Magnetic field growth in large-scale structures

Franco Vazza

franco.vazza2@unibo.it

OUTLINE

In this lesson we will (briefly) see:

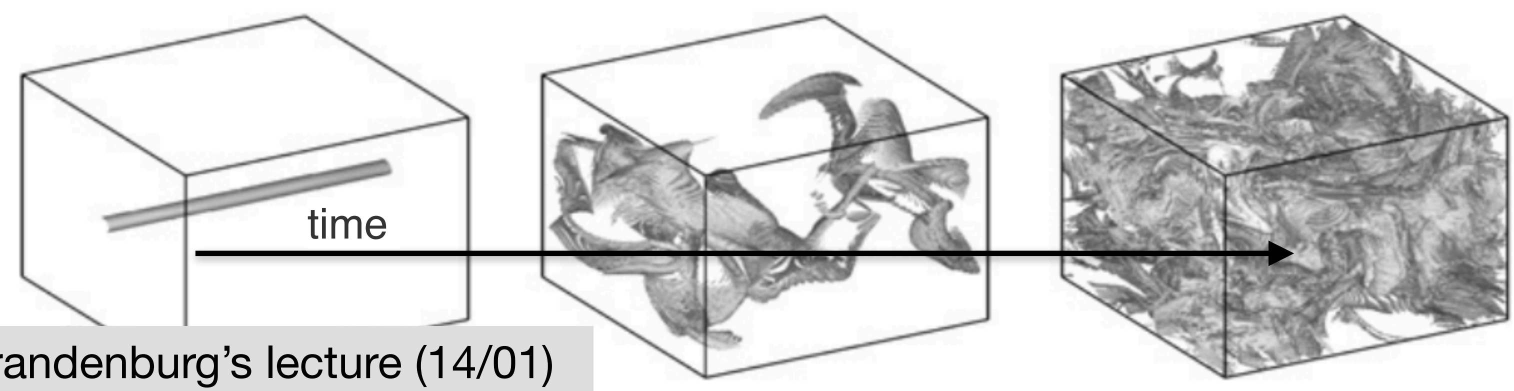
- a recap on small scale dynamo amplification in the intracluster medium

- how magnetic fields are introduced and evolved in MHD cosmological codes (and their numerical challenges)

AMPLIFICATION OF COSMIC MAGNETIC FIELDS

- Supposing we have a weak initial seed B-field, how is this amplified by structure formation, and up to which level?
- Motions in highly conductive fluids (~ideal MHD) produce currents which can sustain the steady growth of $U_B = \int B^2/(8\pi) dV$ through the transfer of kinetic into magnetic energy.

- **A small scale dynamo**: magnetohydrodynamical process converting turbulent kinetic energy ($\langle \vec{V} \rangle = 0,\ \langle V^2 \rangle^{1/2} > 0$) into magnetic energy
- First introduced by Kazantsev (1968), see also Brandenburg & Subramanian (2005) and Rincon (2019) for reviews



see Brandenburg's lecture (14/01)

Cho 2015

AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO

Stochastic gas motions driven by turbulence stretch and fold $\overrightarrow{B}$-lines
At every stretch and fold cycle the magnetic field will grow exponentially: $B \propto e^{\gamma t}$

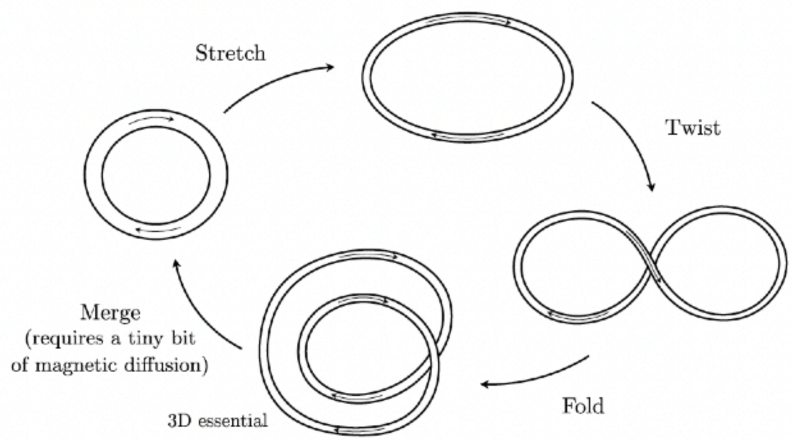
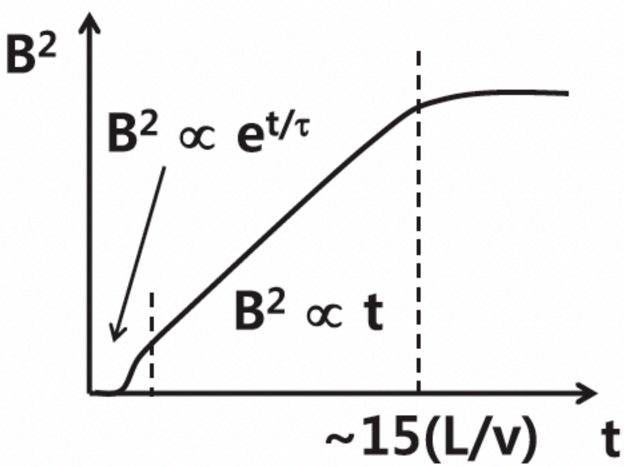


FIGURE 9. The famous stretch-twist-fold dynamo cartoon, adapted from Vainshtein & Zel'dovich (1972) and many others.

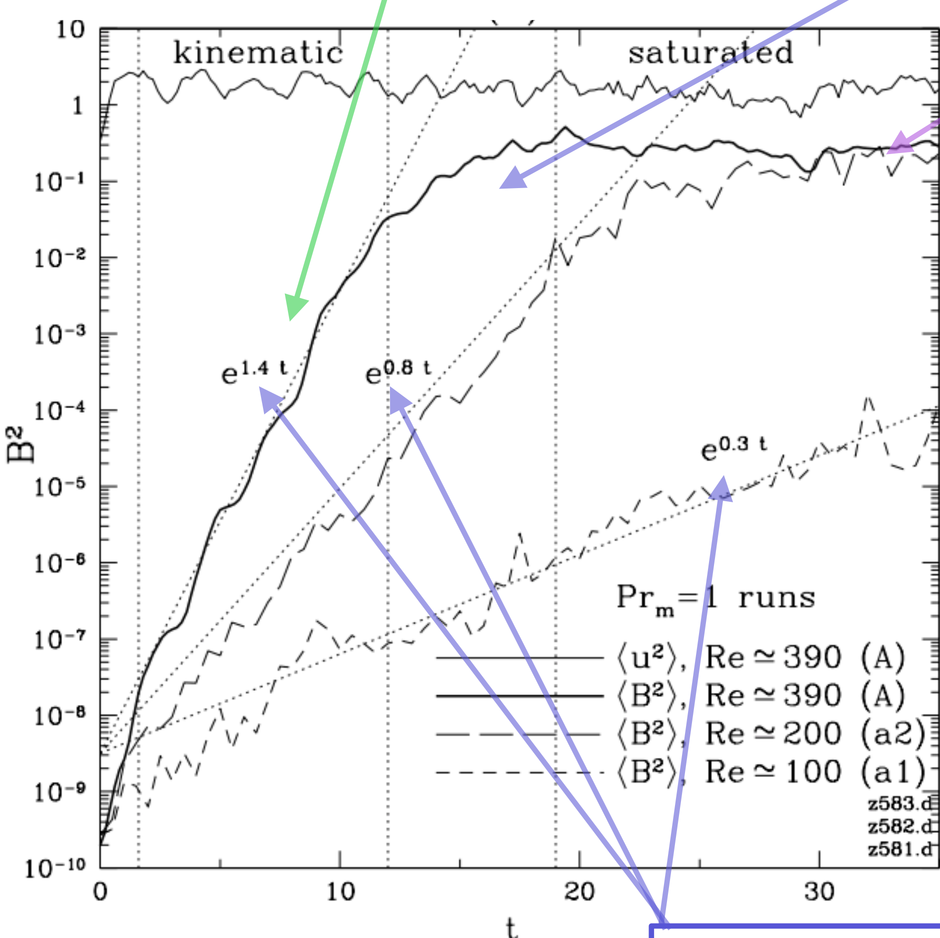


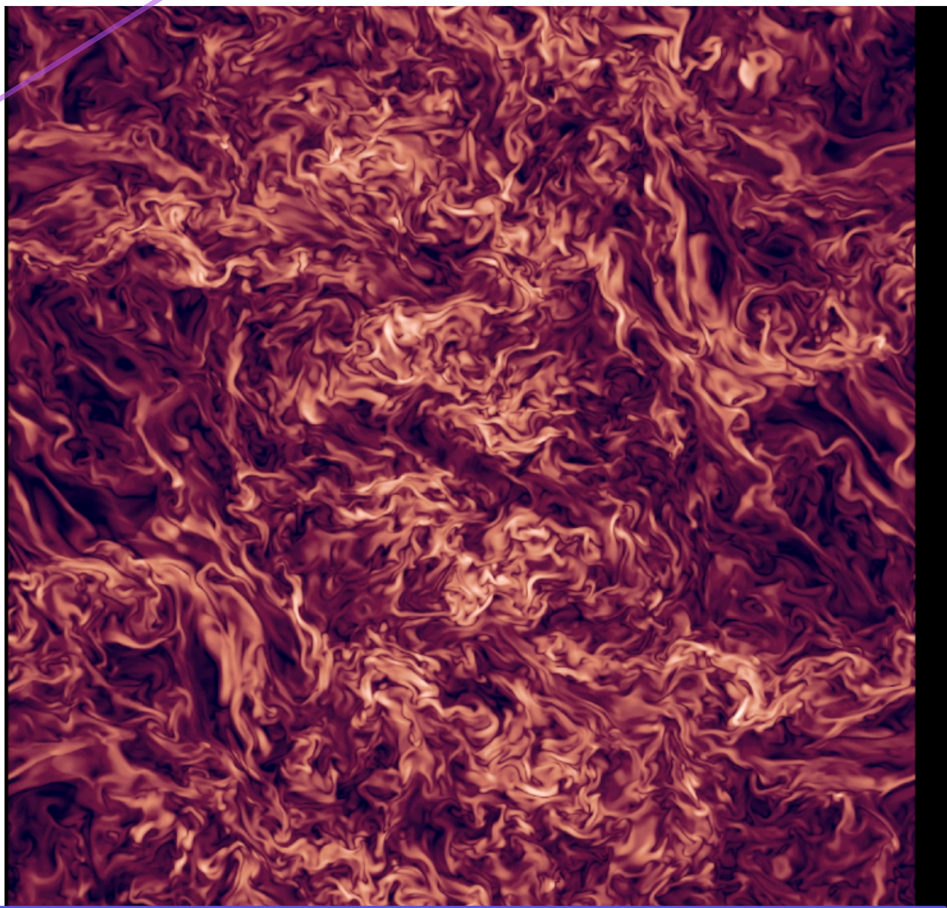
It can operate in systems that are turbulent and that do not have a large organised motion (rotation) e.g. clusters of galaxies/ISM/…

AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO

Also through the use of numerical simulations, 3 main s stages have been identified:

kinematic exponential (fast) , non-linear (slower), saturated (end)





Schekochihin+2005

exponential growth depends on the Reynolds Number $R_e \approx L/\lambda$

AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO

What decides saturation? The end of the kinematic phase occurs at the scale where stretching by turbulence is balanced by magnetic tension $\mathbf{B}\cdot\nabla\overrightarrow{\mathbf{B}} \sim \mathbf{u}\cdot\nabla\mathbf{u} \sim u^2/l \sim B^2/l \;\rightarrow u^2 \sim B^2$ so approximate equipartition between turbulent kinetic and magnetic energy at this scale

From induction equation, one can approximately derive that now:

$$\frac{d\langle\mathbf{B}\rangle^2}{dt} \sim \gamma^*(t)\langle\mathbf{B}\rangle^2 \sim u_{l*}^3/l^*$$

where $\gamma^*(t) = u_{l*}/l^*$ are the largest processed eddies by dynamo, and $\epsilon_K = u_{l*}^3/l_* = u^3/l$ is a constant in Kolmogorov turbulence.

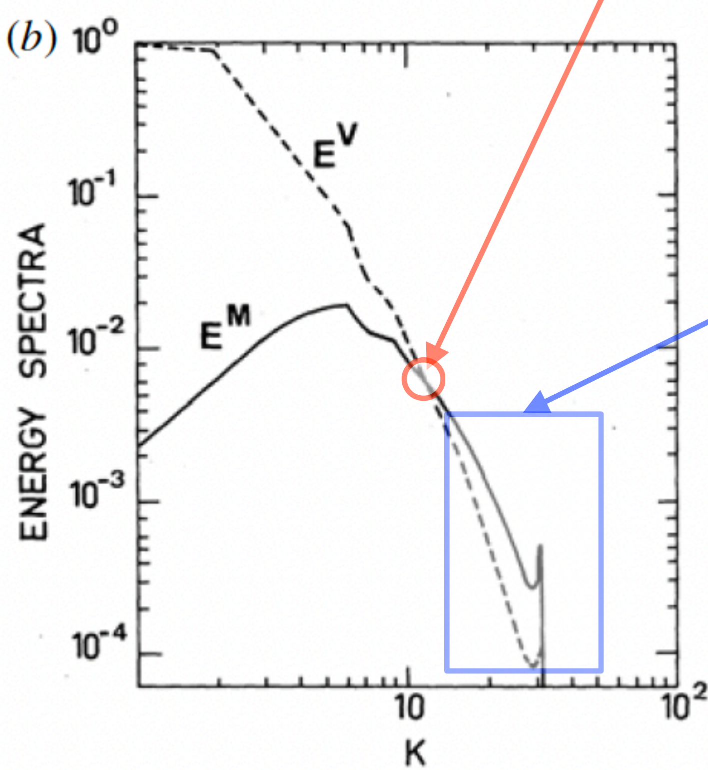
So we finally get: $\langle\,|B^2|\,\rangle \approx \eta\epsilon_K t$ i.e. the total magnetic energy is a tiny fraction ($\eta \sim 10^{-2}$) of the total kinetic energy processed by the cascade.

