

A vibrant, multi-colored visualization of the cosmic web, showing a complex network of blue filaments and green structures with numerous bright yellow and orange galaxy clusters scattered throughout.

Clusters of galaxies

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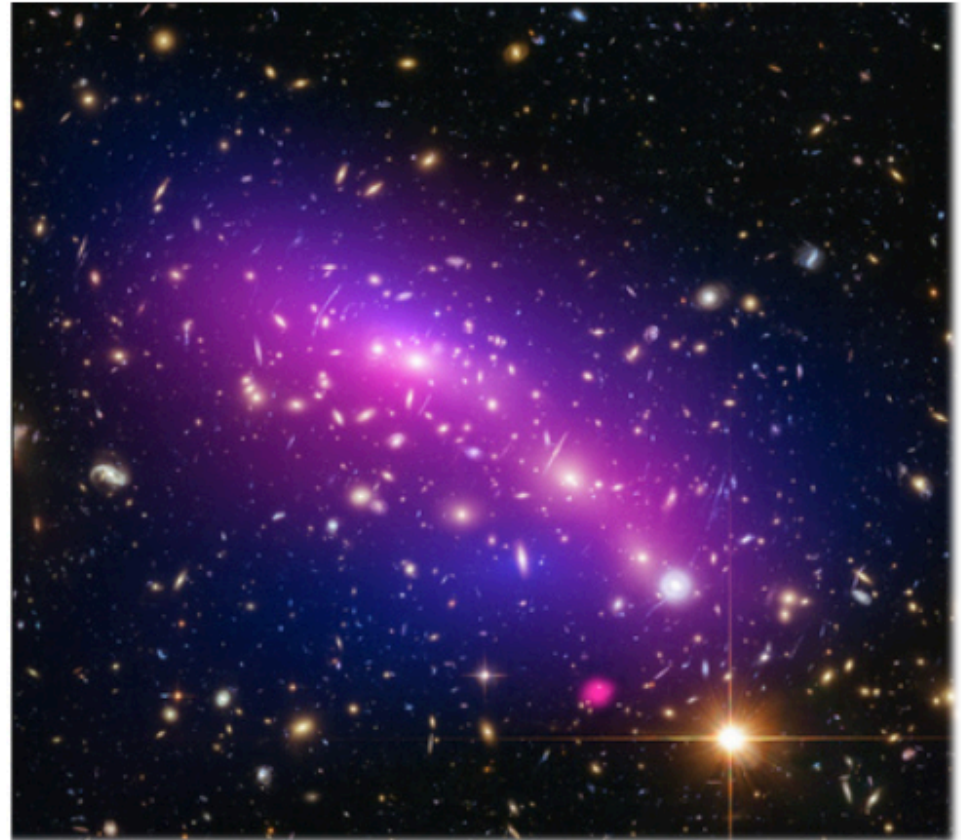
OUTLINE

In this lesson we will (briefly) see:

- the main physical **thermal** and **non-thermal** properties of galaxy clusters
- important **signatures of magnetic fields** in galaxy clusters, which we can also apply to the study of magnetic fields in filaments
- why clusters are not suitable to study cosmic magnetism

GALAXY CLUSTERS : IN BRIEF

they are:



GALAXY CLUSTERS : IN BRIEF

they are:

“the largest gravitationally bound structures in the Universe”

~ 84 % dark matter

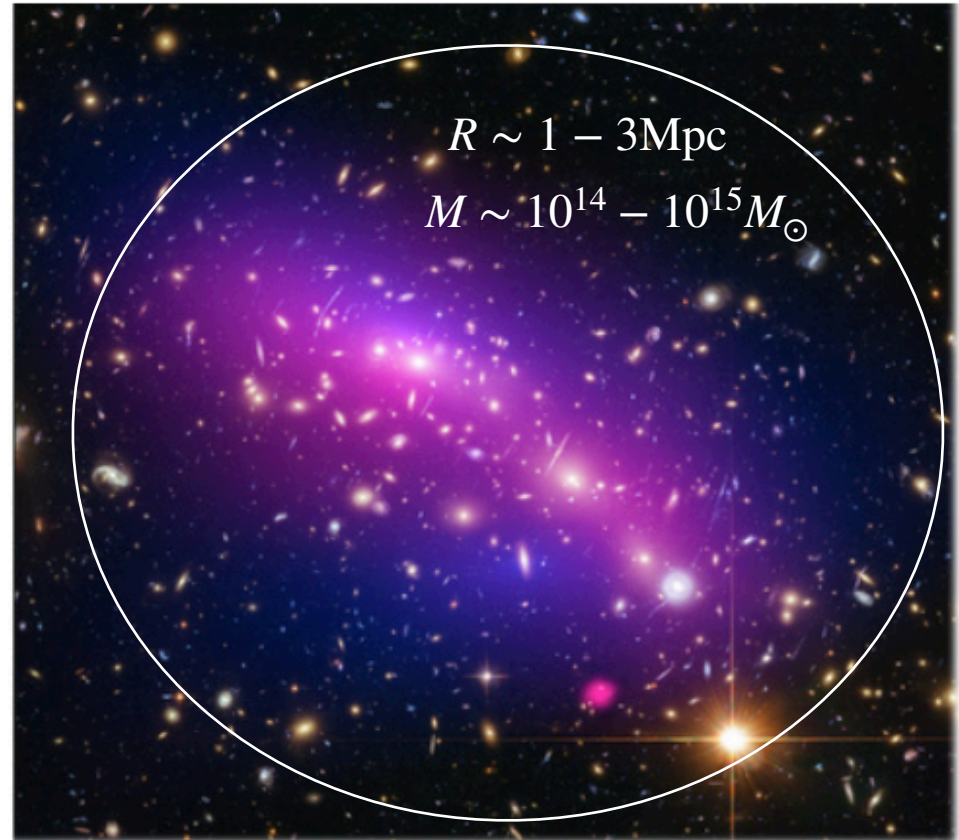
~ 16 % ordinary matter

(~ 1 % stars)

(approximate) virialisation:

$$2E_k = U \rightarrow \sigma_v^2 \sim \frac{GM}{R_{\text{vir}}} \sim \frac{5k_b T}{3\mu m_p} \sim 10^3 \text{ km/s}$$

$$R_{\text{vir}} = \sqrt{GM/\sigma_v^2} \sim 1 - 3 \text{ Mpc}$$



→ large volumes

GALAXY CLUSTERS : IN BRIEF

they are:

**“the most perfect *plasma* laboratory
in the Universe”**

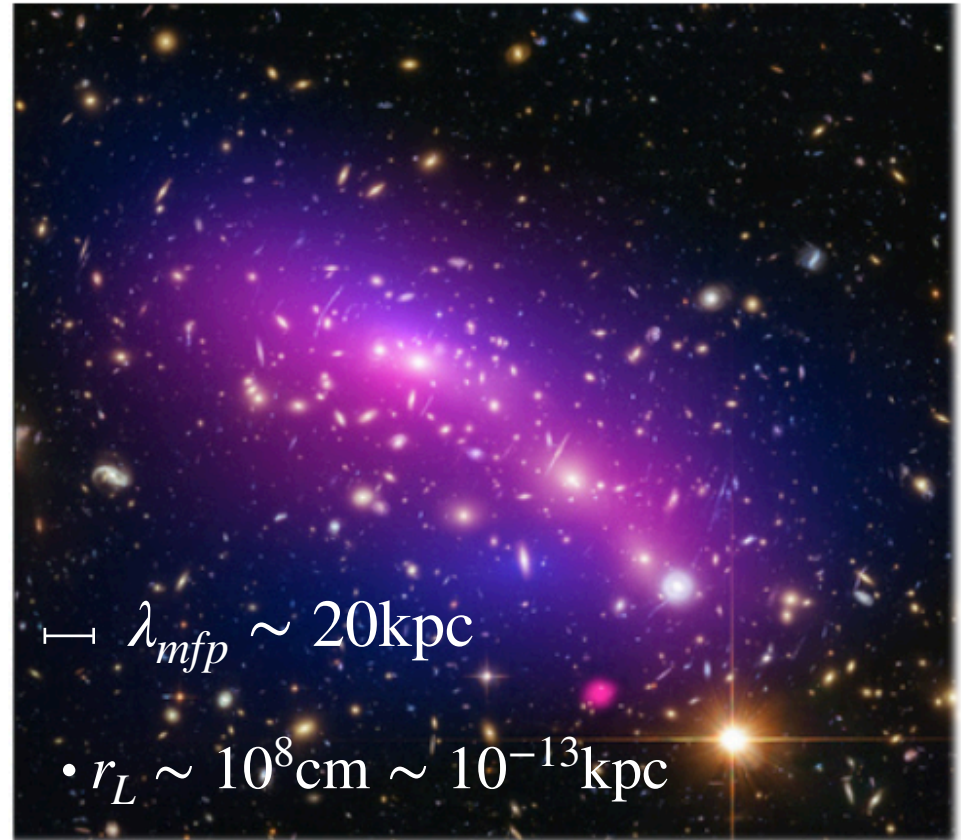
λ_{mfp} collisional mean free path ($\sim 20\text{kpc}$)

r_L gyroradius of protons ($\sim 10^{-13}\text{kpc}$)

$$r_L \ll \lambda_{mfp} \ll R_{vir}$$

the dynamics of the intracluster
medium is ruled by **collective plasma
processes.**

**If turbulence is injected on $\sim R_{vir}$ scales,
it can develop over a huge dynamical
range**



GALAXY CLUSTERS : IN BRIEF

they are:

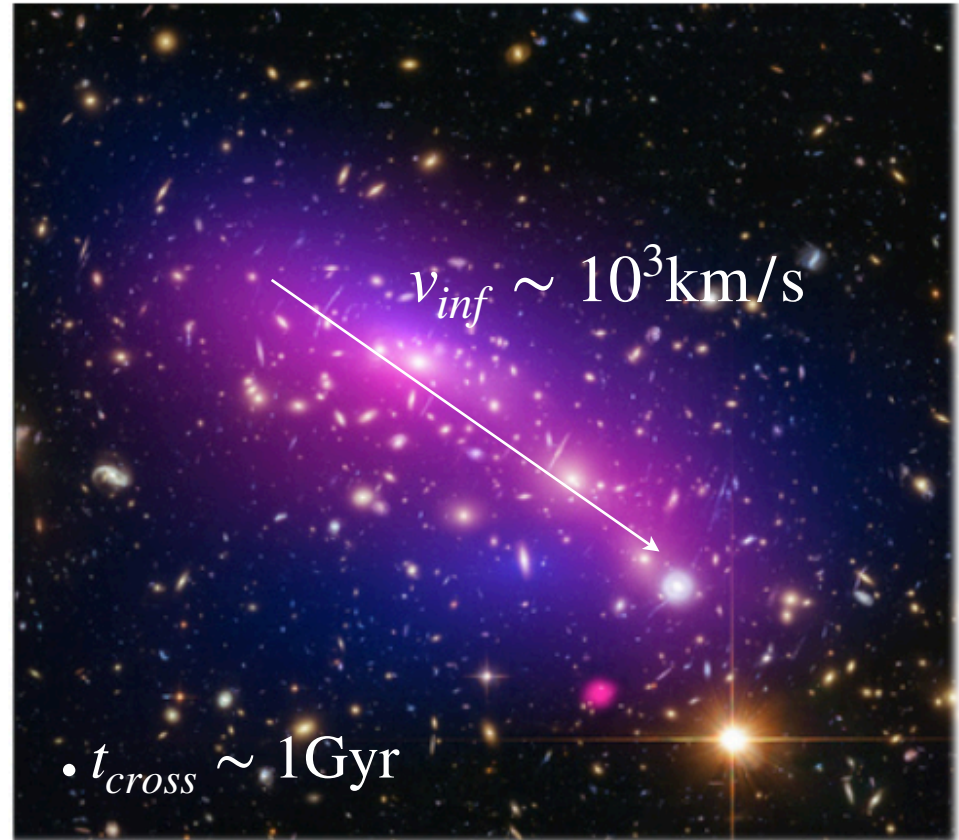
**“formed by the most energetic events
in the Universe (cluster mergers)”**

$$E_k \sim U \sim 10^{64} \text{erg}$$

$$P_{\text{kin}} \sim E_k / t_{\text{cross}} \lesssim 10^{46} \text{erg/s}$$

A fraction $\sim 10\%$ P_{kin}

is channeled into turbulence,
cosmic rays and **B-field**



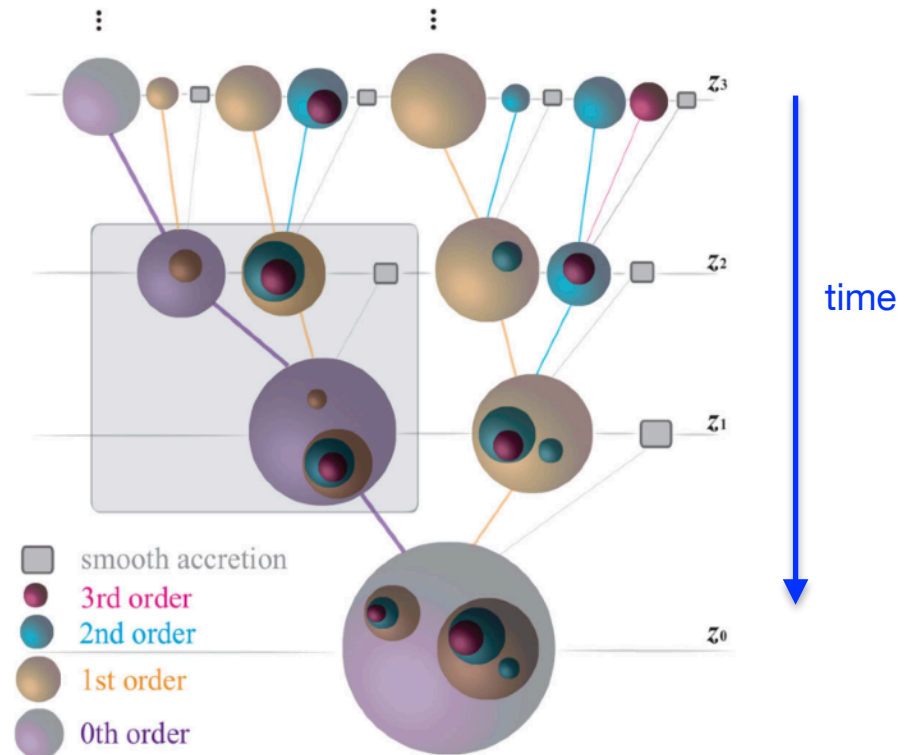
→ non-stationary conditions

GALAXY CLUSTERS : EVOLUTION & MERGERS

- Galaxy Clusters are among the last structures to form in a hierarchical (Λ CDM) Universe.
- They accrete mass both via
 - a) “smooth” accretion on cold gas, like in the Zeldovich model and
 - b) by accreting already formed halos of smaller size, via “minor” or “major” mergers.

