer Schober (Schober et al., 2018). Finally, the little I know about inflationary magnetogenesis stems from my work with (former) Nordita postdoc Ramkishor Sharma (Brandenburg, & Sharma, 2021) and Oksana Iarygina (Iarygina et al., 2024).

At Nordita, we also had relevant programs on Quantum Anomalies and Chiral Magnetic Phenomena (<https://indico.fysik.su.se/event/6140/>) in 2018, Gravitational Waves from the Early Universe (<https://indico.fysik.su.se/event/6554/>) in 2019, Towards a Comprehensive Model of the Galactic Magnetic Field (<https://indico.fysik.su.se/event/7914/>) in 2023, Axions in Stockholm 2025 (<https://indico.fysik.su.se/event/8514/>) in 2025.

[ps://indico.fysik.su.se/event/8808/](https://indico.fysik.su.se/event/8808/)) in 2025, and finally Numerical Simulations of Early Universe Sources of Gravitational Waves (<https://indico.fysik.su.se/event/8805/>), also in 2025. Of major importance was also the Program on the Generation, evolution, and observations of cosmological magnetic fields in Lausanne (<https://indico.cern.ch/event/1334236/>).

3 Caveats

The present material has been written under time pressure and probably contains lots of mistakes. You are welcome to report those to me, so I can fix them for future usage.

References

Brandenburg, A., Enqvist, K., & Olesen, P., “Large-scale magnetic fields from hydromagnetic turbulence in the very early universe,” *Phys. Rev. D* **54**, 1291–1300 (1996).

Brandenburg, A., & Sharma, R., “Simulating relic gravitational waves from inflationary magnetogenesis,” *Astrophys. J.* **920**, 26 (2021).

Frisch, U., Pouquet, A., Léorat, J., & Mazure, A., “Possibility of an inverse cascade of magnetic helicity in hydrodynamic turbulence,” *J. Fluid Mech.* **68**, 769–778 (1975).

Iarygina, O., Sfakianakis, E. I., Sharma, R. & Brandenburg, A., “Backreaction of axion-SU(2) dynamics during inflation,” *J. Cosm. Astrop. Phys.* **04**, 018 (2024).

Kahniashvili, T., Brandenburg, A., Tevzadze, A. G., & Ratra, B., “Numerical simulations of the decay of primordial magnetic turbulence,” *Phys. Rev. D* **81**, 123002 (2010).

Parker, E. N., “The generation of magnetic fields in astrophysical bodies. II. The galactic field,” *Astrophys. J.* **163**, 255–278 (1971).

Pencil Code Collaboration: Brandenburg, A., Johansen, A., Bourdin, P. A., Dobler, W., Lyra, W., Rheinhardt, M., Bingert, S., Haugen, N. E. L., Mee, A., Gent, F., Babkovská, N., Yang, C.-C., Heinemann, T., Dintrans, B., Mitra, D., Candelaresi, S., Warnecke, J., Käpylä, P. J., Schreiber, A., Chatterjee, P., Käpylä, M. J., Li, X.-Y., Krüger, J., Aarnes, J. R., Sarson, G. R., Oishi, J. S., Schober, J., Plasson, R., Sandin, C., Karchniwy, E., Rodrigues, L. F. S., Hubbard, A., Guerrero, G., Snodin, A., Losada, I. R., Pekkilä, J., & Qian, C., “The Pencil Code, a modular MPI code for partial differential equations and particles: multipurpose and multiuser-maintained,” *Journal of Open Source Software* **6**, 2807 (2021).

Pouquet, A., Frisch, U., & Léorat, J., “Strong MHD helical turbulence and the nonlinear dynamo effect,” *J. Fluid Mech.* **77**, 321–354 (1976).

Rogachevskii, I., Ruchayskiy, O., Boyarsky, A., Fröhlich, J., Kleeorin, N., Brandenburg, A., & Schober, J., “Laminar and turbulent dynamos in chiral magnetohydrodynamics. I. Theory,” *Astrophys. J.* **846**, 153 (2017).

Roper Pol, A., Mandal, S., Brandenburg, A., Kahniashvili, T., & Kosowsky, A., “Numerical simulations of gravitational waves from early-universe turbulence,” *Phys. Rev. D* **102**, 083512 (2020).

Schober, J., Rogachevskii, I., Brandenburg, A., Boyarsky, A., Fröhlich, J., Ruchayskiy, O., & Kleeorin, N., “Laminar and turbulent dynamos in chiral magnetohydrodynamics. II. Simulations,” *Astrophys. J.* **858**, 124 (2018).