(e.g. Rincon 2019)

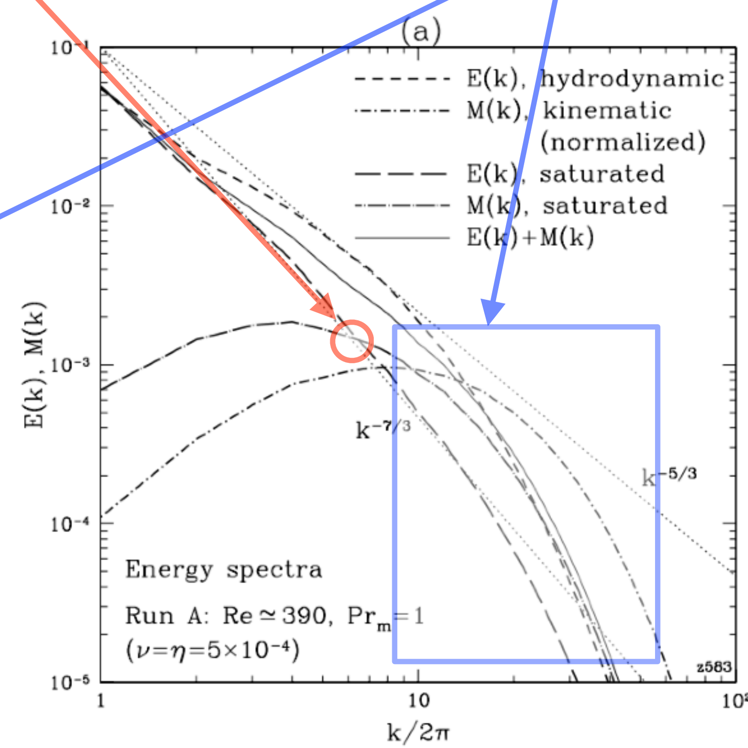
AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO

At saturation:

- the magnetic spectrum reaches ~equipartition with the kinetic spectrum at the larger processed eddy scale, $k_* = 2\pi/l_*$
- magnetic tension has modified the turbulent cascade for $k \geq k_*, l \leq l_*$



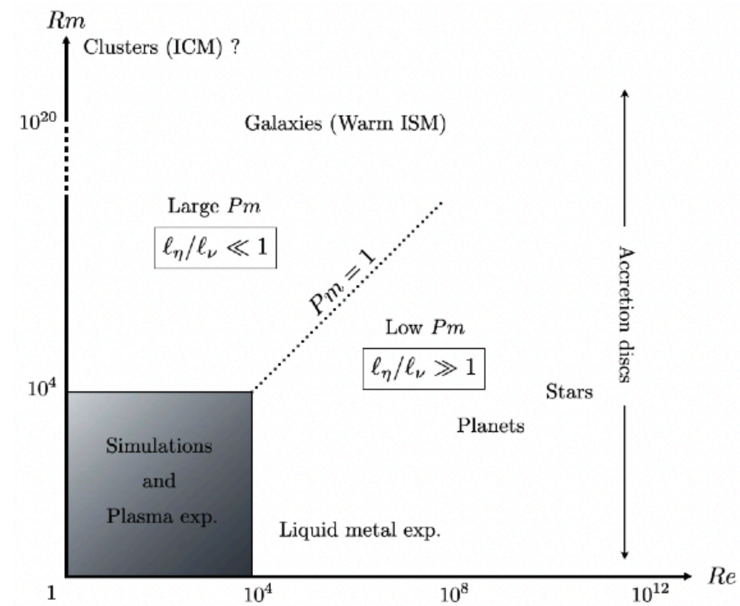
(Meneguzzi+1981)



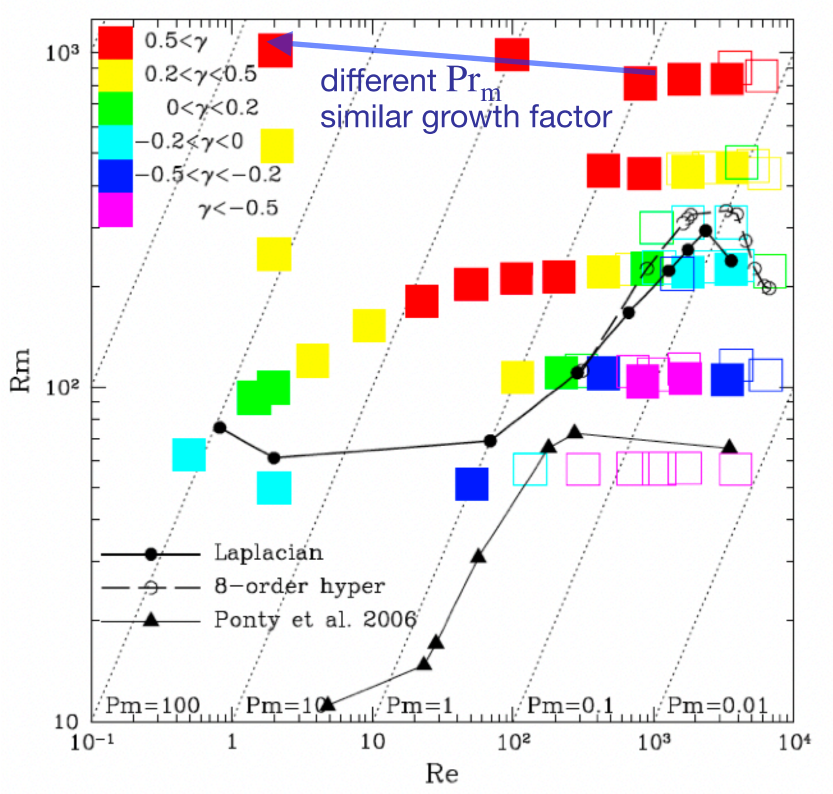
(Schekochihin+2005)

IMPORTANT CAVEAT ON THE PRANDTL NUMBER!

A key parameter in dynamos is the Prandtl number: $\mathrm{Pr_m} = \nu/\eta = \mathrm{Re_m}/\mathrm{Re}$, the ratio between viscosity and resistivity. It depends on the fluid, not on the flow. The ICM likely has $\mathrm{Pr_m} \gg 1$!!



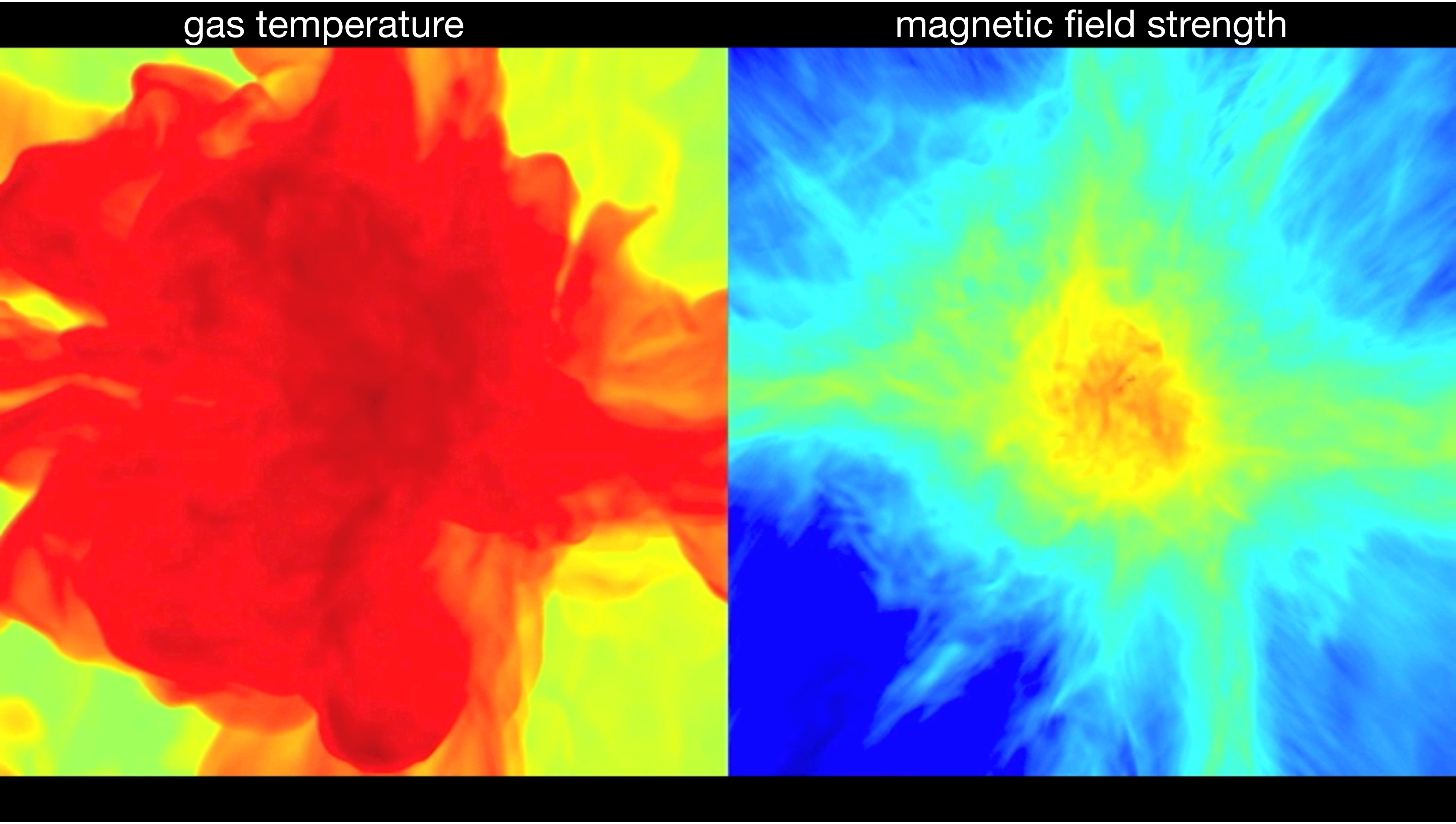
results on growth factors from simulations with different Re and Rm



- "due to computing power limitations implying finite numerical resolutions, most virtual MHD fluids of computer simulations have $0.1 \le \mathrm{Pr_m} \le 10$. Hence, it is and will remain impossible in a foreseeable future to simulate magnetic-field amplification in any kind of regime found in nature. The best we can **hope** for is that simulations of largish or smallish $\mathrm{Pr_m}$ regimes can provide glimpses of the asymptotic dynamics"

(e.g. Rincon 2019)

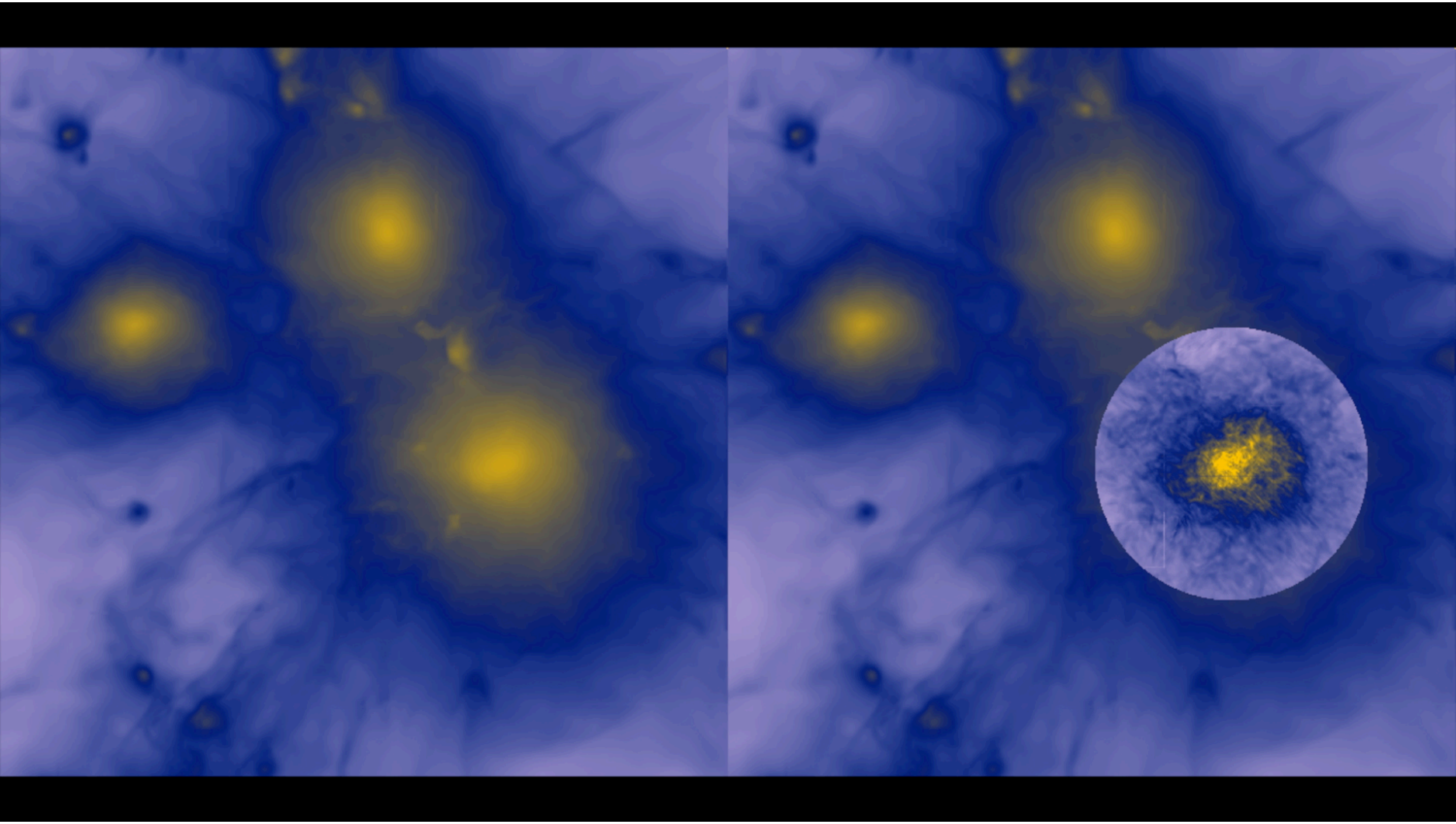
AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO



Example of a small-scale dynamo in a simulated cluster of galaxies

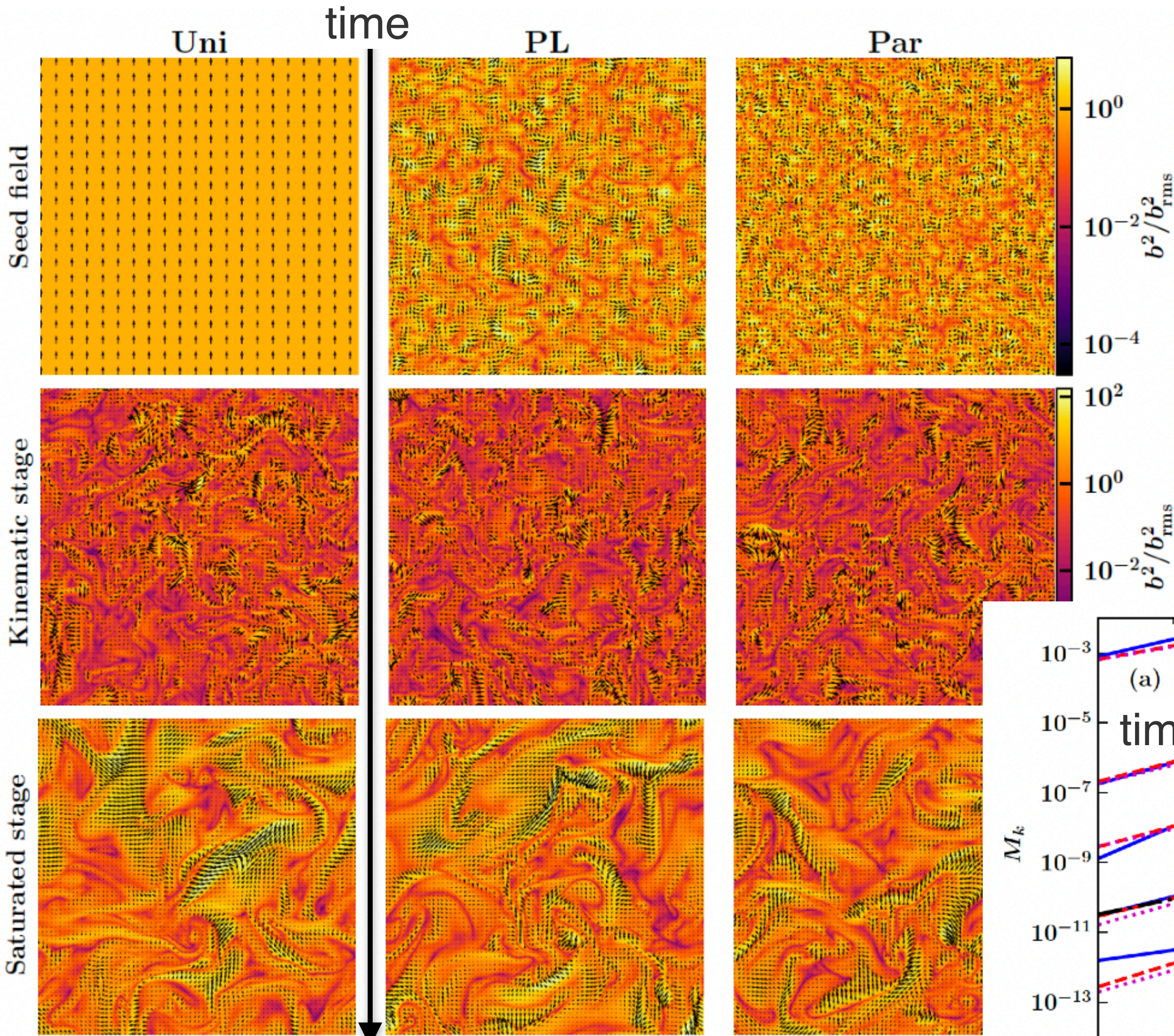
$(R_e \sim 100,\ Pr_M = 1)$

AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO



Example of a small-scale dynamo in a simulated cluster of galaxies
$(R_e \sim 300,\ Pr_M = 1)$

AMPLIFICATION OF COSMIC MAGNETIC FIELDS : SMALL SCALE DYNAMO



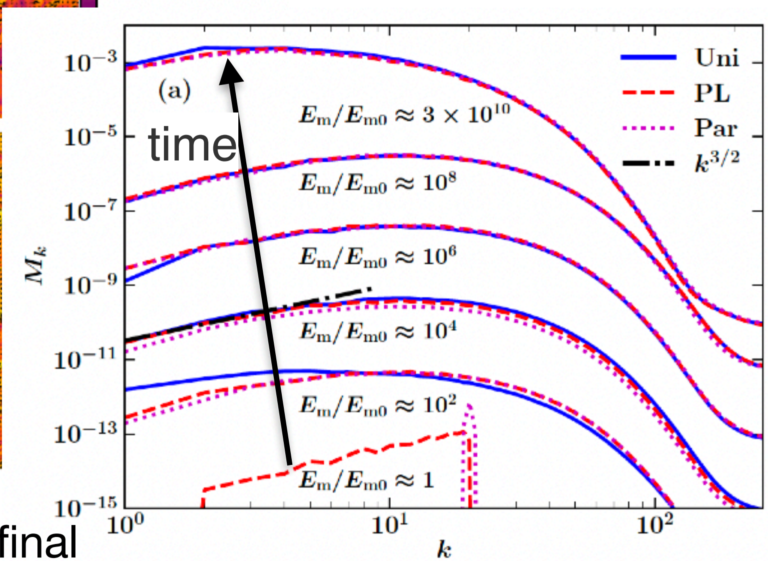
Simulated dynamo “in a box”,

$R_e \sim 3 \cdot 10^2, Pr_M = 2$

ICM-like, starting from 3 different seed fields:

Uniform

PL and Par : two different stochastic B models



Small-scale dynamo evolves all fields to the same final spectra, any memory of seed fields is lost!