Minor mergers = when the accreted mass is $<20\%$ of the halo mass

Major mergers = when the accreted mass is $>20\%$ of the halo mass



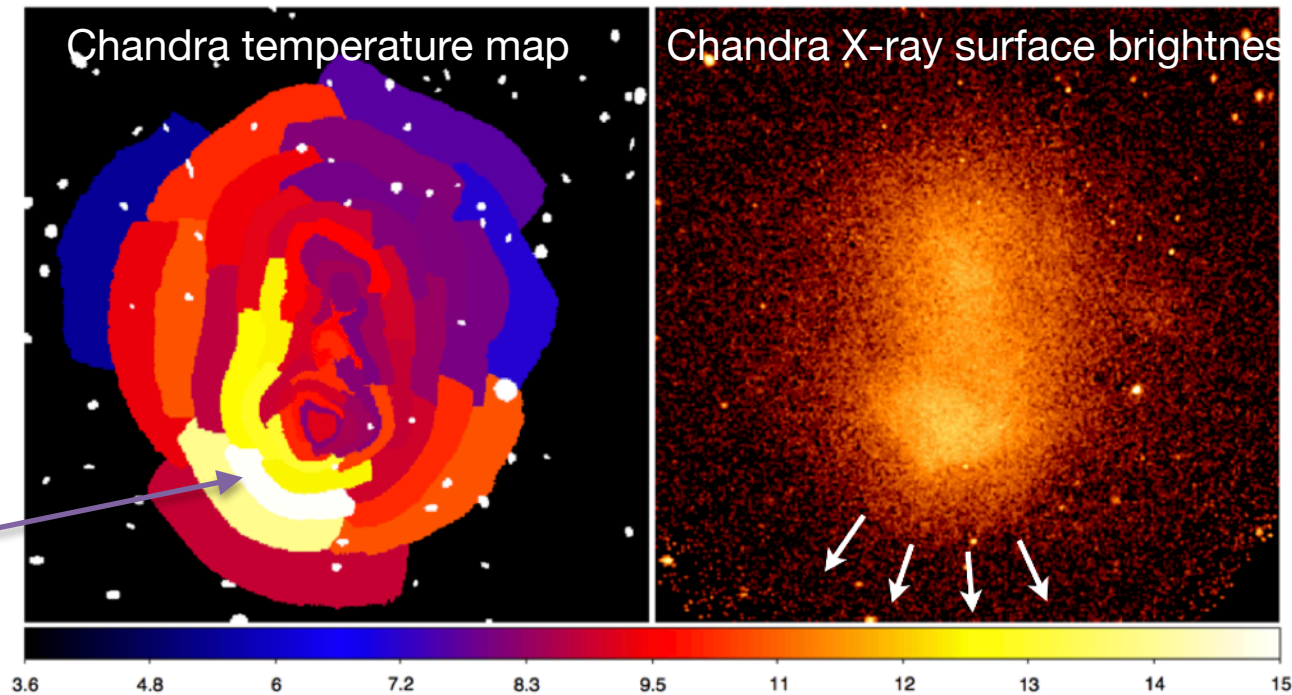
GALAXY CLUSTERS : EVOLUTION & MERGERS

- **Major mergers** involve $E_k \sim U \sim 10^{63} - 10^{64}$ erg of kinetic/potential energy.
- The collision velocity is the free fall one: $v_{inf} \sim \sqrt{2GM/R_{vir}} \sim 1000$ km/s
- This energy is mostly dissipated into heat in a “crossing time”: $t_{cross} \sim 2R_{vir}/v_{inf} \sim 2$ Gyr
- This yields a kinetic power of $P_k \sim E_k/t_{cross} \sim 10^{45}$ erg/s

- Since the sound speed of the ICM is close to the circular velocity, the induced gas motions are transonic, i.e. they release only **weak shocks**:

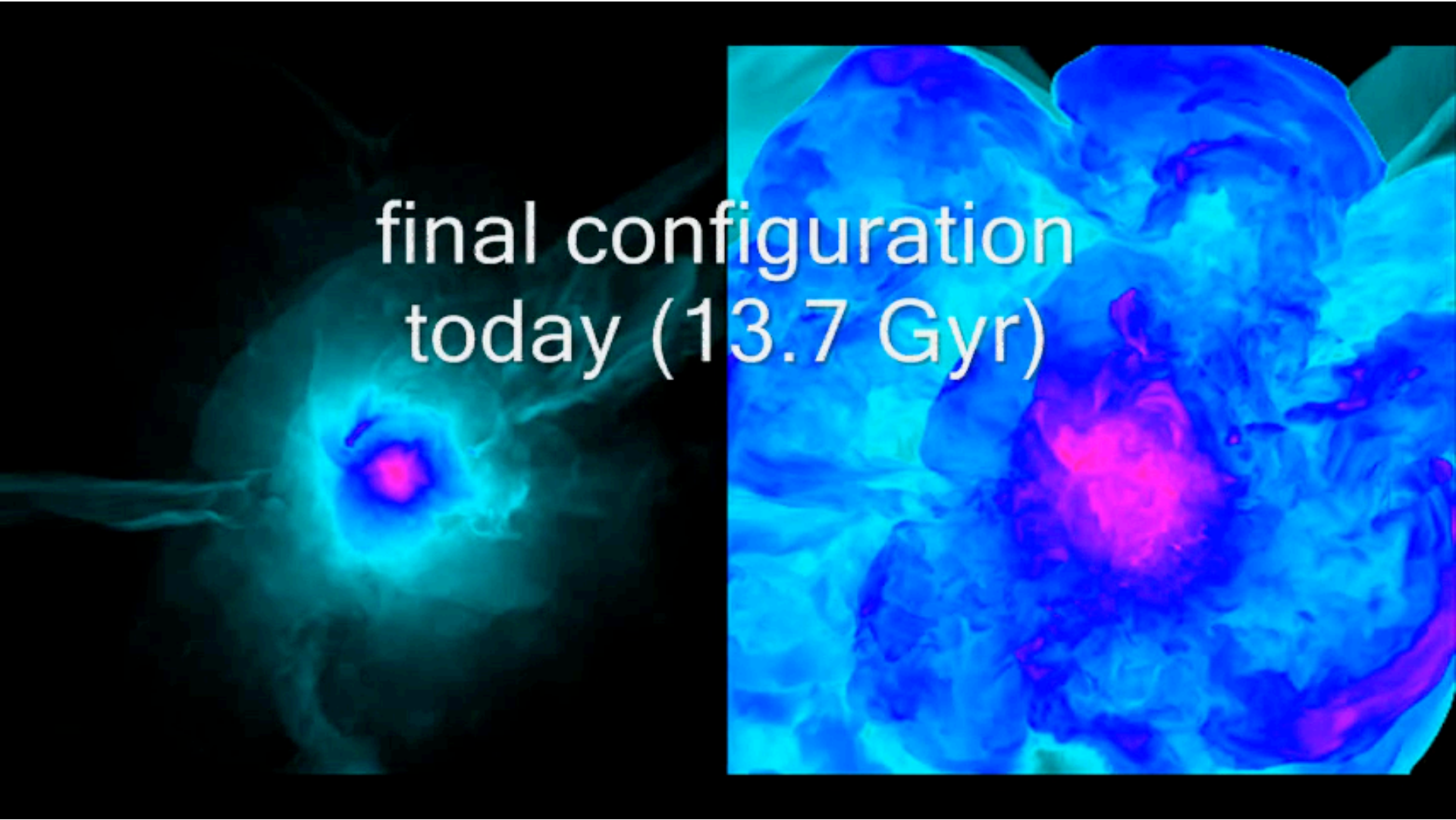
$$\mathcal{M} \approx v_{inf}/c_s \approx \sqrt{2}$$

shock measured in
X-ray ($\mathcal{M} \sim 2.5$)



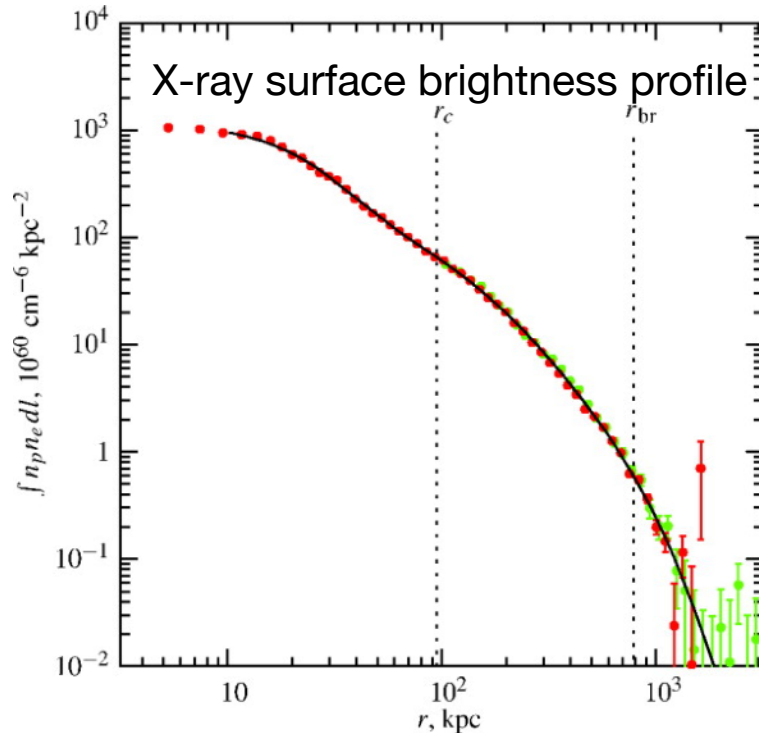
GALAXY CLUSTERS : EVOLUTION & MERGERS

final configuration
today (13.7 Gyr)



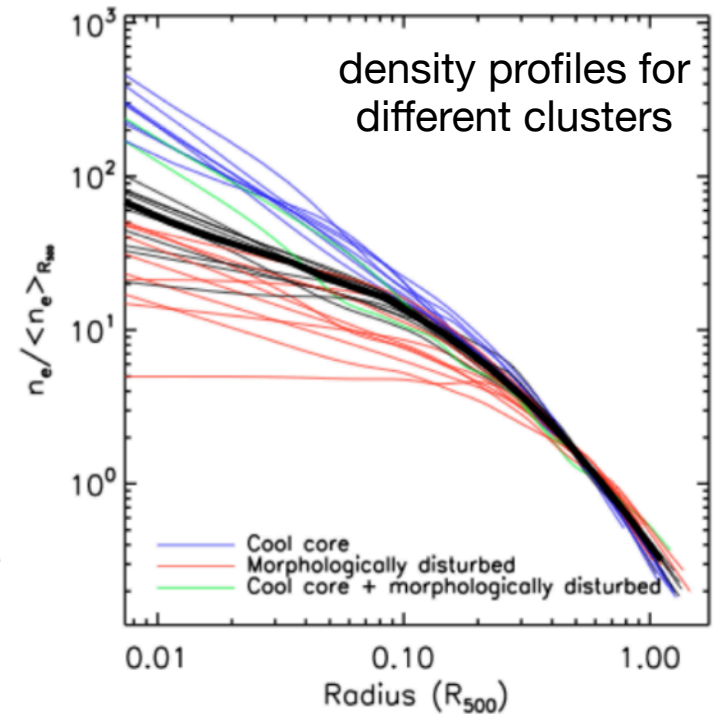
INTRACLUSTER MEDIUM DISTRIBUTION

From the projected distribution of the **X-ray surface brightness** and the temperature distribution (~isothermal in the centre) one can derive the **gas density distribution**, which is well fitted by the “**beta-model**”:



$$n_e(r) = n_e(0) \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-3/2\beta},$$

$$\beta = \frac{\mu m_p \sigma_r^2}{k_B T} \sim 0.76 \left(\frac{\sigma_r}{10^3 \text{ km/s}} \right)^2 \left(\frac{T}{10^8 \text{ K}} \right)^{-1}$$



- clusters with different dynamical state have more or less peaked profiles (i.e. “cool core” vs “non-cool-core”)
-

SELF-SIMILARITY

- If the formation of clusters of galaxies is only ruled by gravity (which is scale-free), every halo must be to a good approximation a “self-similar” rescaled version of all other clusters.
- If the system is in equilibrium: **gravity force = pressure force**. Thus by imposing hydrostatic equilibrium one derives:

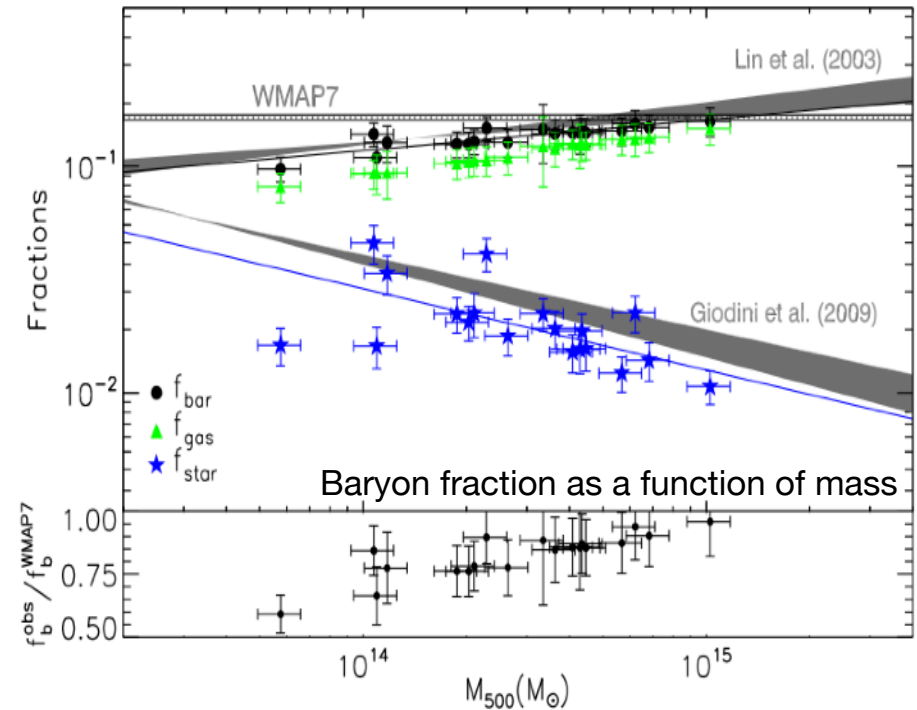
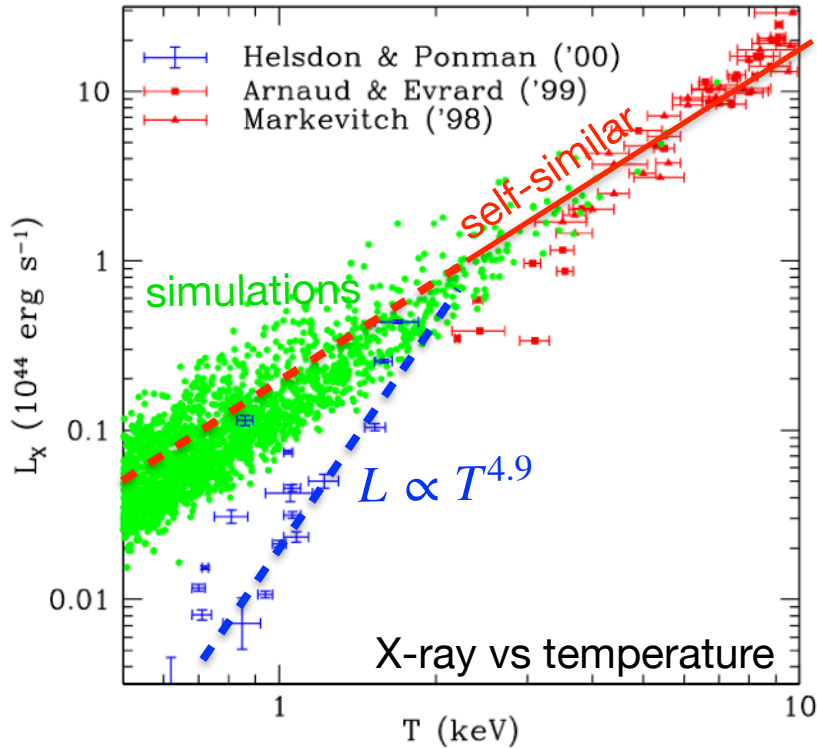
$$M_{HE}(< r) = -\frac{rk_B T(r)}{G\mu m_p} \left[\frac{d \ln \rho_g(r)}{d \ln r} + \frac{d \ln T(r)}{d \ln r} \right]$$

- This implies the following **self-similar relations**, in which global key quantities are only related to the total mass :

$$\begin{aligned} T &\propto M/R \propto M^{2/3} && \text{for the temperature} \\ L_{X,bol} &\propto \rho^2 T^{1/2} R^3 \propto M^{4/3} \propto T^2 && \text{for the X-ray luminosity} \\ K &= k_B T / n_e^{2/3} \propto \rho^{-2/3} T \propto M^{2/3} && \text{for the ICM “entropy”} \\ Y &= M_{gas} T \propto M^{5/3} && \text{for the ICM pseudo-pressure} \end{aligned}$$

BREAKING OF THE SELF-SIMILARITY

- However, real observations of a large number of clusters showed however that self-similarity does not really hold for $M \leq 10^{14} M_{\odot}$, i.e. for groups of galaxies

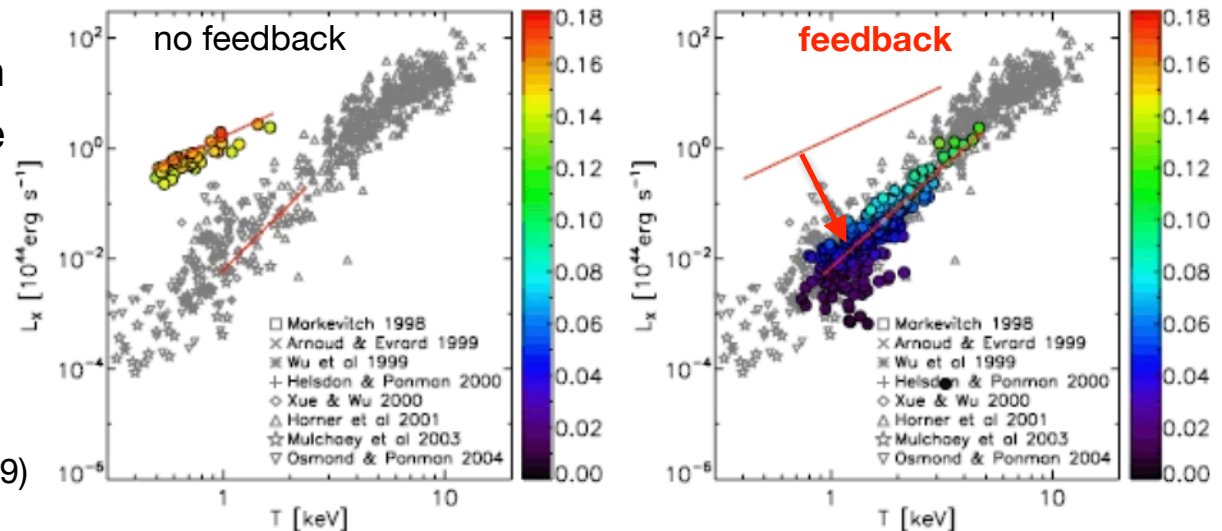
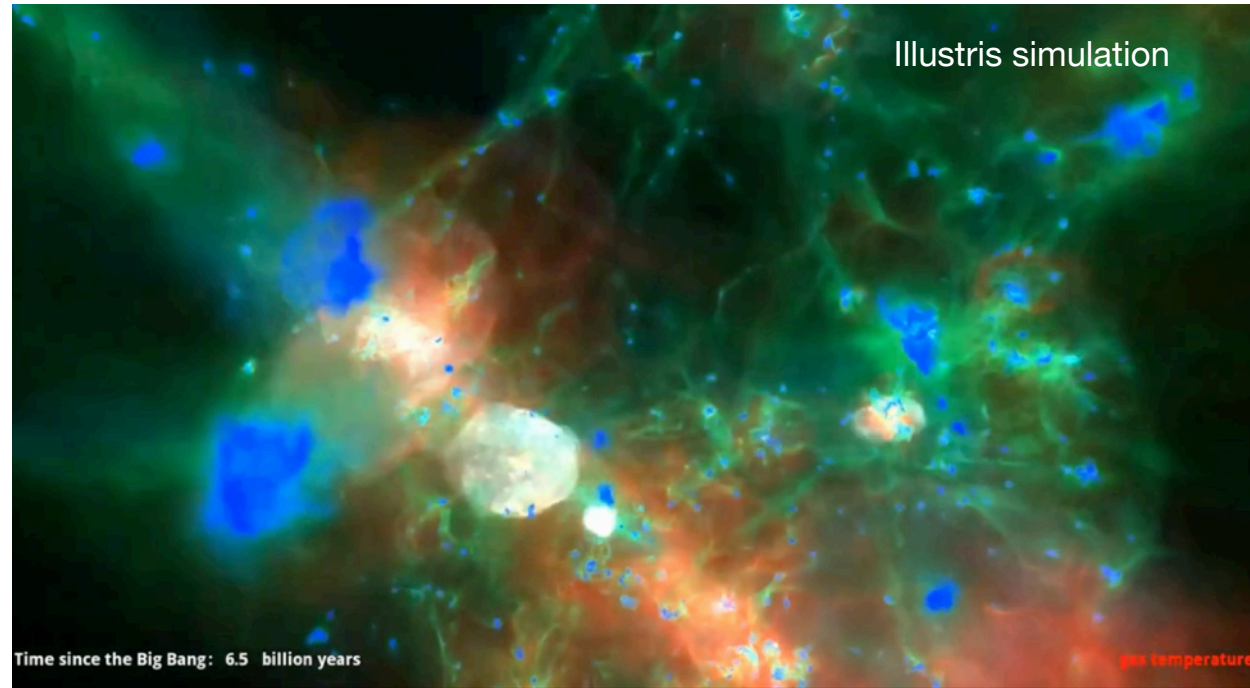


- Groups have proportionally less baryons than halos \rightarrow gas expulsion in the past
- Radiative cooling of gas and feedback from star formation and active galactic nuclei (AGN) break the self-similarity imposed by gravity

AGN FEEDBACK

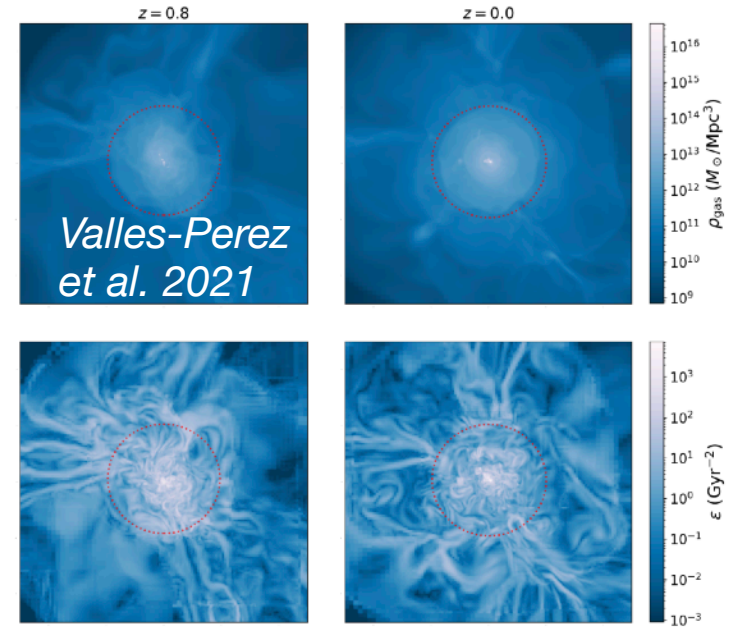
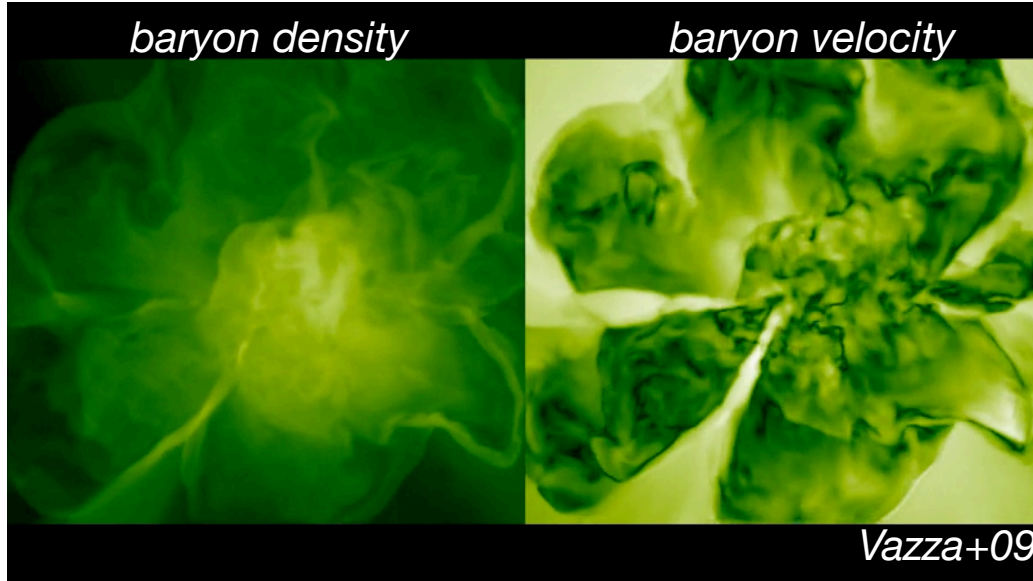
- Cooling promotes the condensation of the ICM into stars and it feeds the growth of Supermassive Black Holes
- The energetic feedback from AGN expels gas from the centre and **deposits baryons in the outer parts of clusters.**
- Cosmological simulation with AGN steepens the $L_x - T$ relation towards observations