Seta & Federrath 2020

HOW TO SIMULATE THE EVOLUTION OF MAGNETIC FIELDS IN COSMOLOGY?

- continuity $$\frac{\partial \rho}{\partial t} + \frac{1}{a}\nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$
- momentum conservation $$\frac{\partial \rho \mathbf{v}}{\partial t} + \frac{1}{a}\nabla \cdot \left(\rho \mathbf{v}\mathbf{v} + \mathbf{I}p^* - \frac{\mathbf{B}\mathbf{B}}{a}\right) = -\frac{\dot{a}}{a}\rho \mathbf{v} - \frac{1}{a}\rho \nabla \phi, \quad (2)$$
- energy conservation $$\frac{\partial E}{\partial t} + \frac{1}{a}\nabla \cdot \left[(E + p^*)\mathbf{v} - \frac{1}{a}\mathbf{B}(\mathbf{B} \cdot \mathbf{v})\right] = -\frac{\dot{a}}{a}\left(2E - \frac{B^2}{2a}\right) - \frac{\rho}{a}\mathbf{v} \cdot \nabla \phi - \Lambda + \Gamma + \frac{1}{a^2}\nabla \cdot \mathbf{F}_{\rm cond}, \quad (3)$$
- magnetic induction $$\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{a}\nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \quad (4)$$

(ideal MHD: single fluid, no resistivity, large conduction)

usually the comoving B is the one evolved: $B = B_{phys} a^2 = B_{phys}/(1+z)^2$

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

- magnetic induction

$$\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{a}\nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \qquad (4)$$

the difficult part of any MHD simulation is to conserve the flux of B, i.e. $\nabla \cdot \mathbf{B} = \mathbf{0}$ no magnetic monopoles!

If magnetic monopoles are forming due to numerical truncation errors, the induction equation doesn't remove them.
Monopoles can accumulate and produce a spurious force parallel to the field lines. div B can grow without bounds (numerical instability).

For long time integration, this lead to inconsistent results and quite often to code crashes or, worse, incorrect solutions.

The goal of computational MHD is to design div B preserving schemes.

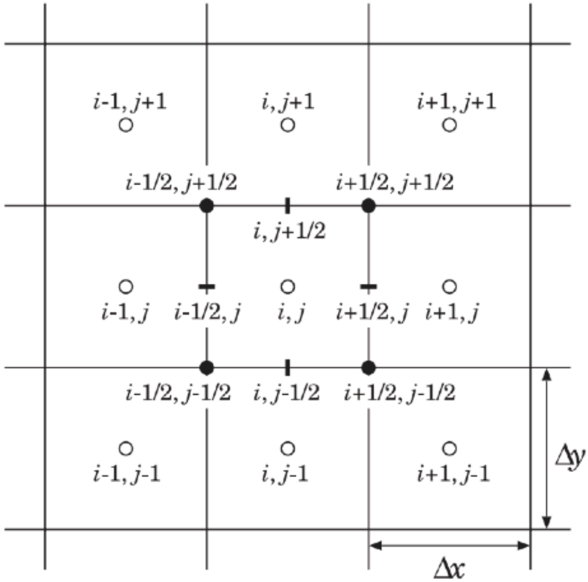
DIV B CLEANING METHODS

As a natural extension of finite-volume schemes to MHD equations, the B-field is defined at the centre of cells/voxels:

$$\overrightarrow{B}_{ikj} = \frac{1}{V}\int_V \overrightarrow{B}(x,y,z)dxdydz$$ with

$$V = [x_{i-1}, x_{i+1}] \times [y_{i-1}, y_{i+1}] \times [z_{i-1}, z_{i+1}]$$

Due to discretisation errors, this representation of $\overrightarrow{B}$ does ensure that $\nabla \cdot \overrightarrow{B} = 0$ and as iteration proceeds, this can lead to the growth of spurious magnetic fields.



The logic of "div B cleaning" schemes was first tested by Brackbill & Barnes (1980) with the projection method:

- compute numerical monopoles as $m_{ij} = (B^{n+1/2}_{x,i+1/2,j} - B^{n+1/2}_{x,i-1/2,j})/\Delta x + (B^{n+1/2}_{y,i,j+1/2} - B^{n+1/2}_{y,i,j-1/2})/\Delta y$

  (example for 2D case)

- solve for the potential with the poisson equations $\Delta\Phi = m$

- correct the cell-centred magnetic field with this $\overrightarrow{B}_{clean} = \overrightarrow{B}^{n+1/2} - \nabla\Phi$

DIV B CLEANING METHODS

Problem with projection method: the Poisson equation is non-local, solving it is time consuming, and the correction to B can lead to large errors in gas pressure when the code the solver is iterated .

The "**Hyperbolic Dedner cleaning method**" (Dedner 2002) introduces an additional scalar field, ψ, with its evolution equation, to be coupled with the induction equation :

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{VB} - \boldsymbol{BV} + \psi \boldsymbol{I}) = 0,$$

$$\frac{\partial \psi}{\partial t} + c_{\mathrm{h}}^2 \nabla \cdot \boldsymbol{B} = -\frac{c_{\mathrm{h}}^2}{c_{\mathrm{p}}^2} \psi,$$

With solution:

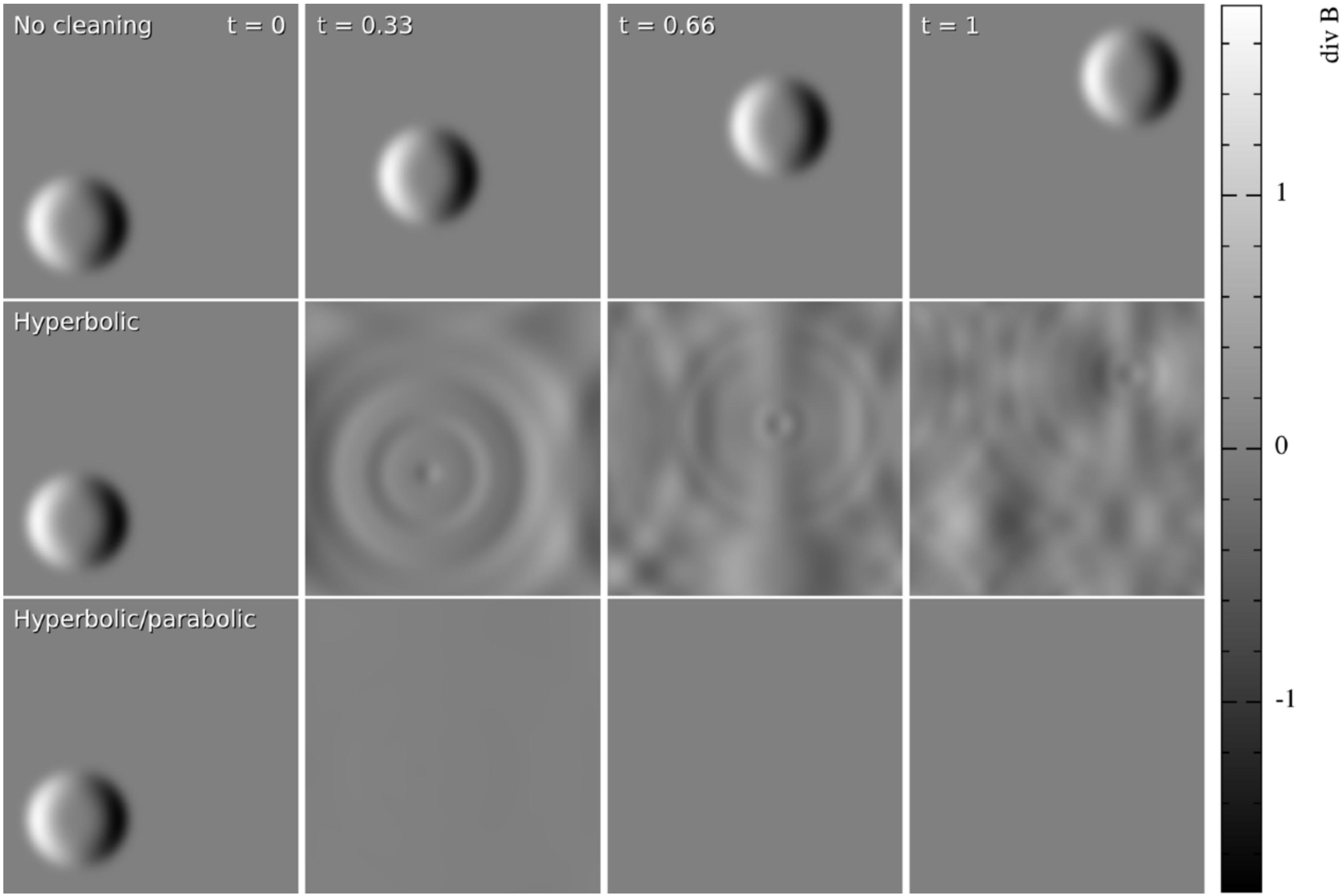
$$\frac{\partial^2 (\nabla \cdot \boldsymbol{B})}{\partial t^2} + \frac{c_{\mathrm{h}}^2}{c_{\mathrm{p}}^2} \frac{\partial (\nabla \cdot \boldsymbol{B})}{\partial t} - c_{\mathrm{h}}^2 \nabla^2 (\nabla \cdot \boldsymbol{B}) = 0,$$ "telegraph equation"

This means that the numerical divergence of the magnetic field is not only **advected outward** with c_h speed, but also **diffused** with a diffusivity c_p^2 .

Downside: the Dedner method is "diffusive" as it diffuse information away from where divB is produced.

DIV B CLEANING METHODS

simple test with advection of a magnetic loop



Tricco et al. 2016

DIV B CLEANING METHODS

Another divB cleaning approach is the **"8-waves cleaning method"** by Powell (1994) which aims to subtracts the unstable $\nabla \cdot \mathbf{B}$ terms from the equation of motion.

This requires a source term proportional to $\nabla \cdot \mathbf{B}$

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{V}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{V}) = -\boldsymbol{V}(\nabla \cdot \boldsymbol{B})$$

$$\frac{\partial (\nabla \cdot \boldsymbol{B})}{\partial t} + \nabla \cdot [\boldsymbol{V} (\nabla \cdot \boldsymbol{B})] = 0.$$

…however, the subtraction necessarily violates momentum conservation, so one would like to minimize the subtracted terms.

Moreover, many studies have shown that certain types of problems, treated only with this method, will converge to the wrong solution

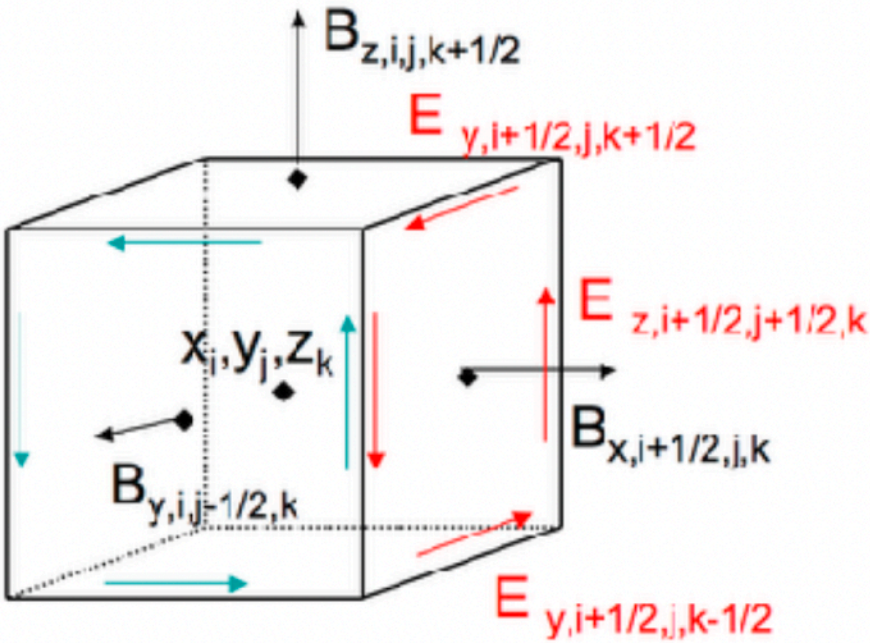
THE CONSTRAINED TRANSPORT METHOD

The induction equation in integral form suggests a surface-average form:

$$\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times \mathbf{u}) = 0 \qquad \text{(Stokes theorem)} \quad \partial_t \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{s} + \int_L (\mathbf{B} \times \mathbf{u}) \cdot \mathrm{d}\mathbf{l} = 0$$

The magnetic field is face-centred while Euler-type variables are cell-centred (staggered mesh approach).

$$(B_x)_{i+1/2,jk} = \frac{1}{S} \int_S B_x(y,z)\mathrm{d}y\mathrm{d}z \qquad S = [y_{i-1/2}, y_{i+1/2}] \times [z_{i-1/2}, z_{i+1/2}]$$



Similar to potential vector methods (Yee 1966; Dorfi 1986; Evans & Hawley 1988).

slide courtesy of R. Teyssier

THE CONSTRAINED TRANSPORT METHOD

Surface-averaged magnetic fields are updated conservatively:

$$B^{n+1}_{z,i,j,k-1/2} = B^{n}_{x,i,j,k-1/2} + \frac{\Delta t}{\Delta x}\left(E^{n+1/2}_{y,i+1/2,j,k-1/2} - E^{n+1/2}_{y,i-1/2,j,k-1/2}\right) - \frac{\Delta t}{\Delta y}\left(E^{n+1/2}_{x,i,j+1.2,k-1/2} - E^{n+1/2}_{x,i,j-1/2,k-1/2}\right)$$

$$B^{n+1}_{y,i,j-1/2,k} = B^{n}_{y,i,j-1/2,k} + \frac{\Delta t}{\Delta z}\left(E^{n+1/2}_{x,i,j-1/2,k+1/2} - E^{n+1/2}_{x,i,j-1/2,k-1/2}\right) - \frac{\Delta t}{\Delta x}\left(E^{n+1/2}_{z,i+1/2,j-1/2,k} - E^{n+1/2}_{z,i-1/2,j-1/2,k}\right)$$

$$B^{n+1}_{x,i-1/2,j,k} = B^{n}_{x,i-1/2,j,k} + \frac{\Delta t}{\Delta y}\left(E^{n+1/2}_{z,i-1/2,j+1/2,k} - E^{n+1/2}_{z,i-1/2,j-1/2,k}\right) - \frac{\Delta t}{\Delta z}\left(E^{n+1/2}_{y,i-1/2,j,k+1/2} - E^{n+1/2}_{y,i-1/2,j,k-1/2}\right)$$

using time-averaged electric fields defined at cell edge center:

$$E^{n+1/2}_{x,i,j-1/2,k-1/2} = \frac{1}{\Delta t \Delta x}\int_{t^n}^{t^{n+1}}\int_{x_{i-1/2}}^{x_{i+1/2}} E_x(x, y_{j-1/2}, z_{k-1/2})\,\mathrm{d}t\,\mathrm{d}x$$

$$E^{n+1/2}_{y,i-1/2,j,k-1/2} = \frac{1}{\Delta t \Delta y}\int_{t^n}^{t^{n+1}}\int_{y_{j-1/2}}^{y_{j+1/2}} E_y(x_{i-1/2}, y, z_{k-1/2})\,\mathrm{d}t\,\mathrm{d}y$$

$$E^{n+1/2}_{z,i-1/2,j-1/2,k} = \frac{1}{\Delta t \Delta z}\int_{t^n}^{t^{n+1}}\int_{z_{k-1/2}}^{z_{k+1/2}} E_z(x_{i-1/2}, y_{j-1/2}, z)\,\mathrm{d}t\,\mathrm{d}z$$

The total flux (div B) across each cell bounding surface vanishes exactly !

But how do we compute the electric field on cell edges ?

slide courtesy of R. Teyssier

THE CONSTRAINED TRANSPORT METHOD

We write Faraday's law $\partial_t \mathbf{B} = \nabla \times \mathbf{E}$ using now the EMF vector $\mathbf{E} = \mathbf{u} \times \mathbf{B}$

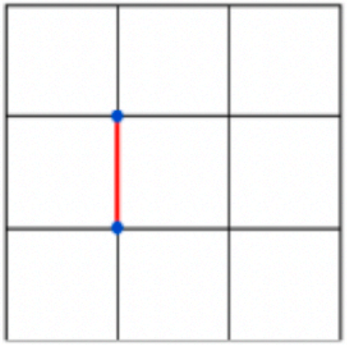
We use a finite-surface approximation for the magnetic field

$$B^n_{x,i+1/2,j} = \frac{1}{\Delta y}\int_{y_{j-1/2}}^{y_{j+1/2}} B_x(x_{i+1/2}, y)\mathrm{d}y \qquad B^n_{y,i,j+1/2} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} B_y(x, y_{j+1/2})\mathrm{d}x$$

Integral form of the induction equation using Stoke's theorem

$$B^{n+1}_{x,i+1/2,j} = B^n_{x,i+1/2,j} + \frac{\Delta t}{\Delta y}\left(E^{n+1/2}_{z,i+1/2,j+1/2} - E^{n+1/2}_{z,i-1/2,j+1/2}\right)$$

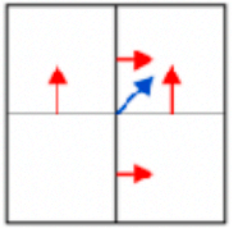
$$B^{n+1}_{y,i,j+1/2} = B^n_{y,i,j+1/2} + \frac{\Delta t}{\Delta x}\left(E^{n+1/2}_{z,i+1/2,j+1/2} - E^{n+1/2}_{z,i+1/2,j-1/2}\right)$$



By construction, div B vanishes exactly:

$$\frac{B^n_{x,i+1/2,j} - B^n_{x,i-1/2,j}}{\Delta x} + \frac{B^n_{x,i,j+1/2} - B^n_{x,i,j-1/2}}{\Delta y} = 0$$

For piece-wise initial constant data, the flux function is self-similar at corner points.



Induction Riemann problem

For pure induction, the exact Riemann solution is:

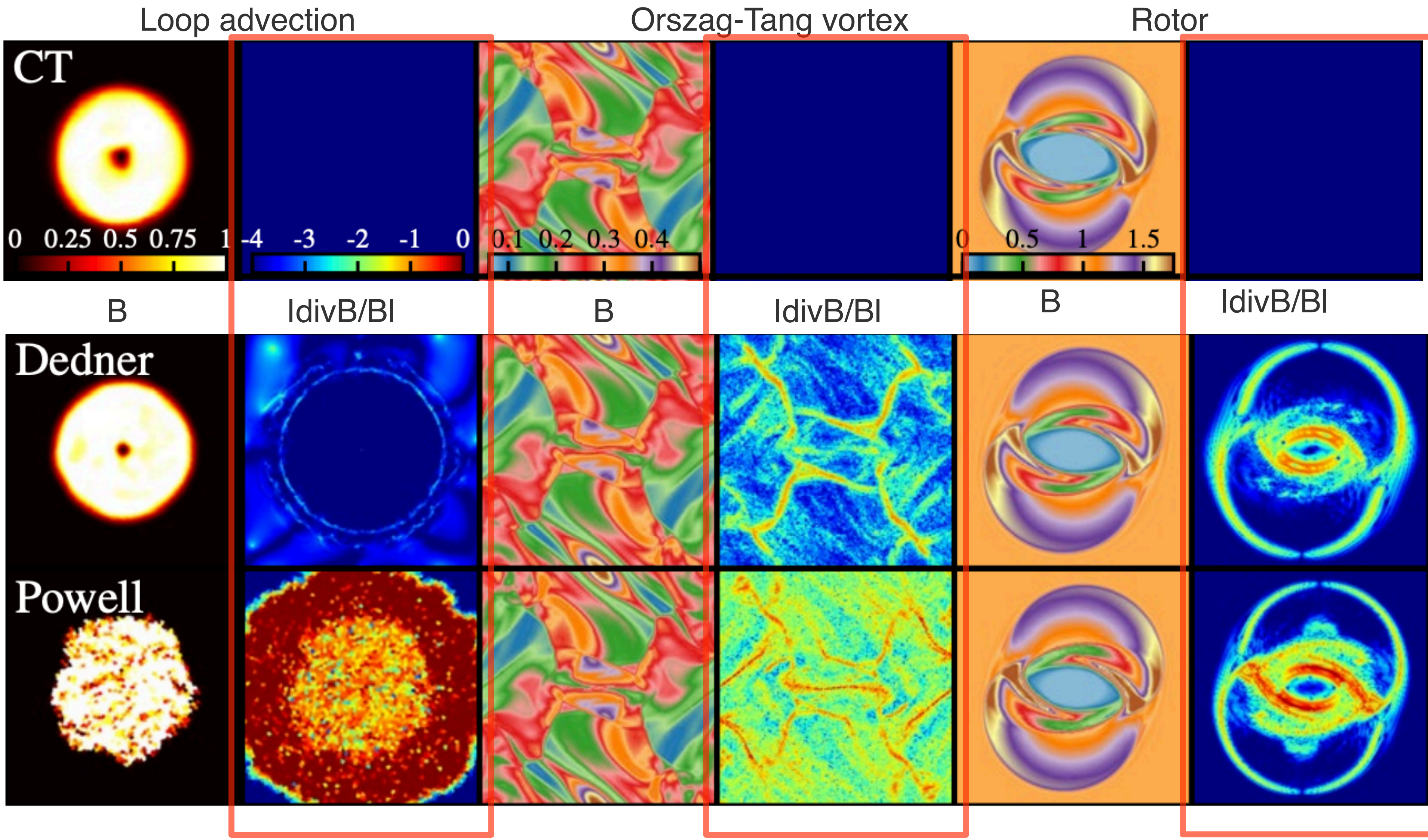
$$E^{n+1/2}_{z,i+1/2,j+1/2} = u\frac{B^n_{y,i+1,j+1/2} + B^n_{y,i,j+1/2}}{2} - v\frac{B^n_{x,i+1/2,j+1} + B^n_{y,i+1/2,j}}{2}$$

$$- |u|\frac{B^n_{y,i+1,j+1/2} - B^n_{y,i,j+1/2}}{2} + |v|\frac{B^n_{x,i+1/2,j+1} - B^n_{y,i+1/2,j}}{2}$$

The CT methods preserves $\nabla \cdot \mathbf{B} = \mathbf{0}$ to machine precision!

slide courtesy of R. Teyssier

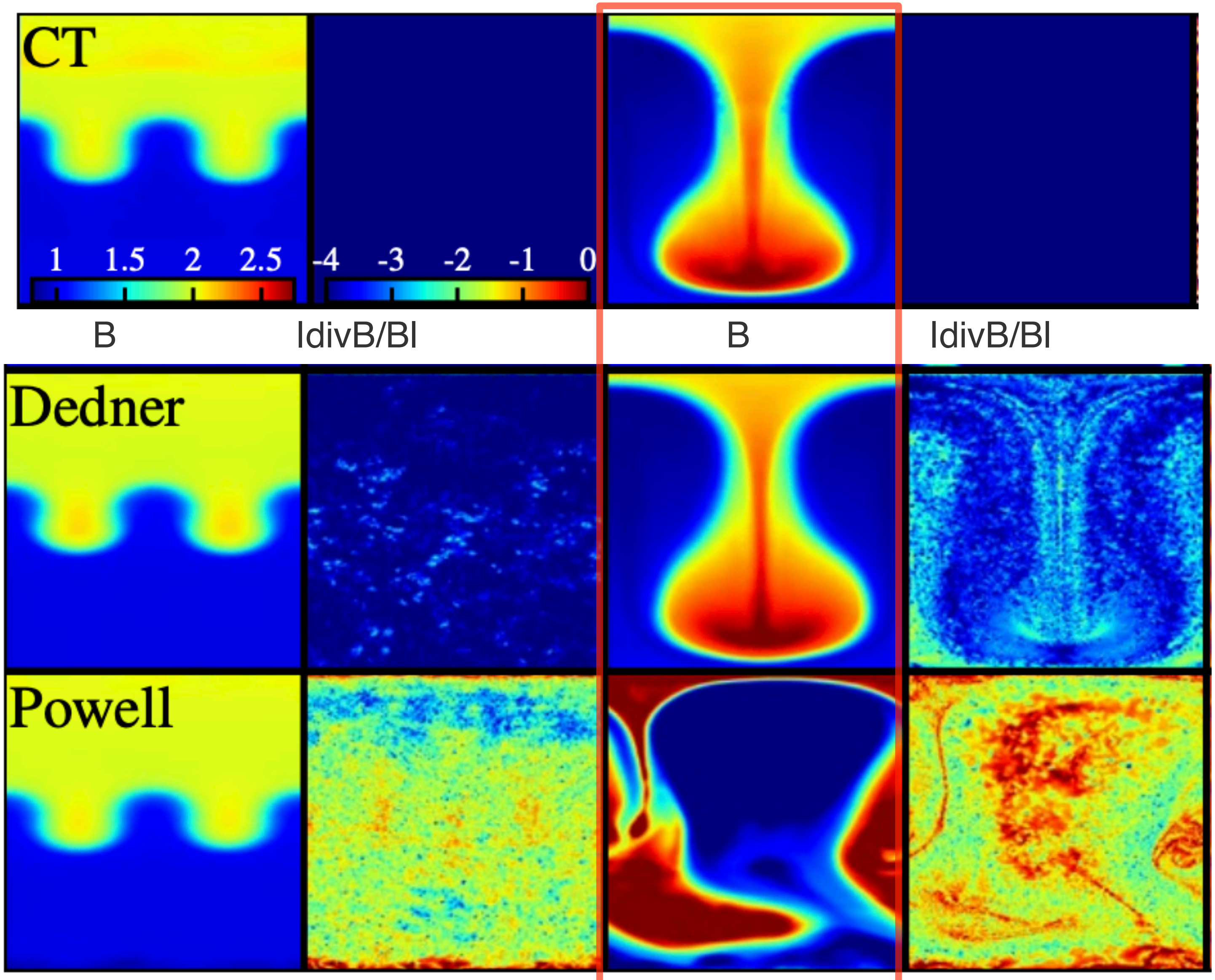
COMPARISON OF METHODS WITH TESTS



Hopkins 2016

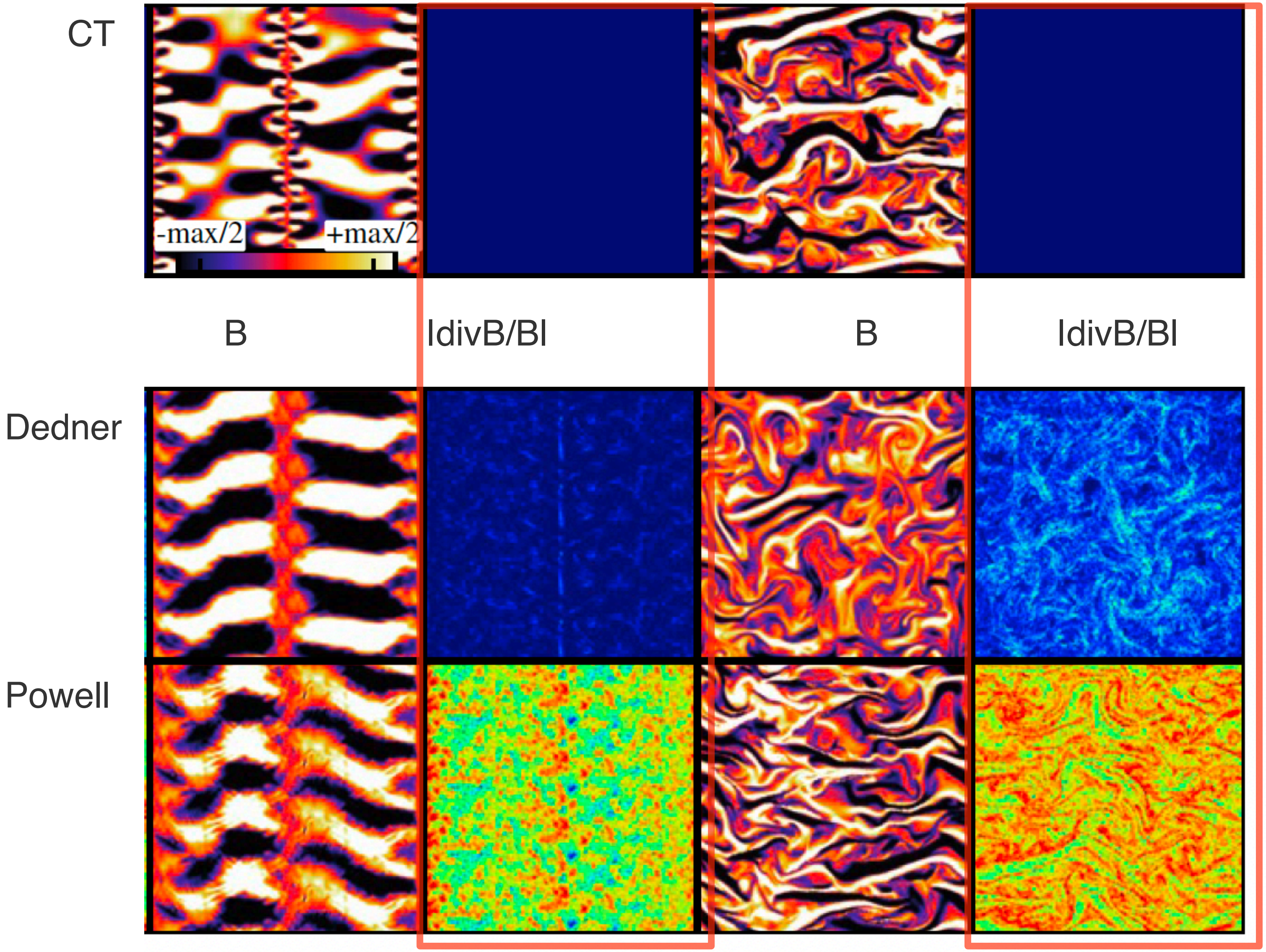
COMPARISON OF METHODS WITH TESTS

Two different time steps of the Taylor Instabilty: **Powell method generates a wrong solution**