(Short & Thomas 2009)

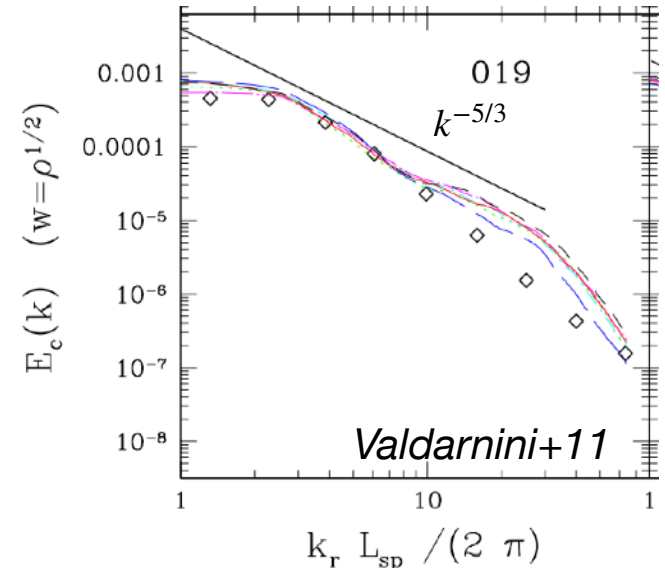
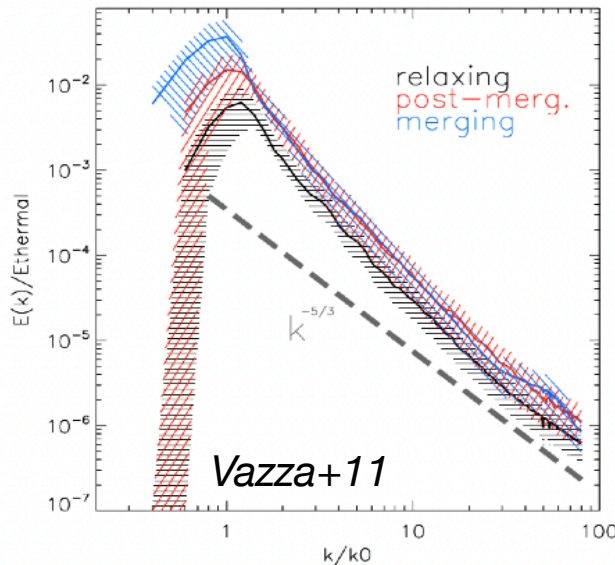


TURBULENCE IN THE ICM

Simulations predict since ~20 years a **substantial non-thermal pressure from subsonic turbulence** (~5-20% of total gas energy)



These motions are overall well described by the **Kolmogorov** model of turbulence

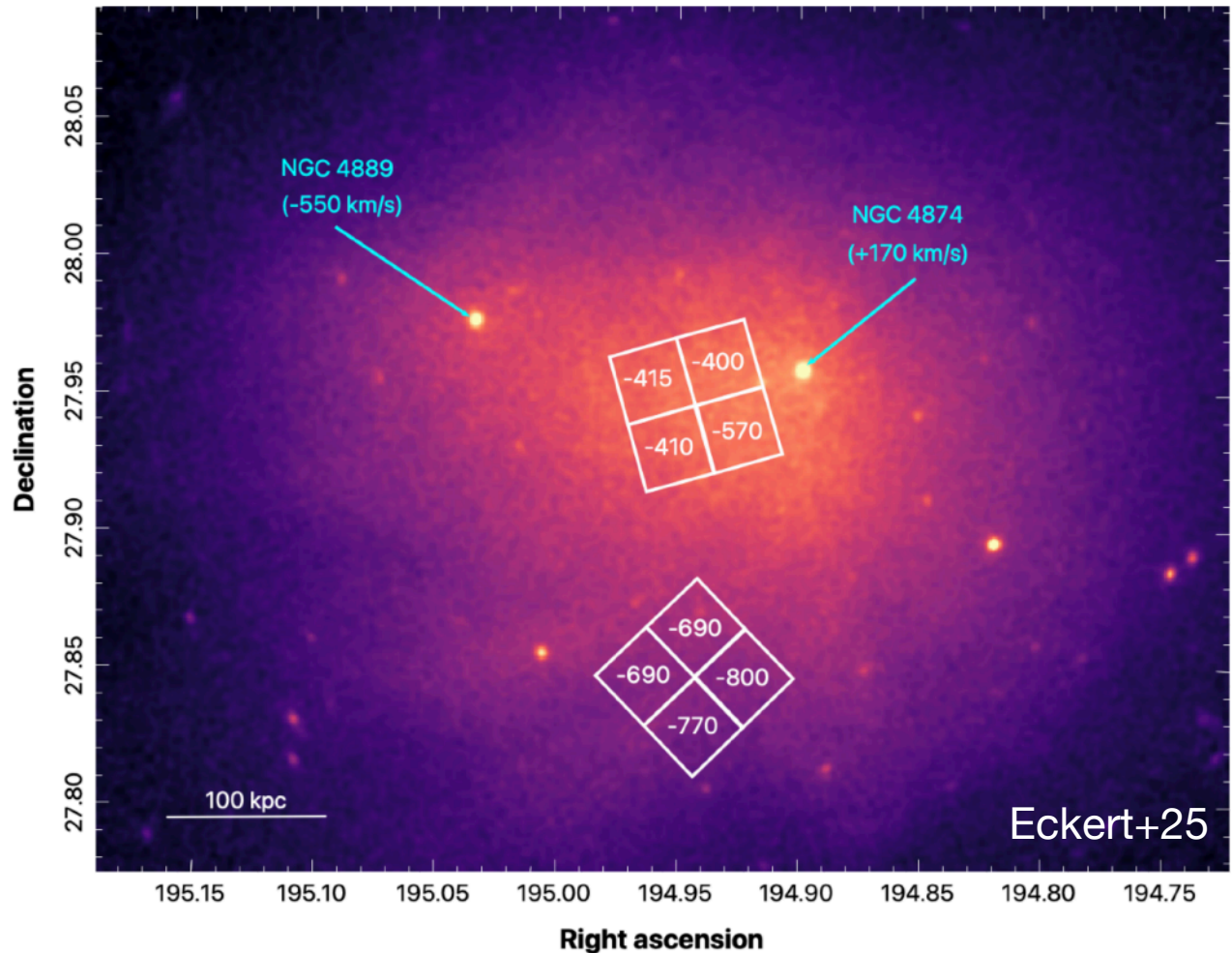
$$P_v(k) \propto k^{-5/3}$$


TURBULENCE IN THE ICM

The XRISM satellite has begun measuring the **Doppler broadening of Iron lines** in small portions of nearby clusters of galaxies, which should scale like $\propto \sigma_v/c$, reporting **quite low values** of non-thermal pressure support in most clusters ($\leq 5\%$)

These measurements may however be affected by a few X-ray biases and their inversion to get volume-wide estimates is difficult... ongoing debate!

(e.g.
XRISM Collab. +25;
FV & Brunetti 25;
Zhang et al. 26..)



GALAXY CLUSTERS : EVIDENCE OF MAGNETIC FIELDS

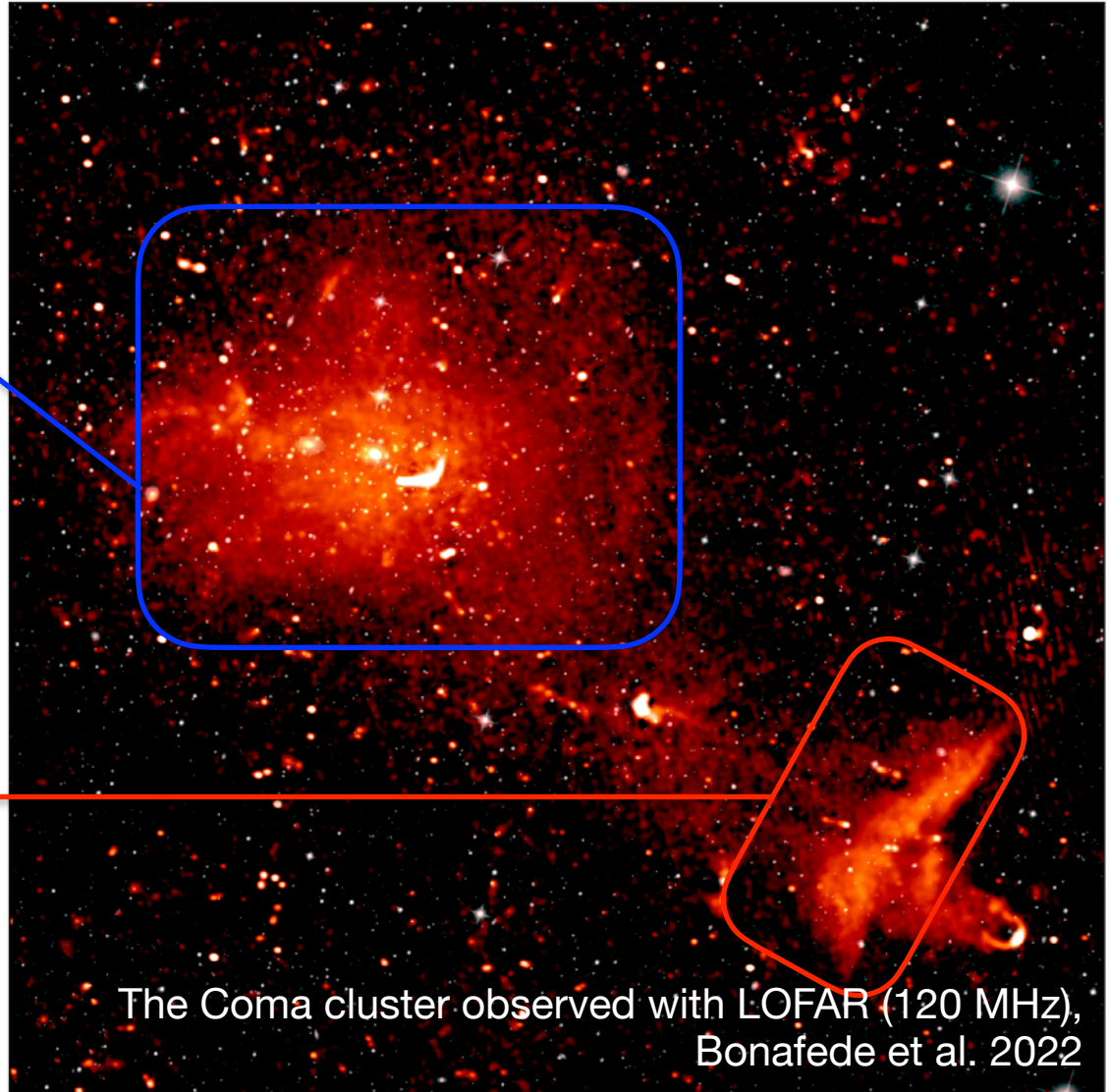
RADIO HALOS :

- size $\sim 0.5-1\text{Mpc}$
- diffuse patchy emission
- centrally located
- spectrum $I(\nu) \propto \nu^{-(1\div 2)}$
- unpolarised.
- Related to **turbulence**?

RADIO RELICS :

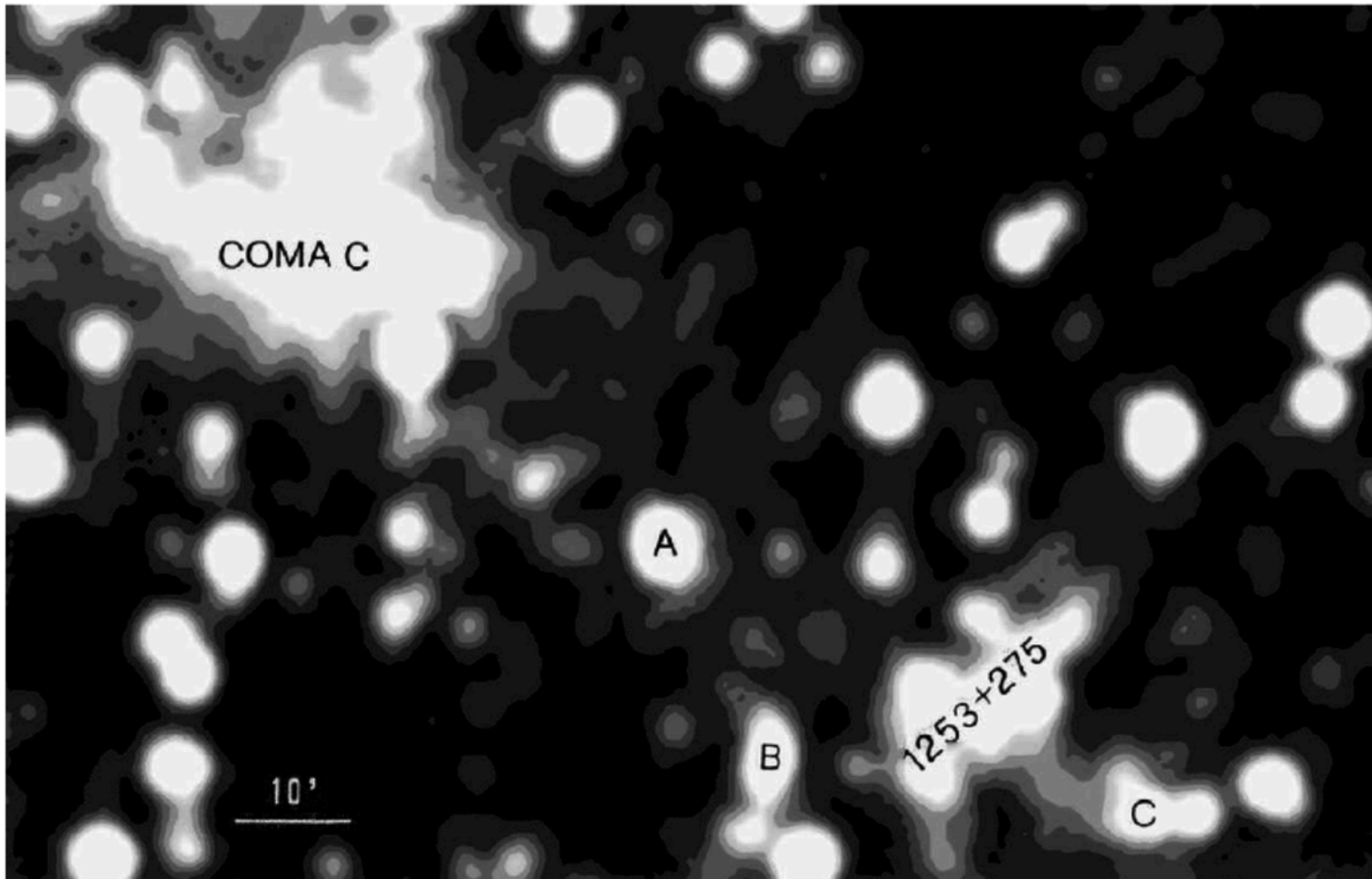
- size $\sim 1-2\text{Mpc}$ wide
- located in cluster periphery
- elongated
- spectrum $I(\nu) \propto \nu^{-(1\div 1.5)}$
- polarised
- *Related to shocks?*