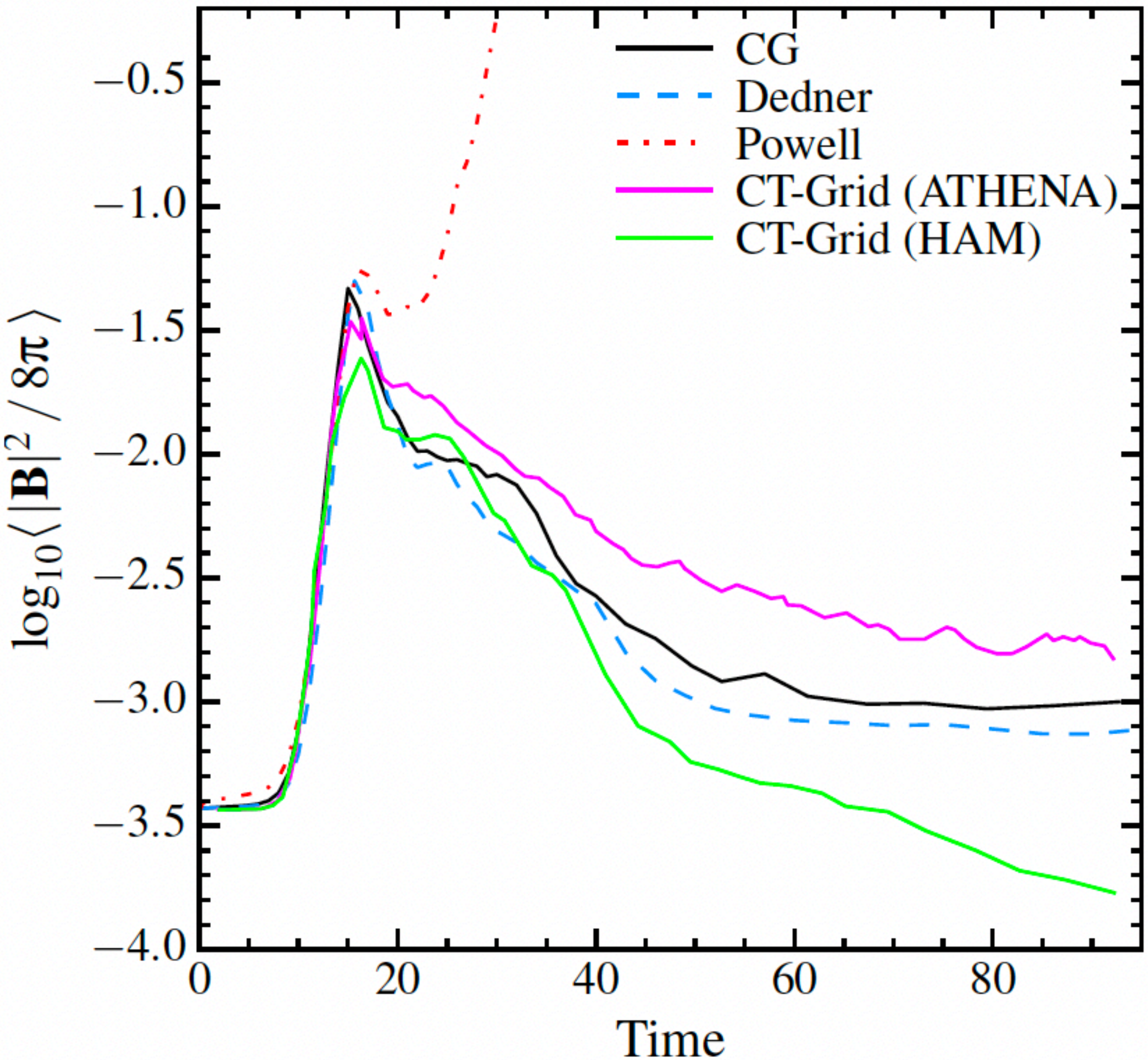
COMPARISON OF METHODS WITH TESTS

Two different time steps of the MRI Instabilty: **Powell method amplifies B a lot**



COMPARISON OF METHODS WITH TESTS

MRI Instabilty: **Powell method spuriously amplifies B a lot**



Hopkins 2016

WHICH METHOD IS THE BEST?

Lagrangian: Moving volume element

- Smears out shocks and discontinuities
- Hard to implement $\nabla \cdot \mathbf{B} = 0$
- Naturally Galilean-invariant

Eulerian: Static volume element

- Riemann solvers are great for capturing shocks!
- Easy to implement $\nabla \cdot \mathbf{B} = 0$
- Truncation errors depend on velocities

Hybrid (moving-mesh)

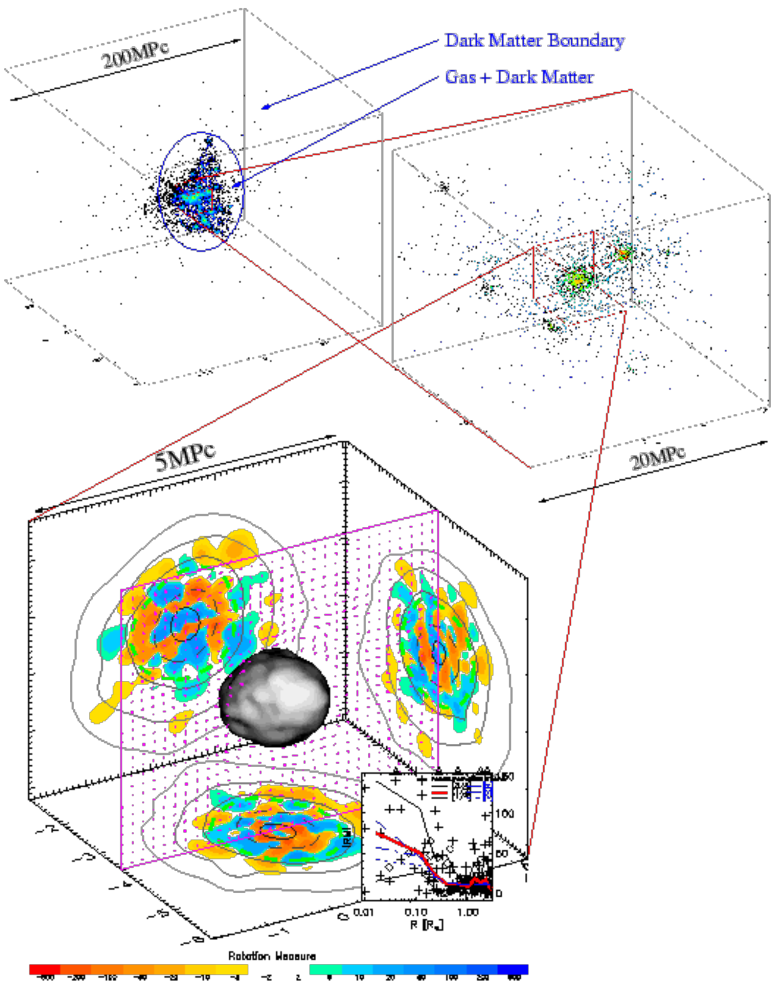
- Riemann solvers are great for capturing shocks!
- Hard to implement $\nabla \cdot \mathbf{B} = 0$
- Naturally Galilean-invariant

Choose a code according to the needs of your problem!

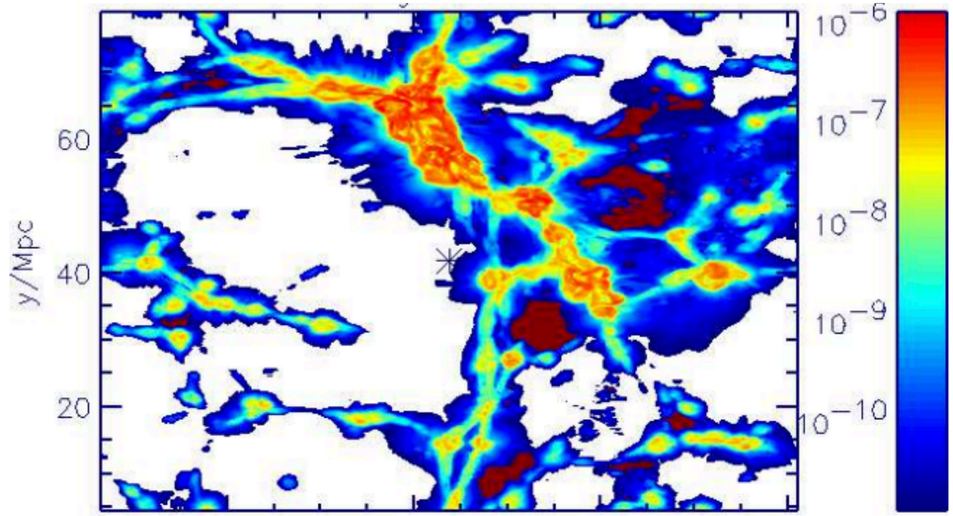
slide courtesy of E. Ntormousi

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

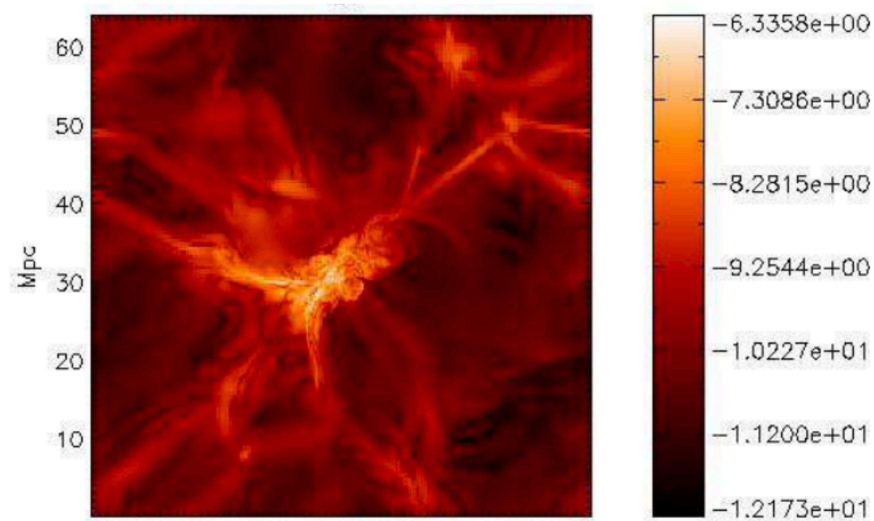
"Historical" simulations



SPH-MHD simulation with seed field 10^{-9}G (Dolag+1999)



non-MHD (passive $\mathbf{B}$) Eulerian simulation (Miniati+2002)



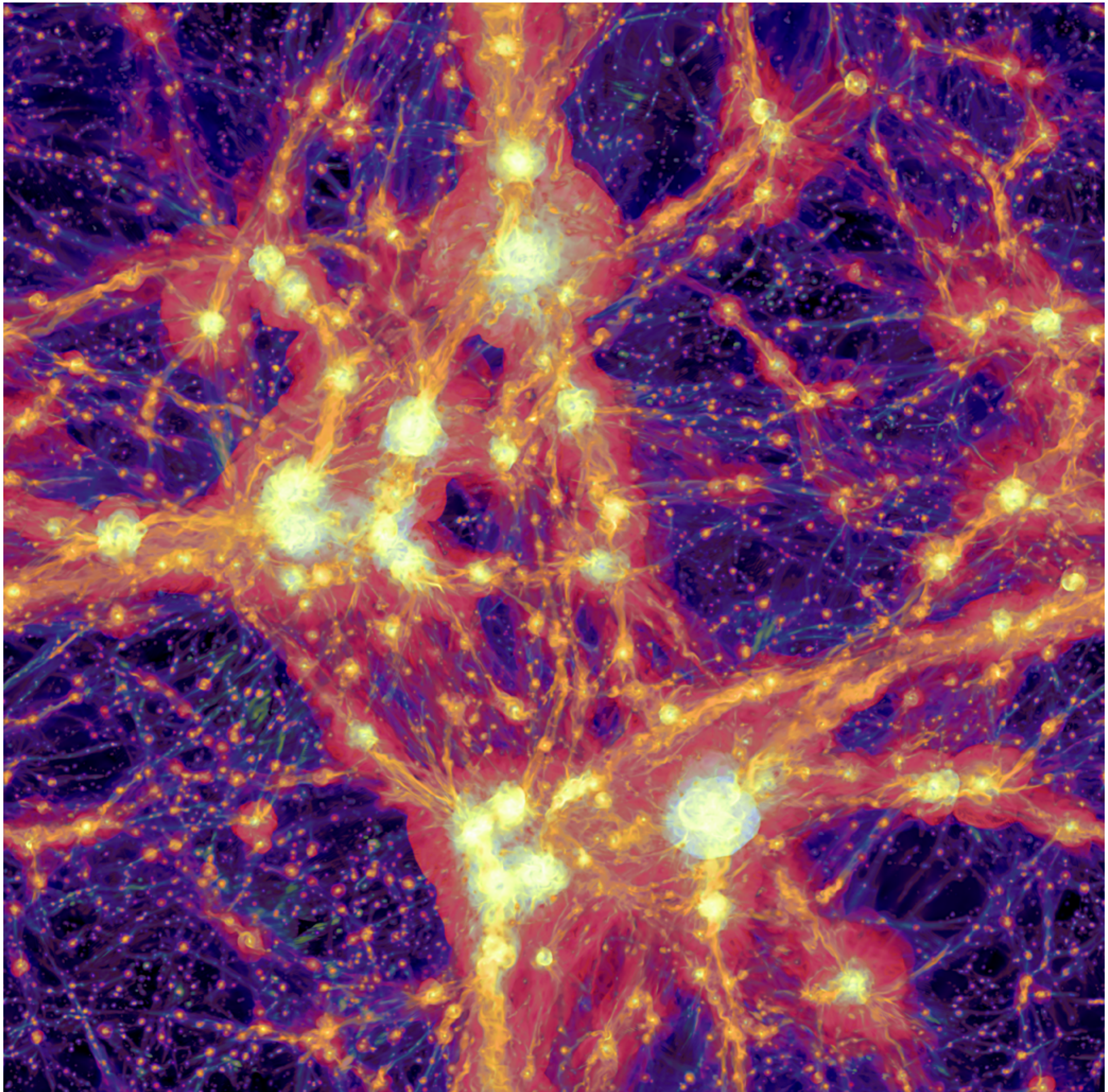
Eulerian-MHD simulation with seed field $3 \cdot 10^{-11}$G (Bruggen+2005)

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

temperature

B-field

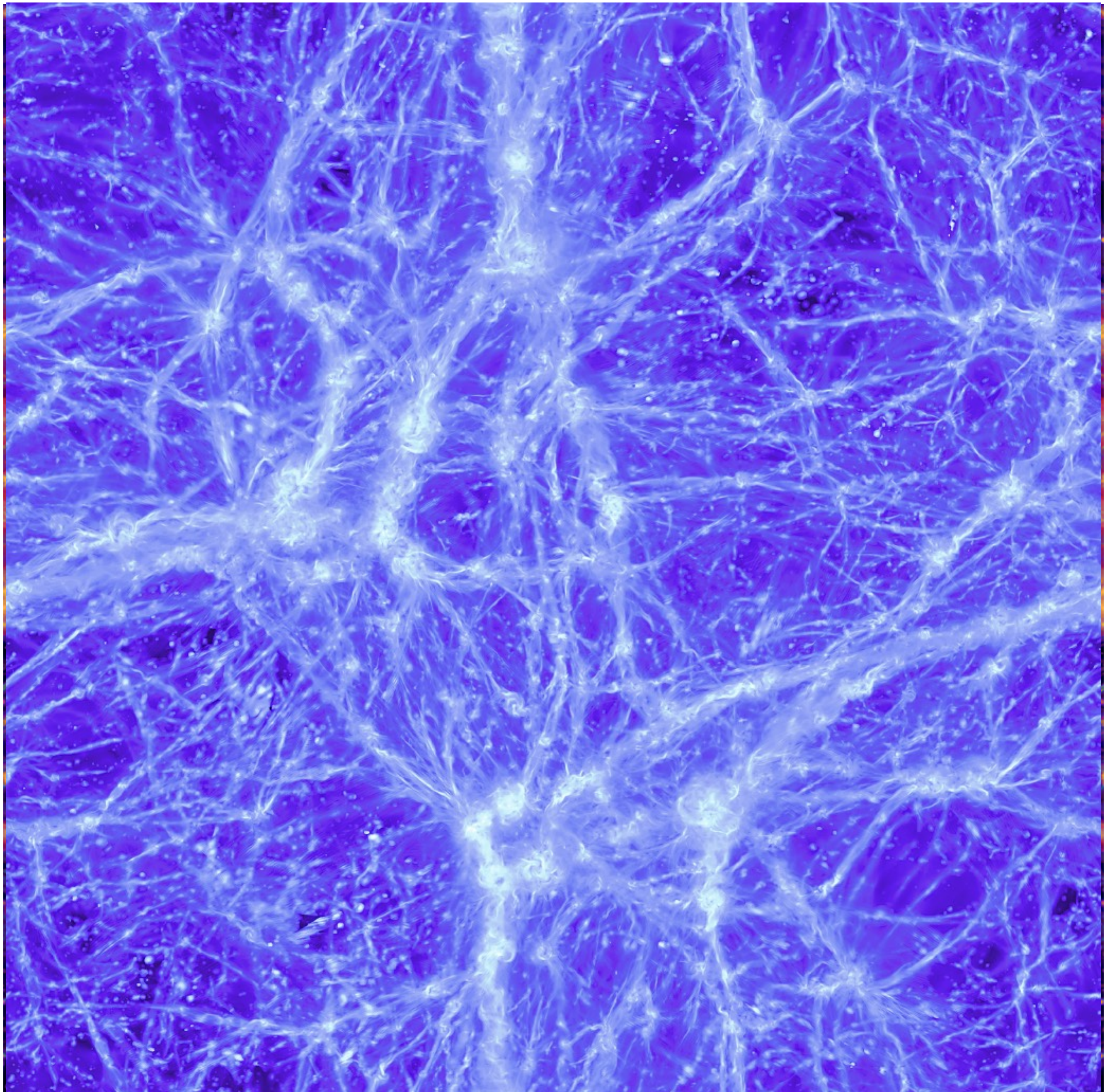
density



FV+15

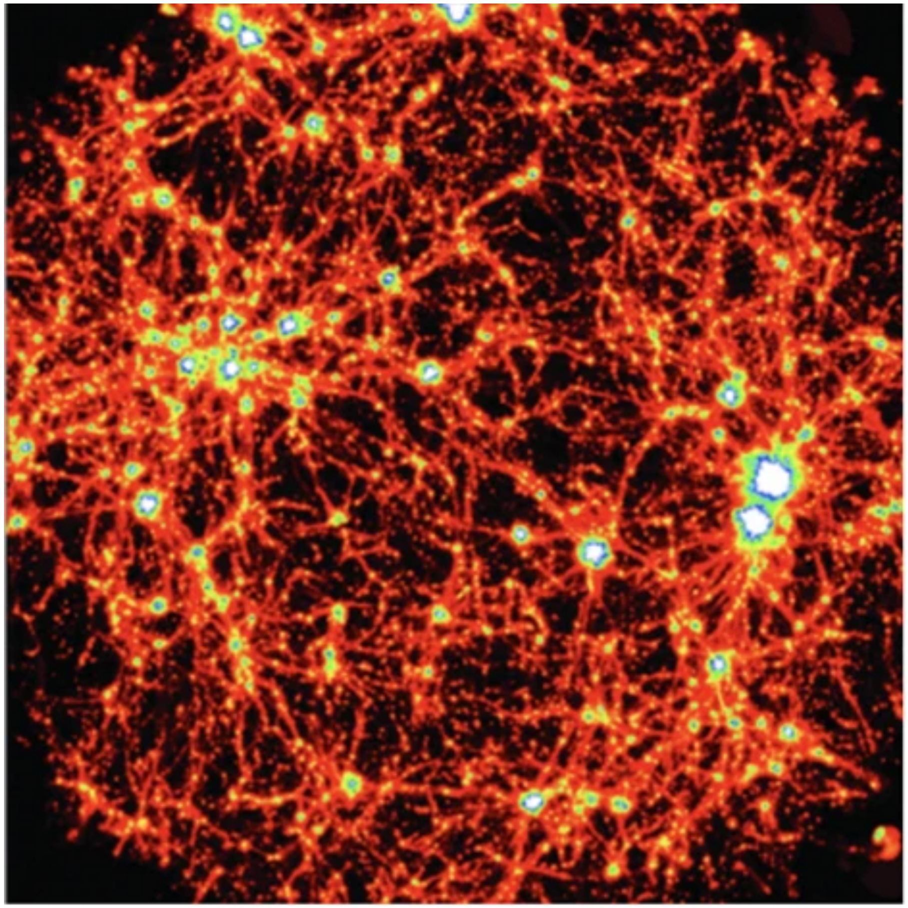
SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

B-field

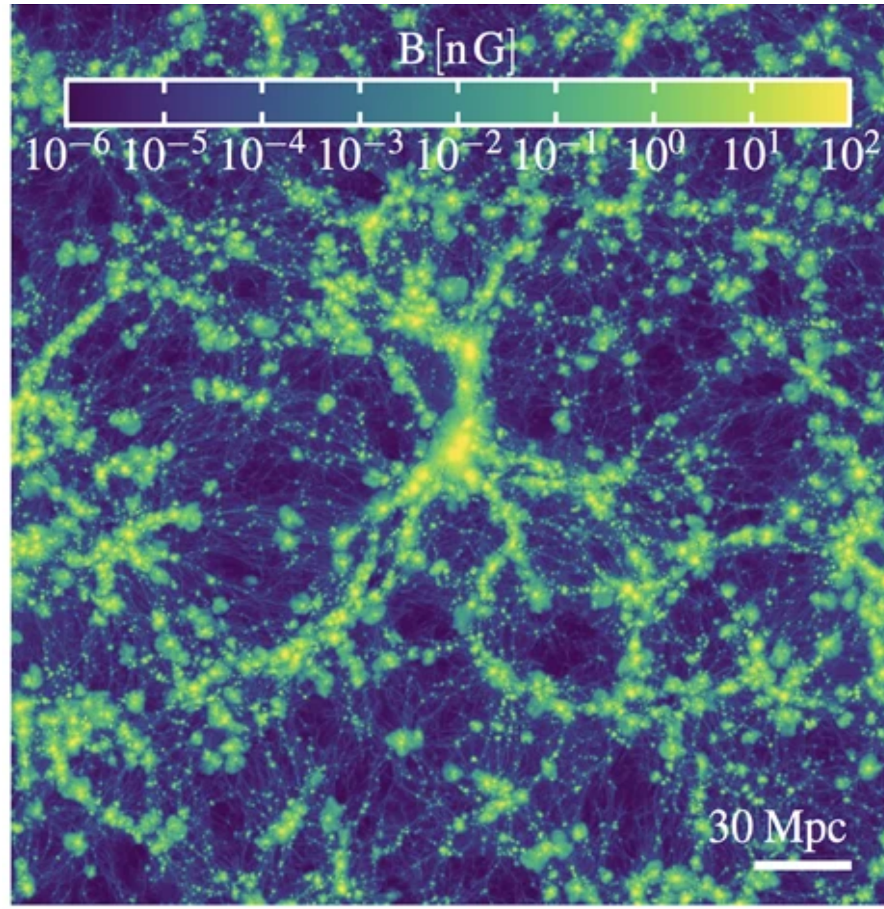


FV+15

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES



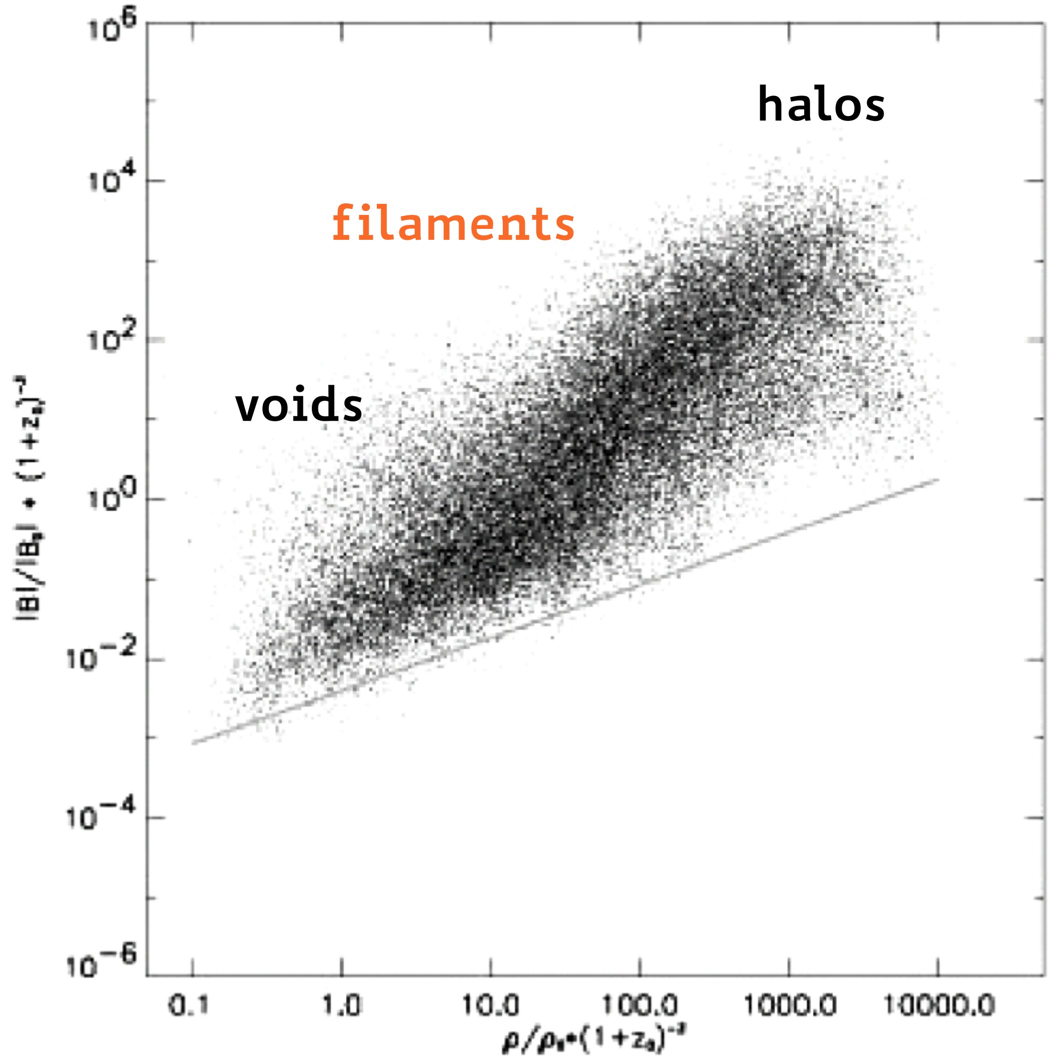
Donnert+09 GADGET-MHD



Marinacci+15 AREPO-MHD

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

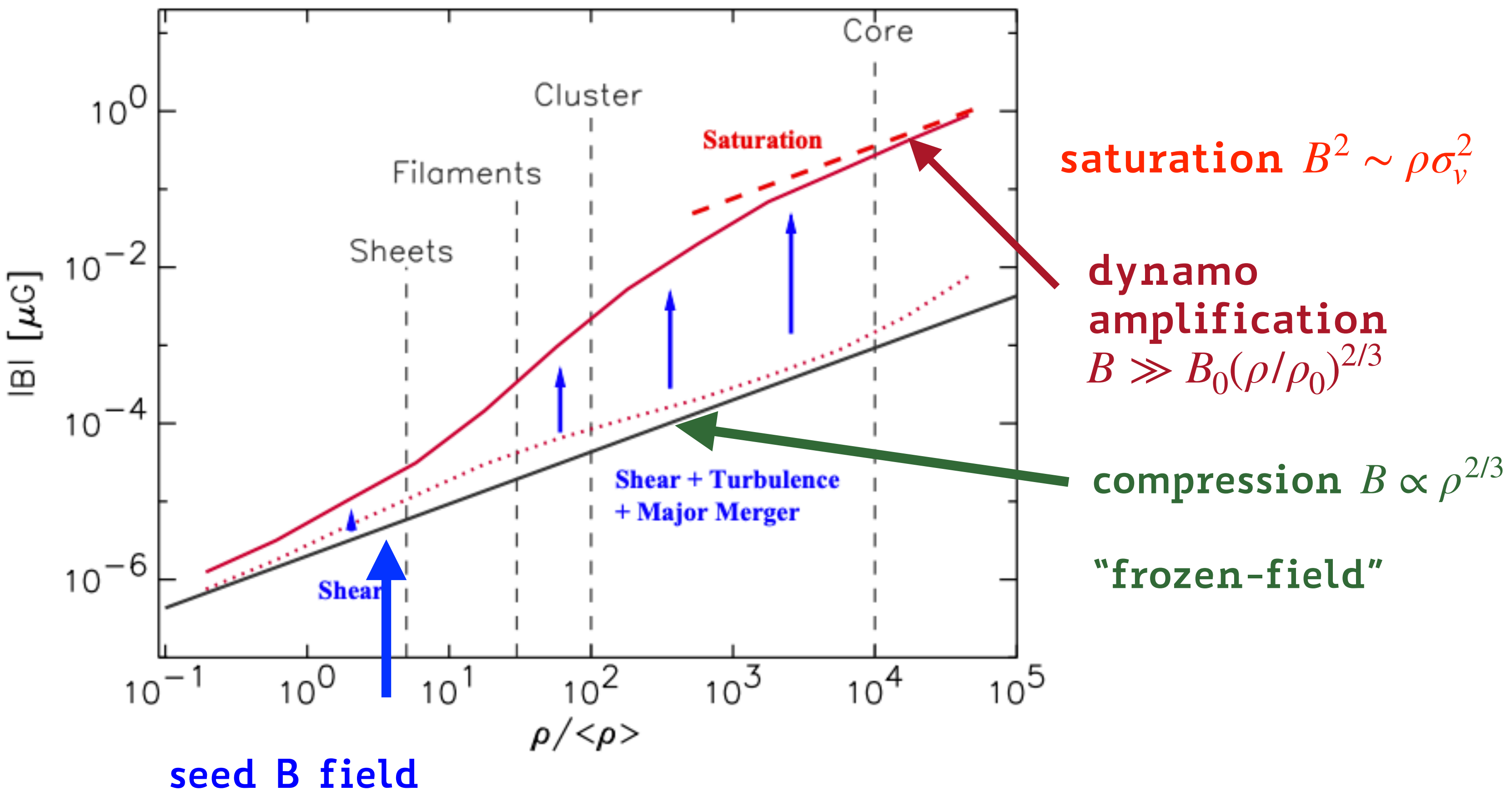
All MHD simulations report a $(|B|, \rho)$ phase diagram similar to the following picture:



Dolag+99
ag+06

SIMULATING MAGNETIC FIELDS IN COSMIC STRUCTURES

All MHD simulations report a $(|B|, \rho)$ phase diagram similar to the following picture:

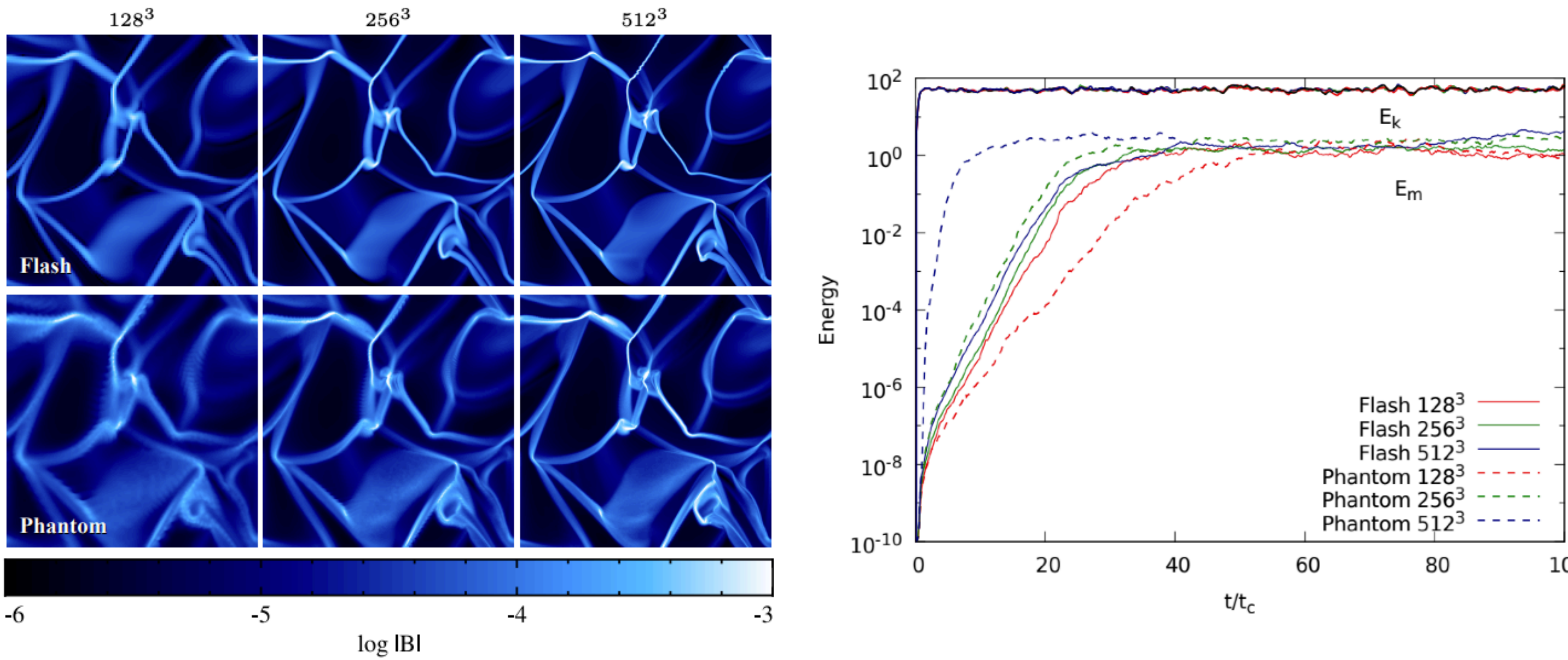


- most of the volume evolves by compression $B \propto B_0(\rho/\langle\rho\rangle)^{2/3}$, with B_0=seed field
- up-turn of the relation at densities > halos due to dynamo amplification

Dolag+06

COMPARISON AND PROBLEMS

Comparison projects for forces small-scale dynamo in a box, grid (FLASH) vs SPH (Phantom)



Quite consistent results in both codes (Phantom uses Dedner cleaning and Flash the Constrainted Transport). Agreement typically improves with resolution.