Both emissions found in **perturbed** (usually post-merger) galaxy clusters



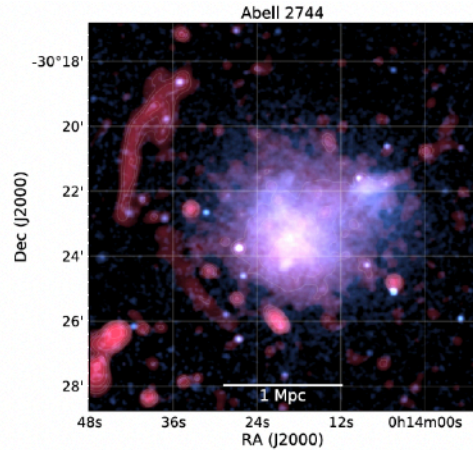
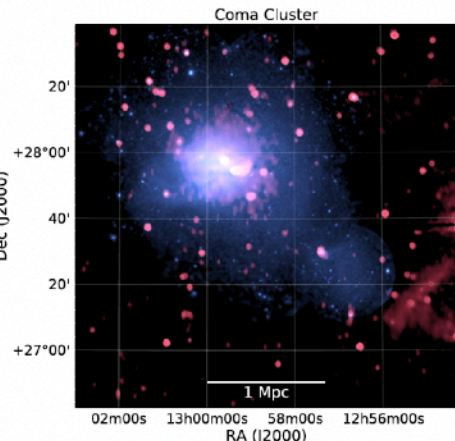
The Coma cluster observed with LOFAR (120 MHz),
Bonafede et al. 2022

GALAXY CLUSTERS : EVIDENCE OF MAGNETIC FIELDS

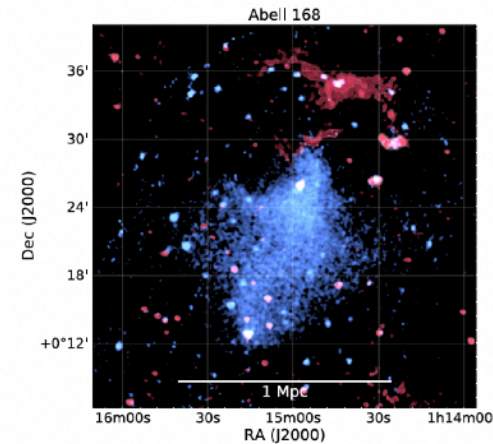
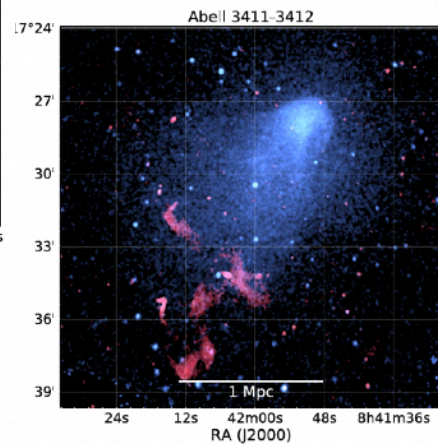
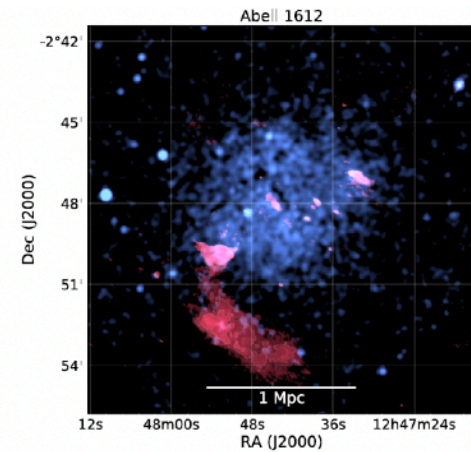
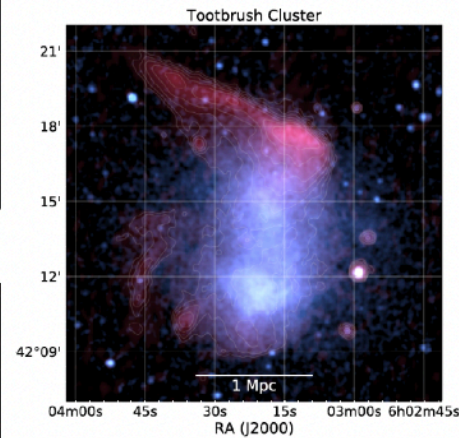
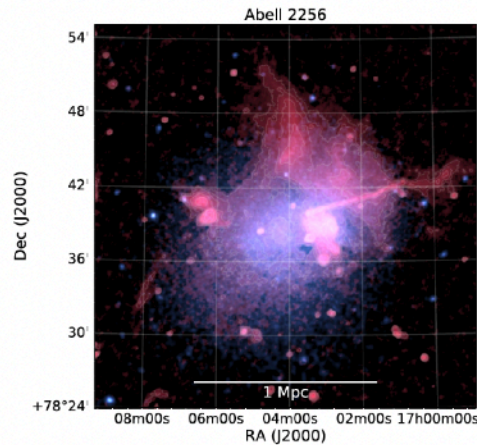
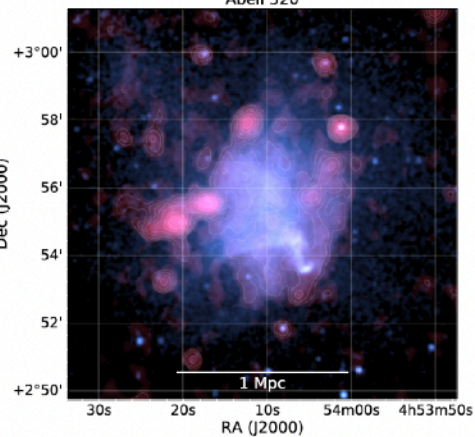


Radio emission from the Coma cluster (Kim et al. 1989, [Giovannini et al. 1991](#)).

GALAXY CLUSTERS : EVIDENCE OF MAGNETIC FIELDS



blue = X-ray emission from hot gas
red = radio synchrotron emission (relativistic electrons and magnetic fields)

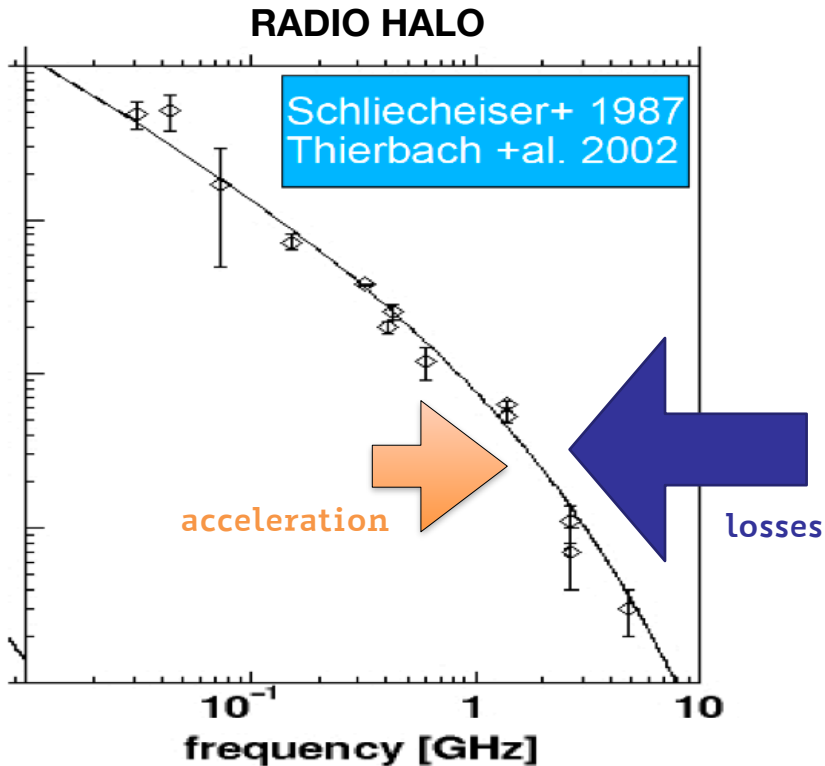


After the first detections in the 80s, we have by now evidence of radio emission on Mpc scales in more than 100 clusters.

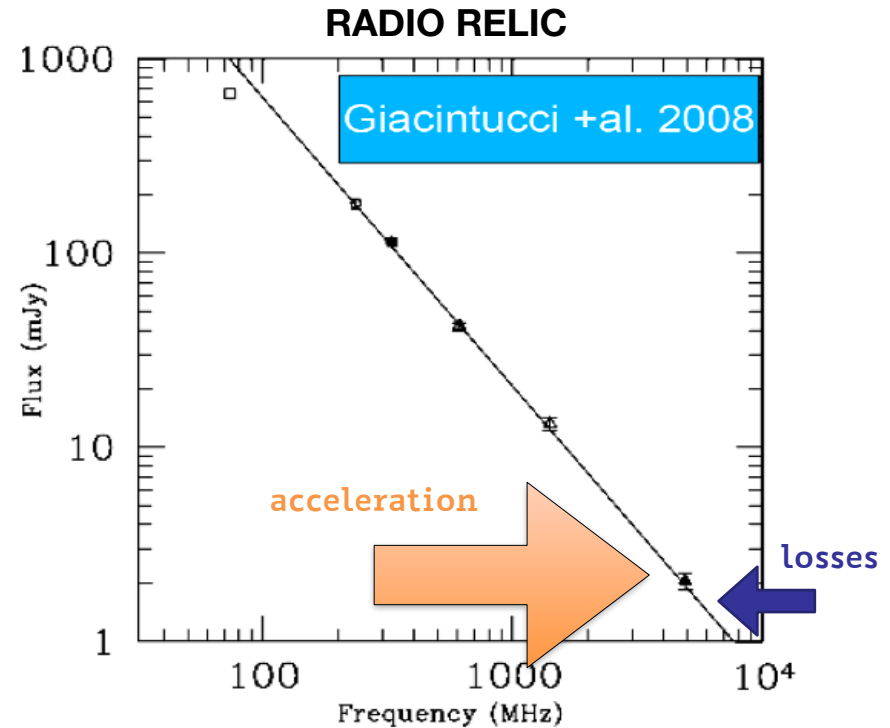
see Lectures by Brunetti, Cuciti, Botteon

RADIO EMISSION IN GALAXY CLUSTERS : TWO MAIN CLASSES

- The tendency of the two classes to show slightly different spectra suggests two different acceleration mechanisms

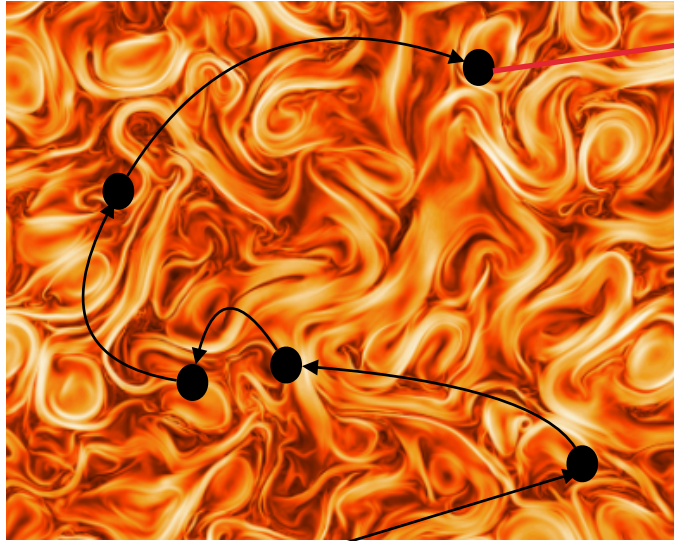


Curved spectrum → losses become dominant after a certain energy
Fermi II mechanism?
(Brunetti + 01,04,07...)