Tricco+2016

COMPARISON AND PROBLEMS

Growth of magnetic field in a spiral galaxy for the same moving mesh simulation (AREPO):

- for this setup, the moving CT leads to a slow amplification in final equipartition with turbulent kinetic energy
- The 8-waves Powell cleaning instead uncorrectly generates spurious large dynamo amplification, much higher than the kinetic energy

Mocz+2016

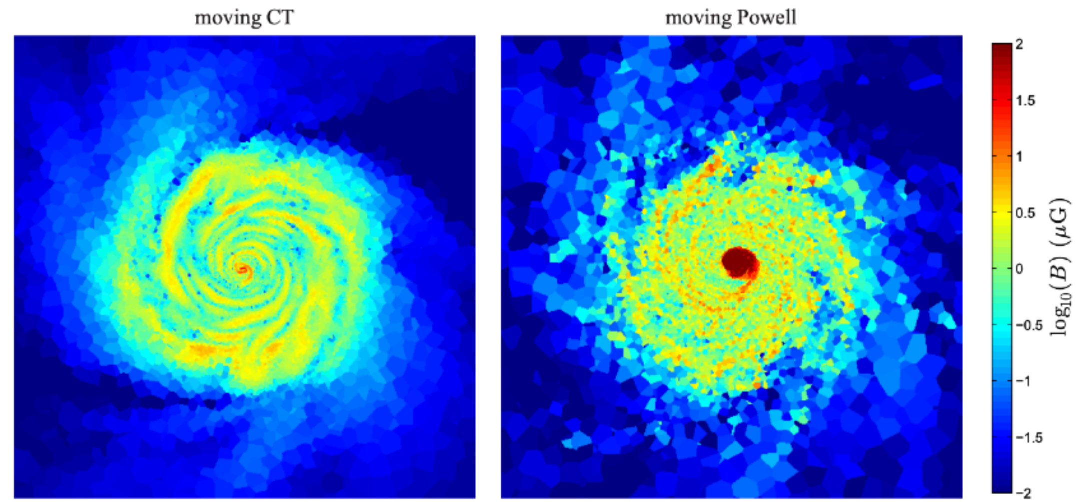


Figure 7. Comparison of the magnetic field strength of the same disc in Fig. 4 at time $t = 0.5$ Gyr in the formation process, simulated using the CT and Powell schemes. The figure displays a physical size of 40 kpc. The CT approach exhibits much better preservation of the topological winding of the magnetic field. The Powell scheme shows substantial divergence error noise seen on the cell level while this is absent to machine precision in CT.

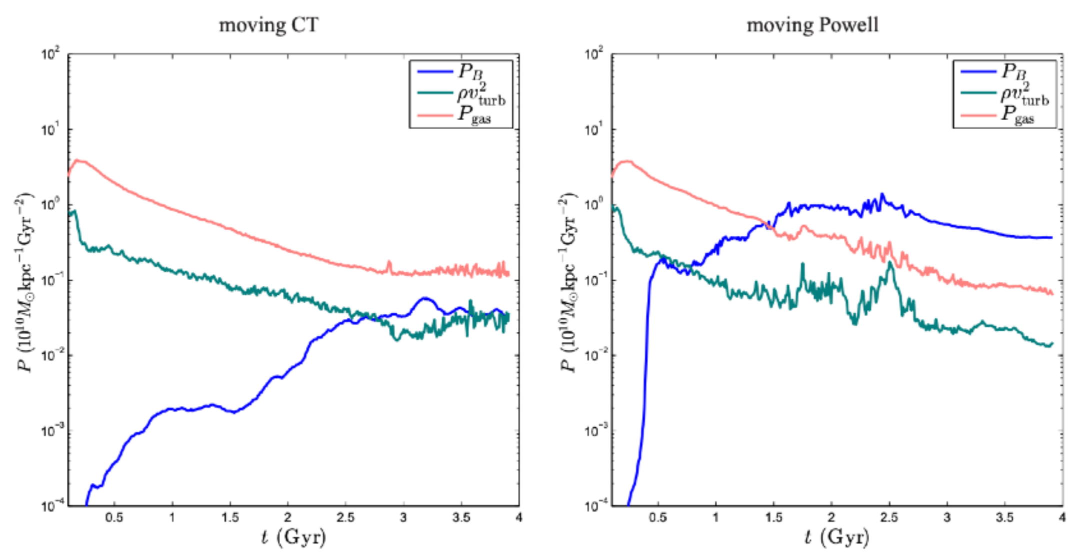
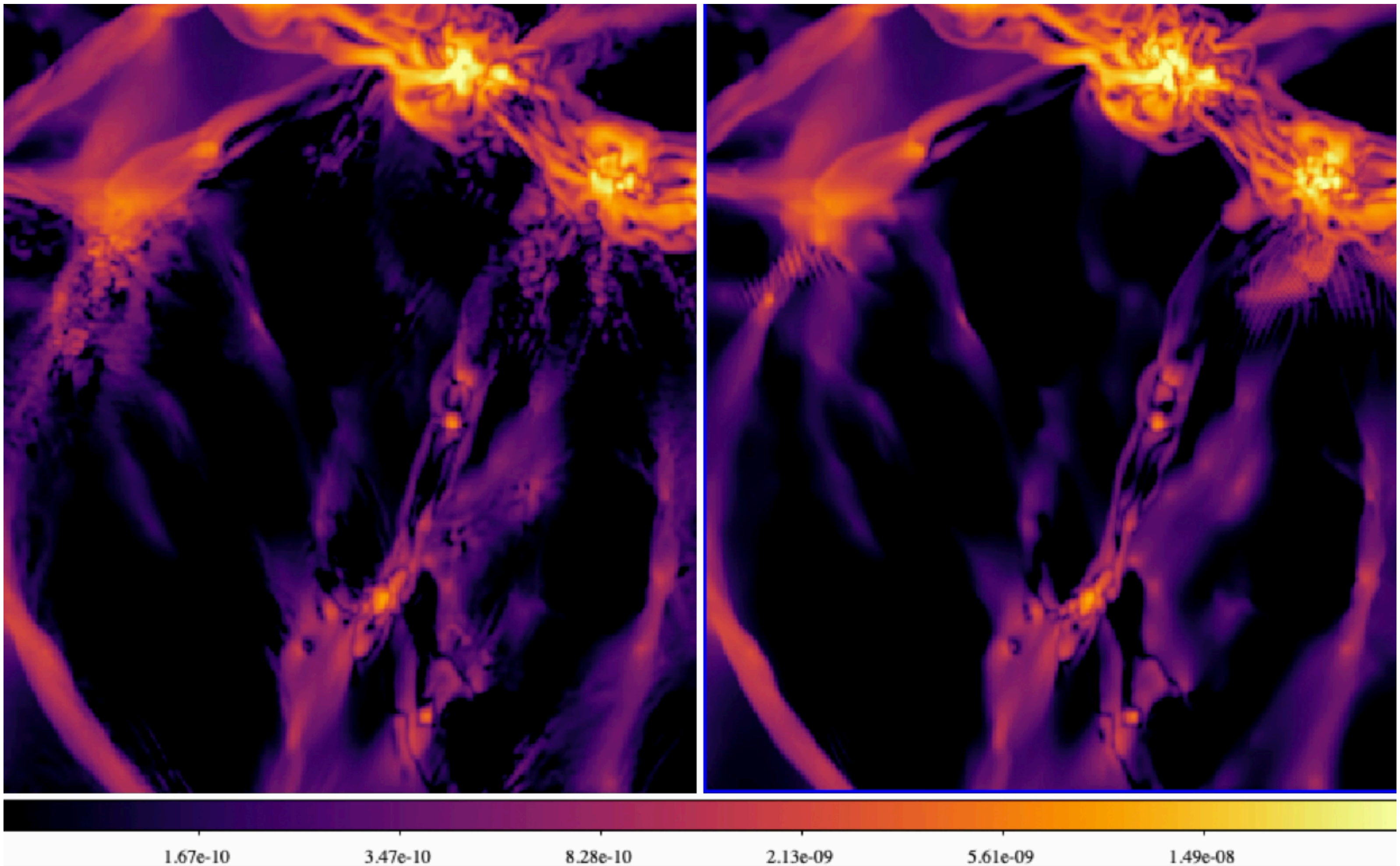


Figure 6. Comparison of magnetic field saturation in the formation of a disc simulated with the CT and Powell schemes. The CT method shows equipartition between magnetic energy density and turbulent kinetic energy density, whereas the Powell technique saturates the field at higher values, exceeding the thermal pressure by about a factor of five.

COMPARISON AND PROBLEMS

ENZO cosmological simulation (fixed grid) with the same B_0

Dedner cleaning vs Constrained transport

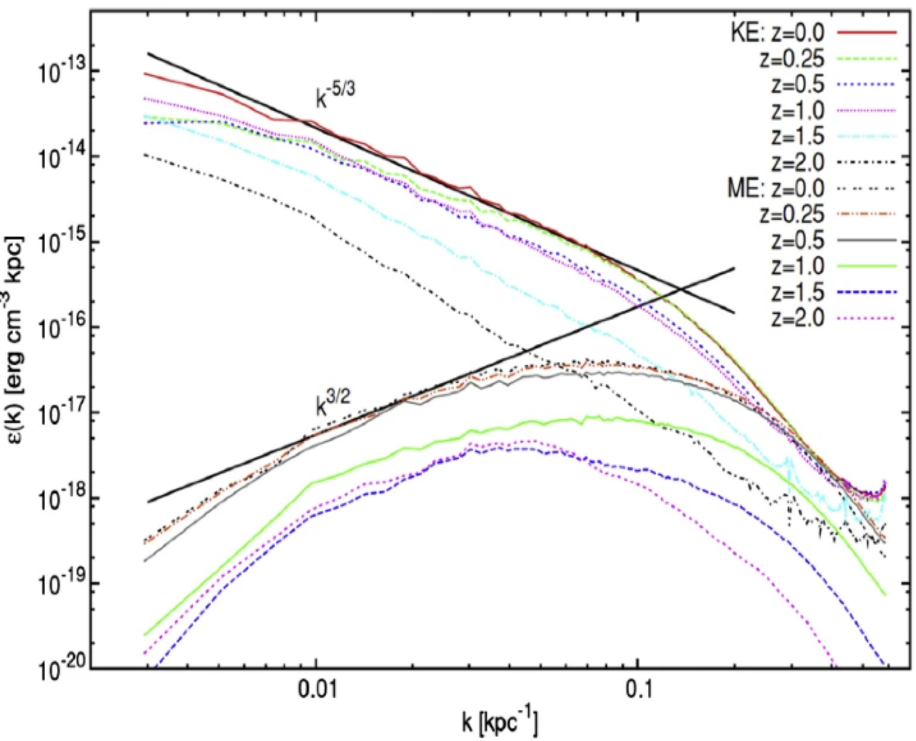


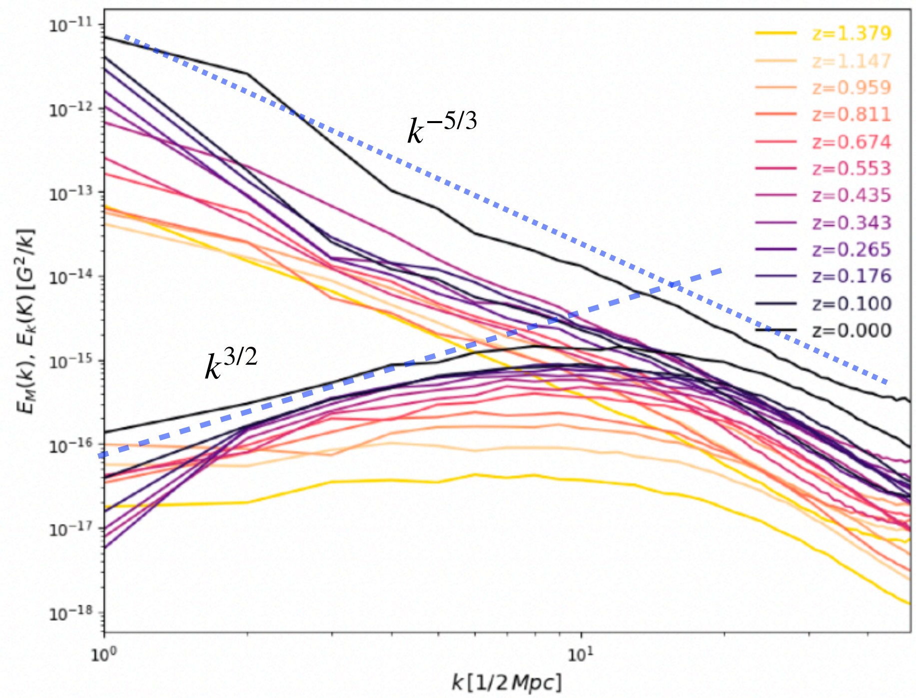
- approximately consistent results for the two solvers across the cosmic web , what about dynamo-dominated regions?

COMPARISON AND PROBLEMS

Turbulent and magnetic spectra in two (different) evolving simulated clusters obtained with Adaptive Mesh Refinement and using:

Constrained Transport (Xu et al. 2012) or **Dedner Cleaning** (Dominguez-Fernandez+2019)

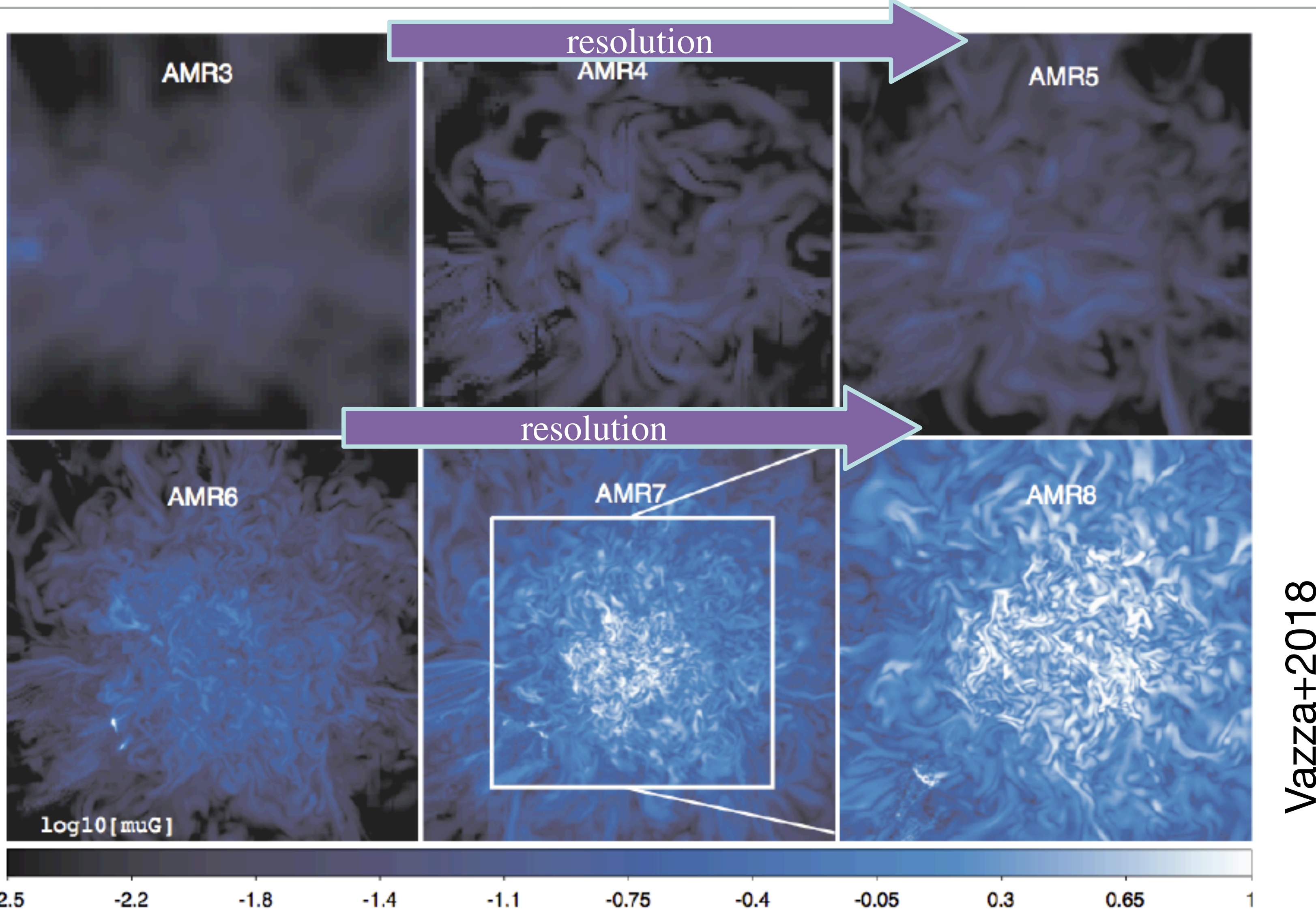




Similar level of amplification in the same timescales (notice: 2 different clusters here!)