Straight spectrum → electrons efficiently accelerated to high energy
Fermi I mechanism?
(Hoeft & Bruggen 07, Kang+12..)

RADIO EMISSION IN GALAXY CLUSTERS : TWO MAIN CLASSES



Fermi II acceleration:

charged particles randomly scattered by magnetic fluctuations with σ_v **turbulent** velocity dispersion.

Systematic acceleration by

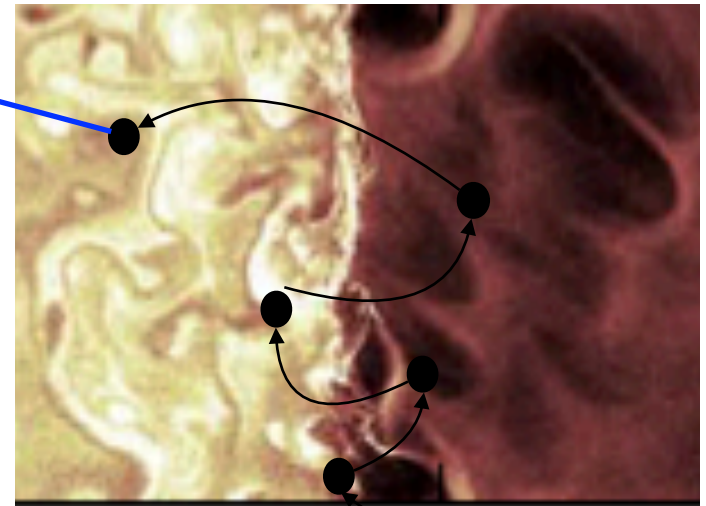
$$\frac{\Delta E}{E} \propto \left(\frac{\sigma_v}{c} \right)^2$$

Fermi I acceleration:

charged particles scattered by magnetic fluctuations across the $|u_2 - u_1|$ velocity jump of a **shock**

Systematic acceleration by

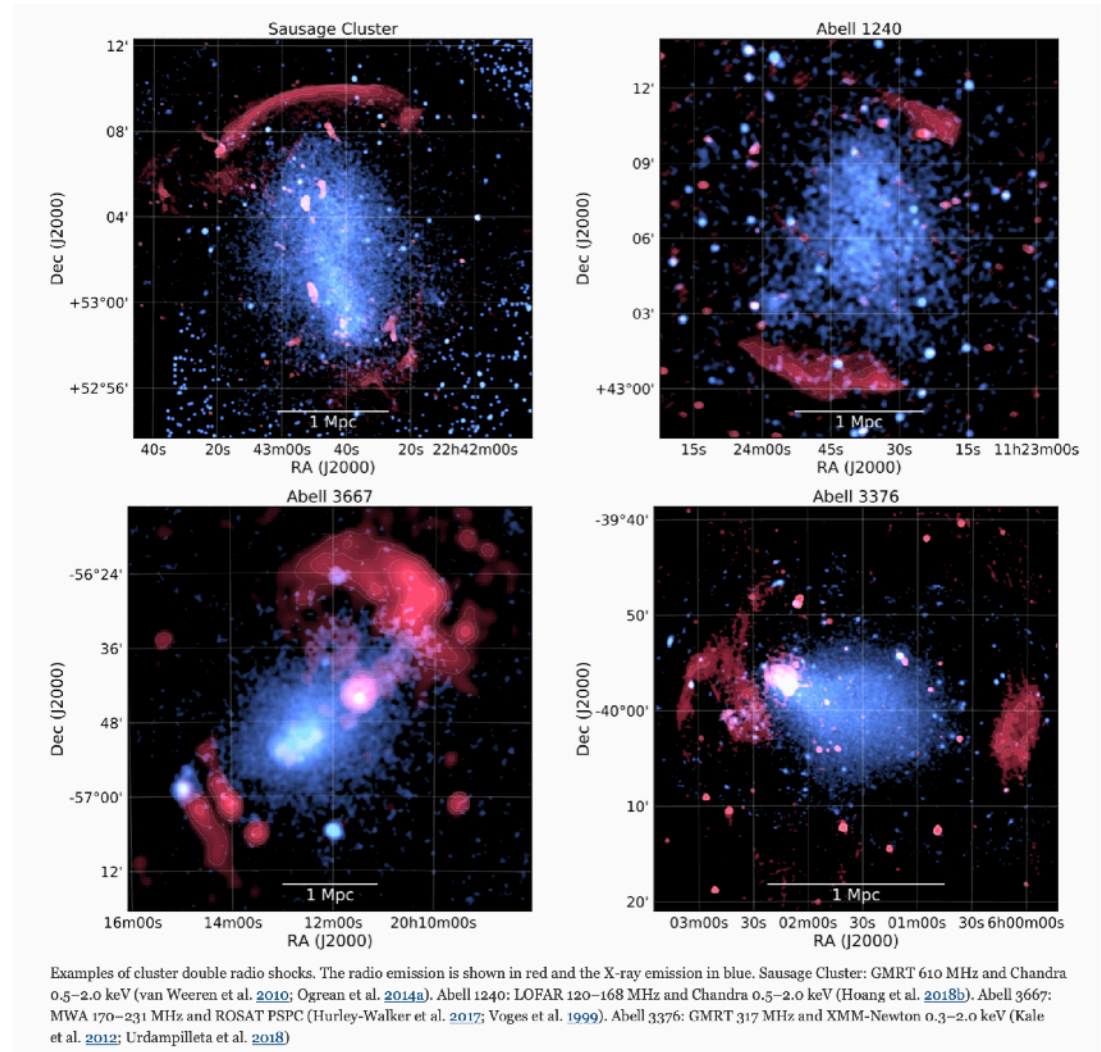
$$\frac{\Delta E}{E} \propto \frac{|u_2 - u_1|}{c}$$



In both cases: strong prediction of a $N(E) \propto E^{-p}$ power law distribution

RADIO RELICS

- We know so far of ~50 relics in the sky (in some cases, 2-3 are found in the same object).
- They are mostly **peripheral** (~0.5-3Mpc from cluster centre), **elongated**, **polarised**
- Their typical radio power is $P_{1.4\text{GHz}} \sim 10^{40} - 10^{42} \text{erg/s}$
- Their spectrum is (usually) flatter than radio halos: $I(\nu) \propto \nu^{-(1 \div 1.2)}$
- Often associated to **visible X-ray shocks**



RADIO RELICS - POLARISATION

Radio relics are **polarised** ($p \sim 70\%$). Polarisation vectors show that:

- the B-field is often \sim parallel to the shock front (\rightarrow consistent with compression of B)
- multiple threads & reversals \rightarrow not simple quasi spherical shocks, but complex surfaces

Notice:

a) these maps all show \vec{E} -vectors. \vec{B} -vectors are perpendicular to them!

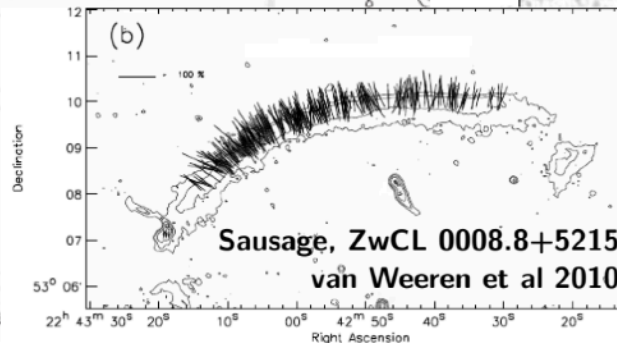
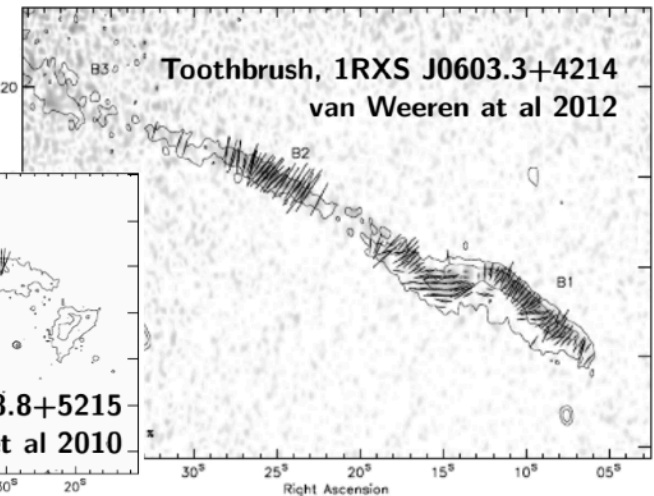
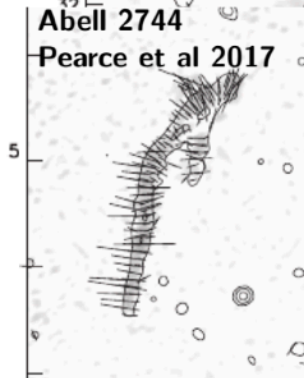
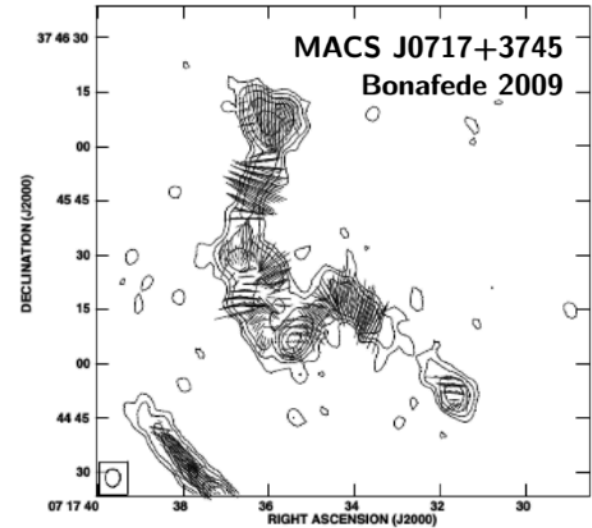
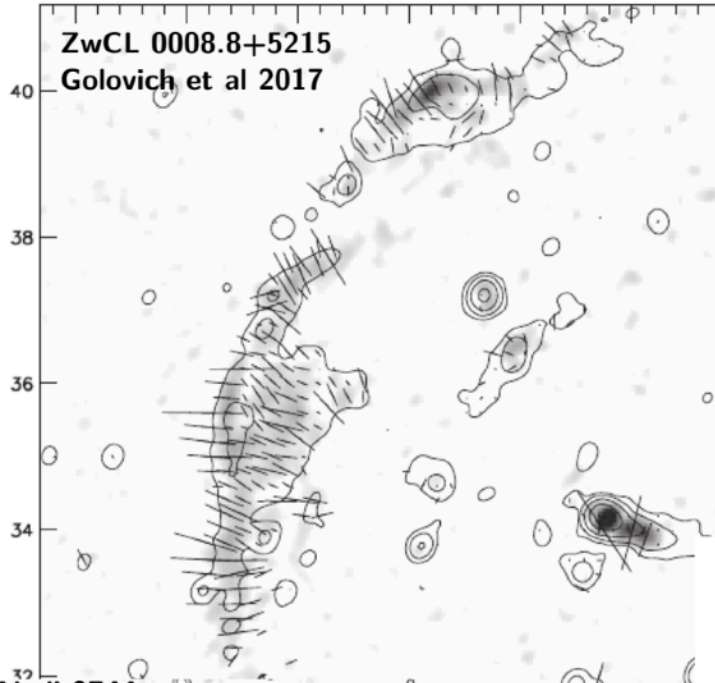
b) polarisation vector can only give the direction - NOT THE VERSE of \vec{B} . i.e.

$\leftarrow \rightarrow \rightarrow \leftarrow$

or

$\leftarrow \leftarrow \leftarrow \leftarrow$

exactly produce the same polarisation!