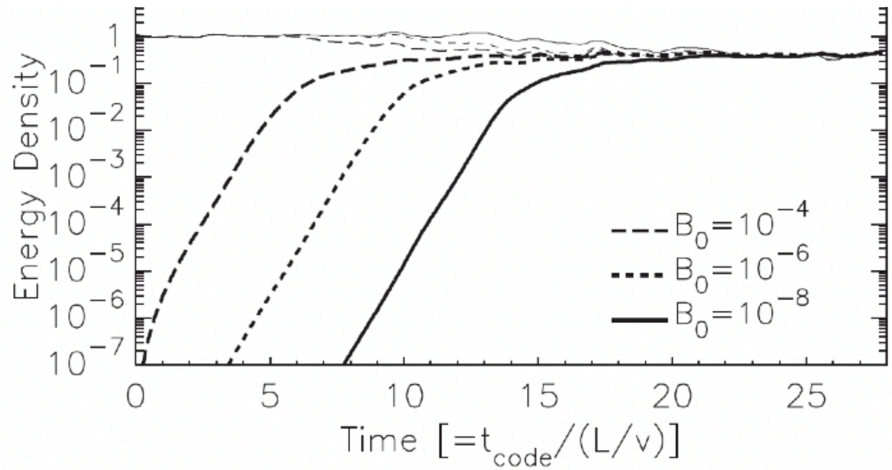
Estimated efficiency of conversion of kinetic energy into magnetic power: $\sim 2-3\,\%$

HOW TO KNOW IF THERE IS REAL SMALL-SCALE DYNAMO IN A SIMULATION?

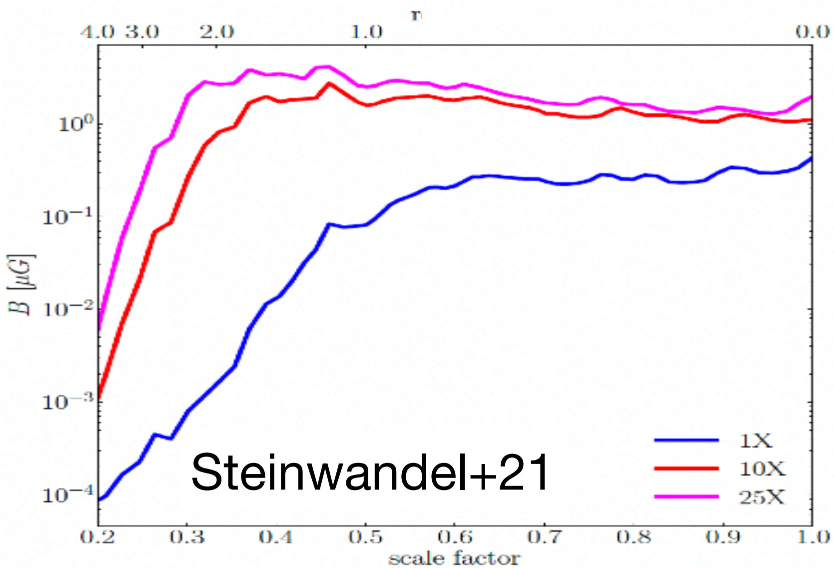


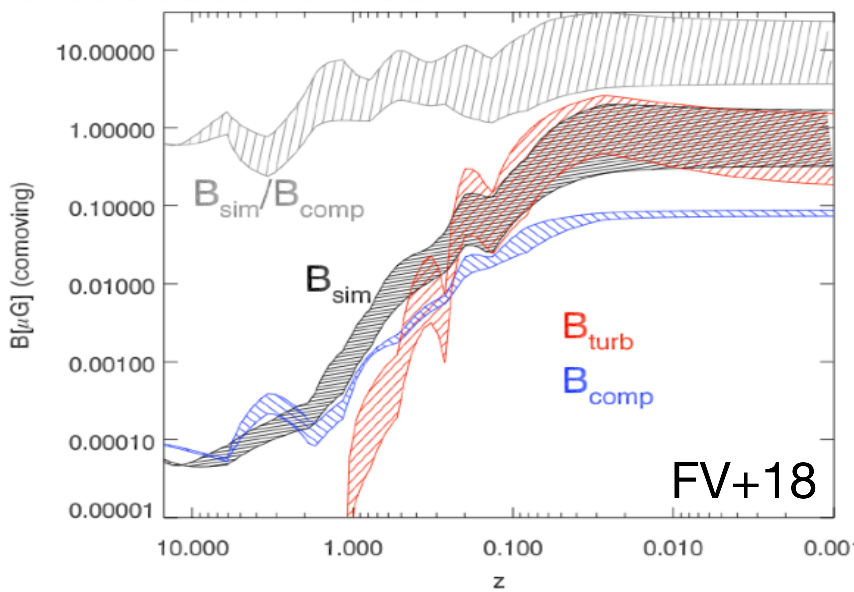
1) evolution with resolution: by increasing resolution the Reynolds number changes, and so must the effect of the dynamo

HOW TO KNOW IF THERE IS REAL SMALL-SCALE DYNAMO IN A SIMULATION?



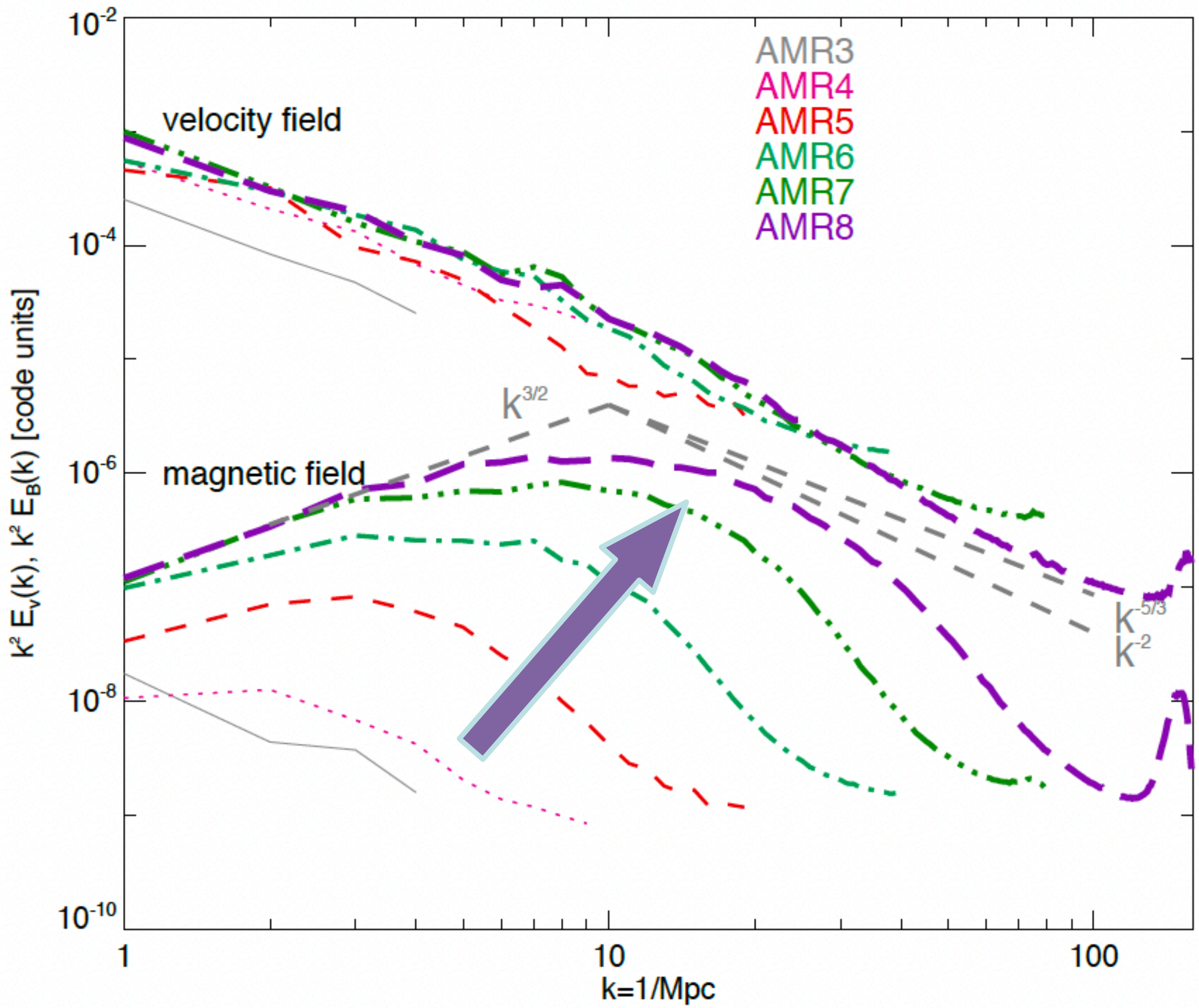
Cho 15





2) time growth and saturation levels: the field must grow final magnetic energy should be a few % of the kinetic energy. The final amplifed field should be independent on the seed field value

HOW TO KNOW IF THERE IS REAL SMALL-SCALE DYNAMO IN A SIMULATION?



Vazza+2018

3) power spectra: magnetic spectra must evolve and approach equipartition with kinetic energy. The kinetic spectrum gets modified

HOW TO KNOW IF THERE IS REAL SMALL-SCALE DYNAMO IN A SIMULATION?

$$\mathbf{K} = \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{|\mathbf{B}^2|}$$

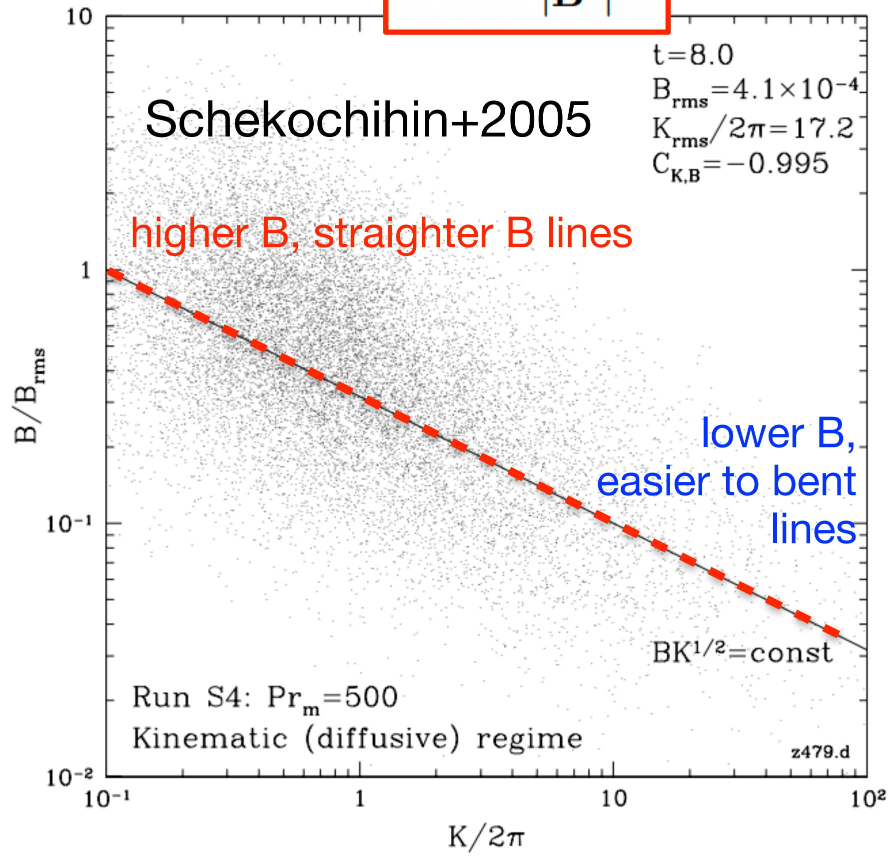
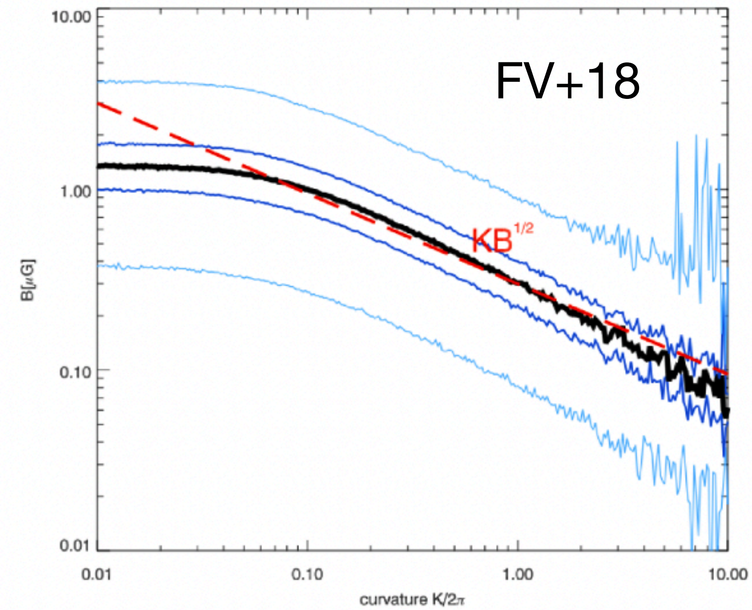
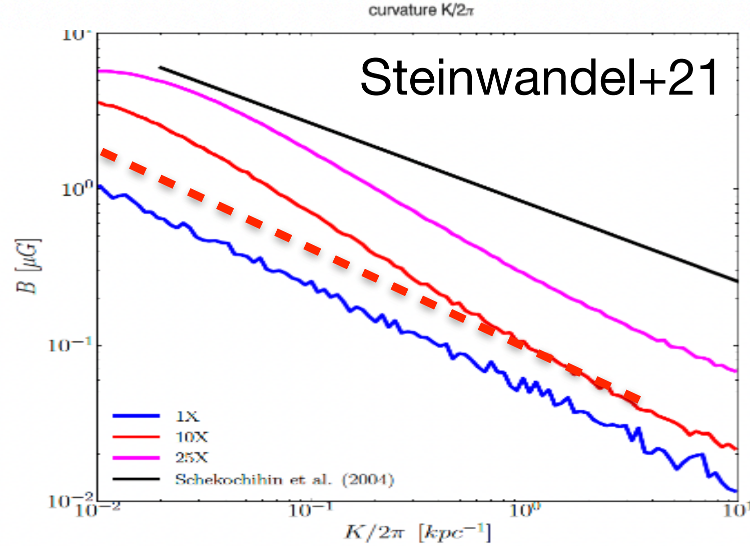


FIG. 8.— Scatter plot of B vs. K at $t = 8$ during the kinematic stage of our run S4 (the 256^3 data were thinned out by a factor of 1000).





4) distribution of "curvature (K)": there must be anti-correlation between magnetic field amplitude and curvature (magnetic tension prevents bending!)

WHAT IS THE ORIGIN OF COSMIC MAGNETISM ?

- magnetic induction $$\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{a}\nabla \times (\mathbf{v} \times \mathbf{B}) = 0.$$

even the most explosive dynamo must start from a non-zero initial seed field B_0

As long as the ideal MHD picture applies to the dynamics of large scale structures we need "seeds" of magnetic field for dynamo to start.

What are the B-field seeds?

Must they be primordial, or also astrophysical seeding scenarios can do the job?

SOME SUGGESTED READING

- *F. Rincon 2019, "LECTURE NOTES on Dynamo theories "https://arxiv.org/pdf/1903.07829"*
- *J. Donnert at al. 2018 "Magnetic Field Amplification in Galaxy Clusters and Its Simulation" https://arxiv.org/pdf/1810.09783*