RADIO RELICS – POLARISATION

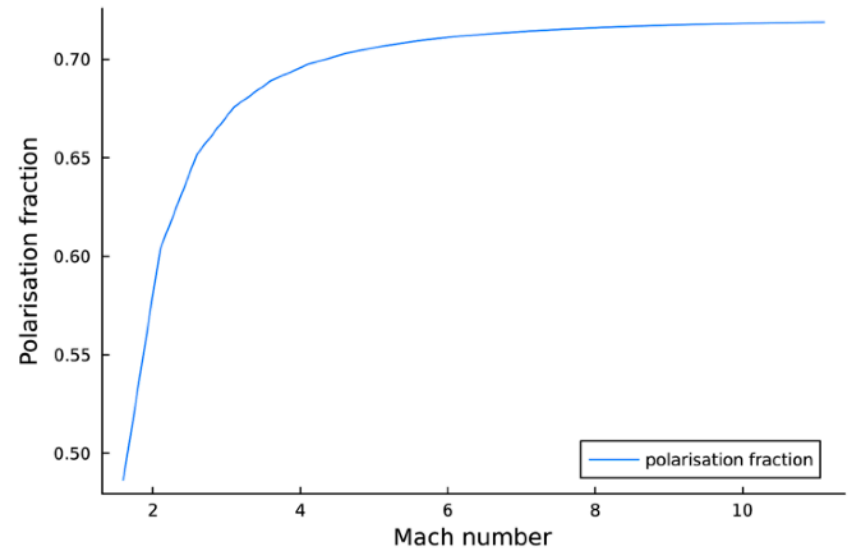
For simple (\sim plane wave) geometry, and “edge-on” view, the polarisation degree of radio emission should scale like:

$$\langle p \rangle = \frac{\delta + 1}{\delta + 7/3} \cdot \frac{1}{2R^2/(R^2 - 1) - 1}$$

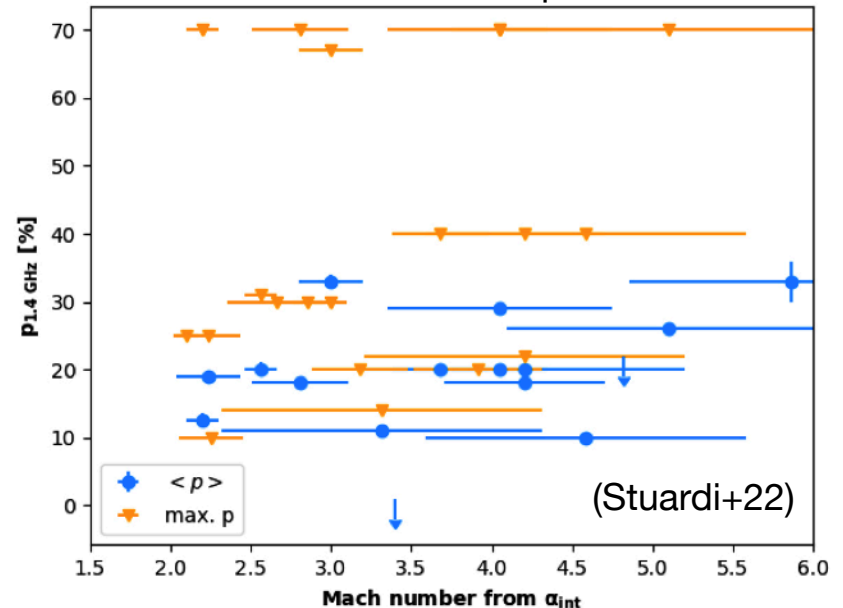
where $\delta = 2\alpha + 1$ is the radio spectrum and $R = 4M^2/(M^2 + 3)$ is the shock compression factor (Ensslin+1998)

So from $\langle p \rangle$ one might get the Mach number

..of course, reality is not so simple!
Radio relics are not simple surfaces, they are not always seen edge on, and their emission can be de-polarised by the ICM inbetween them and the observer.

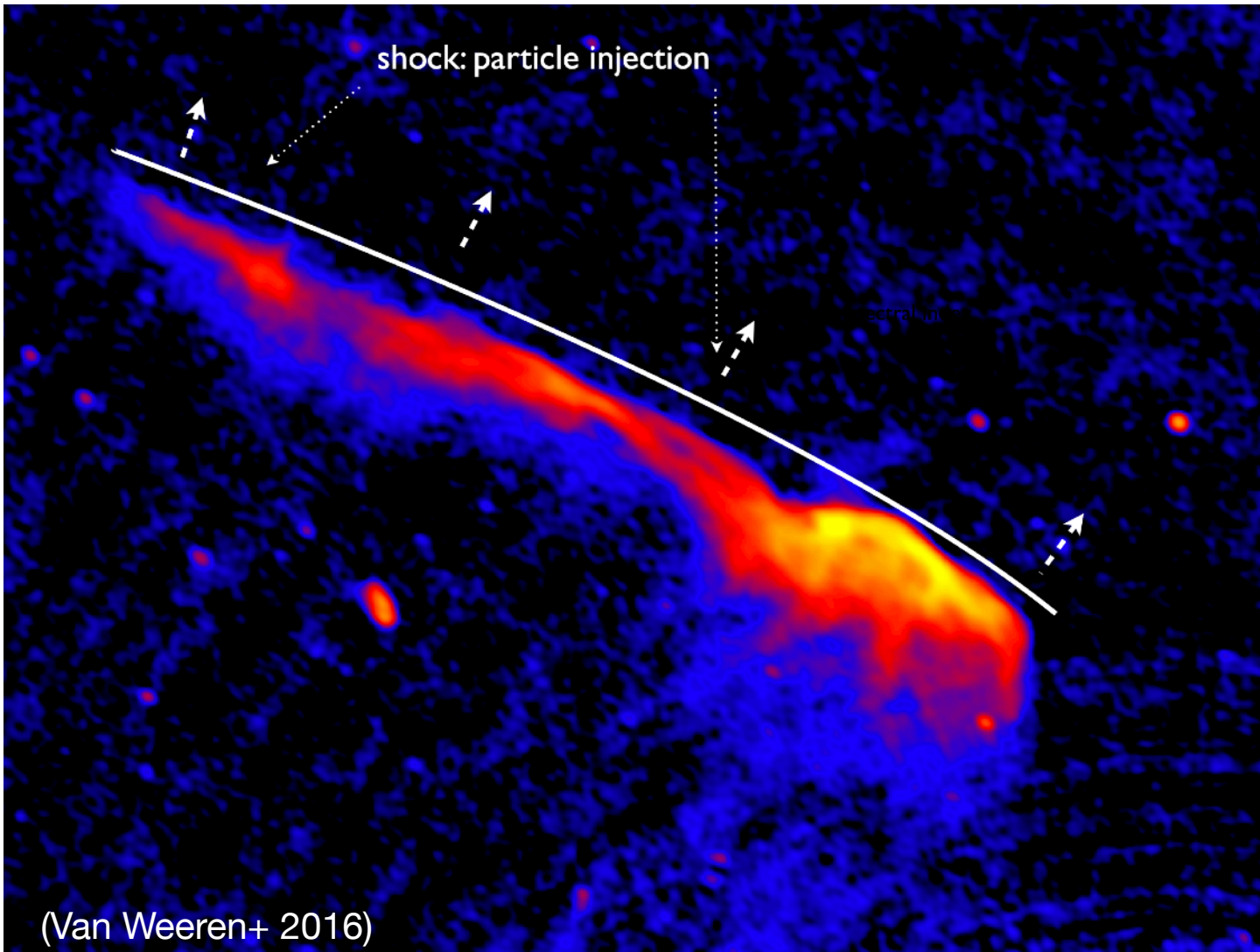


radio measurements on a sample of radio relics:



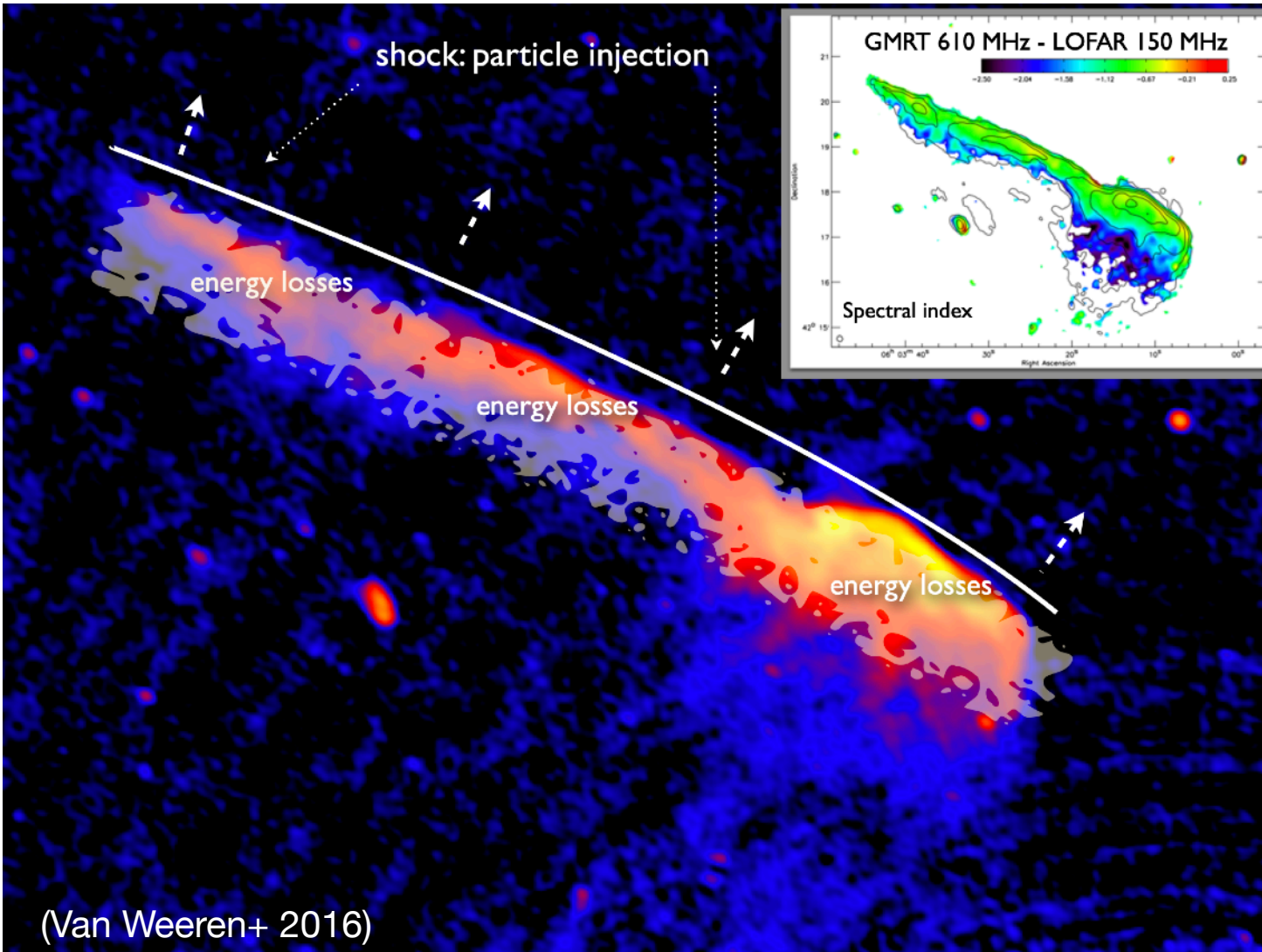
(Stuardi+22)

THE DIFFUSIVE SHOCK ACCELERATION MODEL FOR RADIO RELICS



1) particles are injected at the shock front

THE DIFFUSIVE SHOCK ACCELERATION MODEL FOR RADIO RELICS



1) particles are injected at the shock front

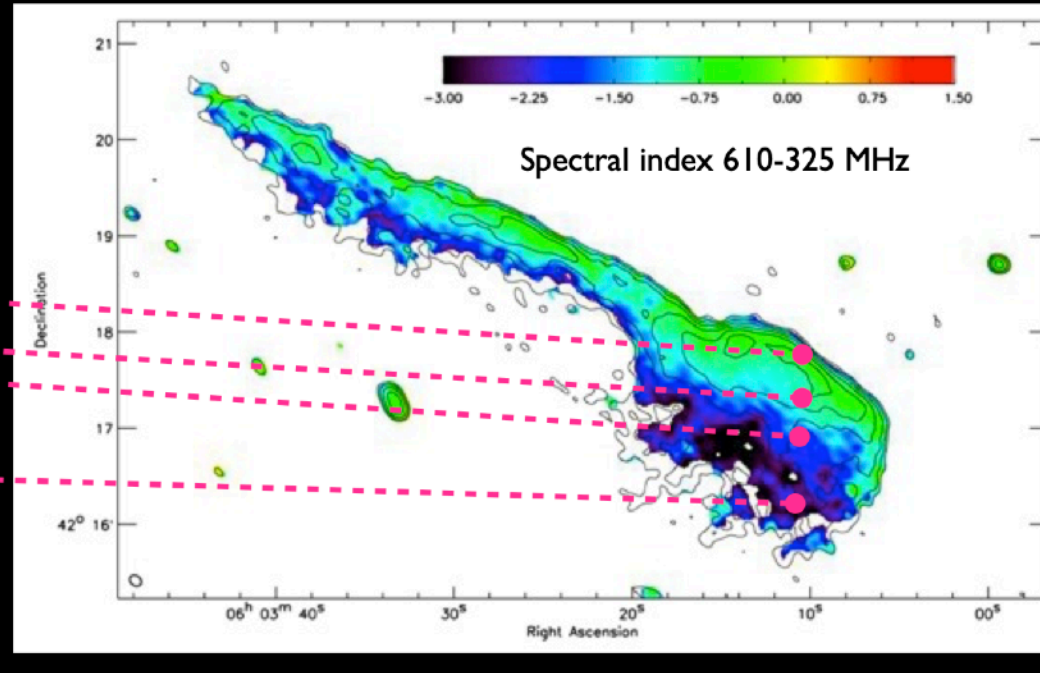
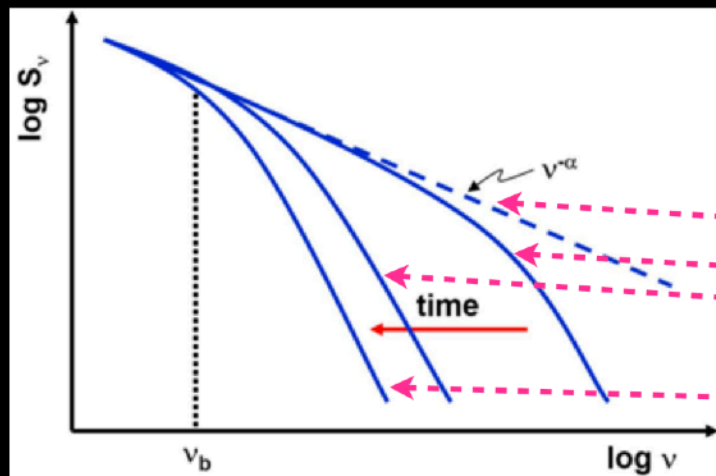
2) as the shock front advances ($V_s \sim 10^3 \text{ km/s}$) the electrons downstream cool down

$$(\tau_{loss, GeV} \leq 10^8 \text{ yr})$$

(Van Weeren+ 2016)

THE DIFFUSIVE SHOCK ACCELERATION MODEL FOR RADIO RELICS

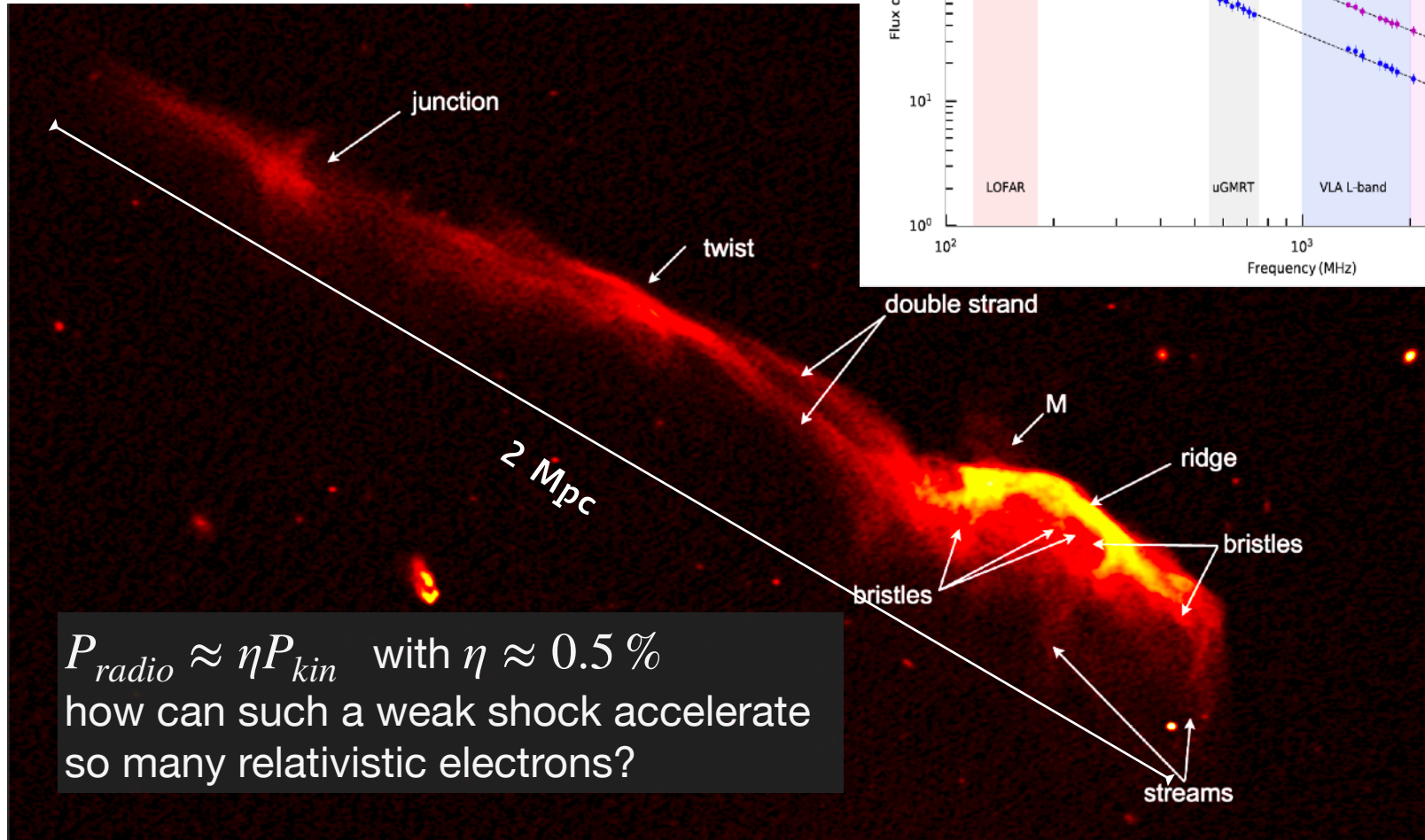
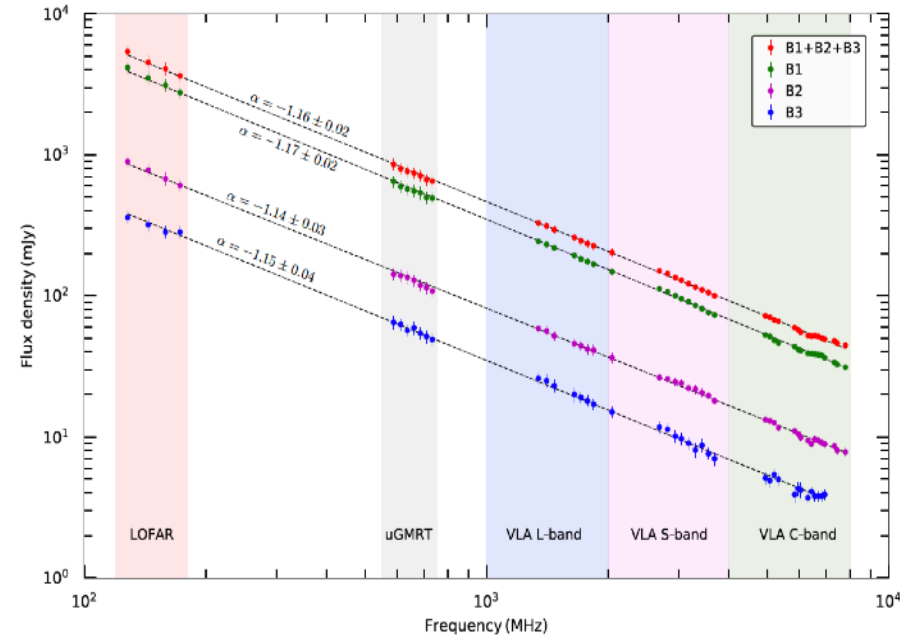
spectral ageing



3) different parts of the relic show increasingly steeper spectra moving further away from the advancing shock edge \rightarrow what we observe always is the convolution of several “families” of particles emitting along the same line of sight.

THE DIFFUSIVE SHOCK ACCELERATION MODEL FOR RADIO RELICS

The Tootbrush relic displays **an unbroken power-law** for 2 decades in frequency. Based on DSA, a radio spectrum of $I(\nu) \propto \nu^{-1.16}$ corresponds to a shock with $\mathcal{M} \sim 3.7$. From the shock velocity and density we get the shock kinetic power: $P_{\text{kin}} = \rho V_s^3 A/2$



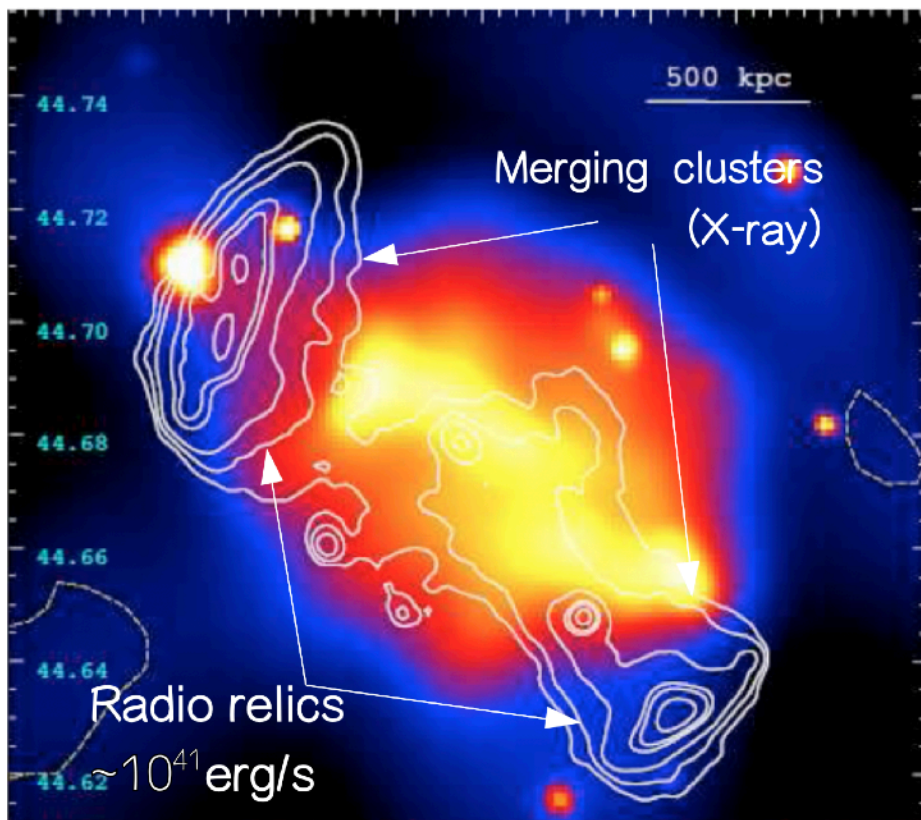
$P_{\text{radio}} \approx \eta P_{\text{kin}}$ with $\eta \approx 0.5\%$
 how can such a weak shock accelerate
 so many relativistic electrons?

(Rajpurohit+17 ...21)

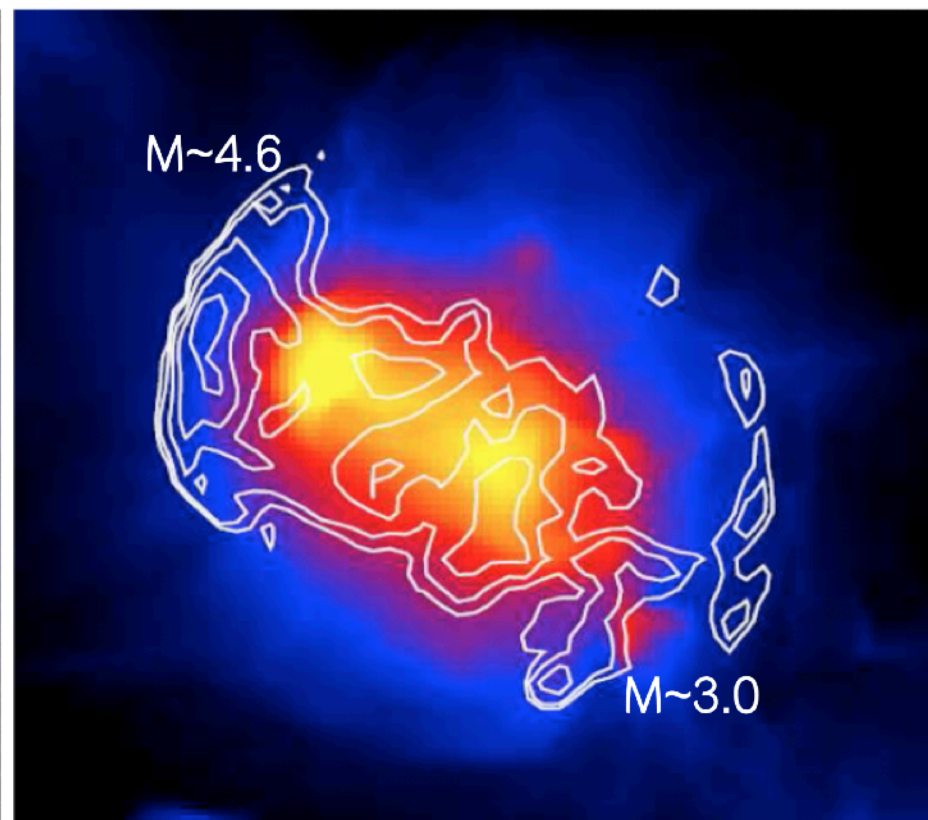
SIMULATED RADIO RELICS

Recipes have been developed to link the **energy flux across shocks** to the energy injected into CR electrons also in simulations (e.g. in Hoeft & Bruggen 2007:

$$P_{\text{radio}} \propto \eta_e(\mathcal{M}) P_{\text{kin}} B^{1+\delta/2} / (B_{\text{CMB}}^2 + B^2), \text{ with } \eta_e = \text{electron accel. efficiency)}$$



Observed (Bonafede+12)



Simulated (Vazza+10)

- an acceleration efficiency of $\eta_e \sim 10^{-5}$ will be sufficient to explain these relics if $B \sim \mu\text{G}$
- but many other relics require $\eta_e \geq 0.01 - 0.1$ at odds with Diffusive Shock Acceleration

DIFFUSIVE SHOCK ACCELERATION AND RADIO RELICS

Numerical simulations: reasonable match of morphologies/power, require multiple (re)acceleration events



THE LONG EVOLUTION OF RADIO EMITTING ELECTRONS

The Diffusion Loss Equation

For a quantitative description of the modification of particle spectra under the effect of radiative and collisional losses, it is convenient to introduce the *diffusion-loss equation* to describe the long-term (\geq Gyr) evolution of electrons

$$\frac{\partial N(E)}{\partial t} = D \nabla^2 N(E) + \frac{\partial}{\partial E} [b(E)N(E)] + Q(E)$$

THE LONG EVOLUTION OF RADIO EMITTING ELECTRONS

The Diffusion Loss Equation

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$$\frac{\partial N(E)}{\partial t} = D \nabla^2 N(E) + \frac{\partial}{\partial E} [b(E)N(E)] + Q(E)$$

diffusion coefficient in energy space

source term of CR:
shocks, AGN, SNR...

$$b(E) = A_1(\ln E/m_e c^2 + \text{const}) + A_2 E + A_3 E^2$$

ionisation losses

bremsstrahlung/
adiabatic losses

synchrotron & inverse compton
losses

THE LONG EVOLUTION OF RADIO EMITTING ELECTRONS

The Diffusion Loss Equation

For a quantitative description of the modification of particle spectra under the effect of radiative and collisional losses, it is convenient to introduce the *diffusion-loss equation* to describe the long-term (\geq Gyr) evolution of electrons

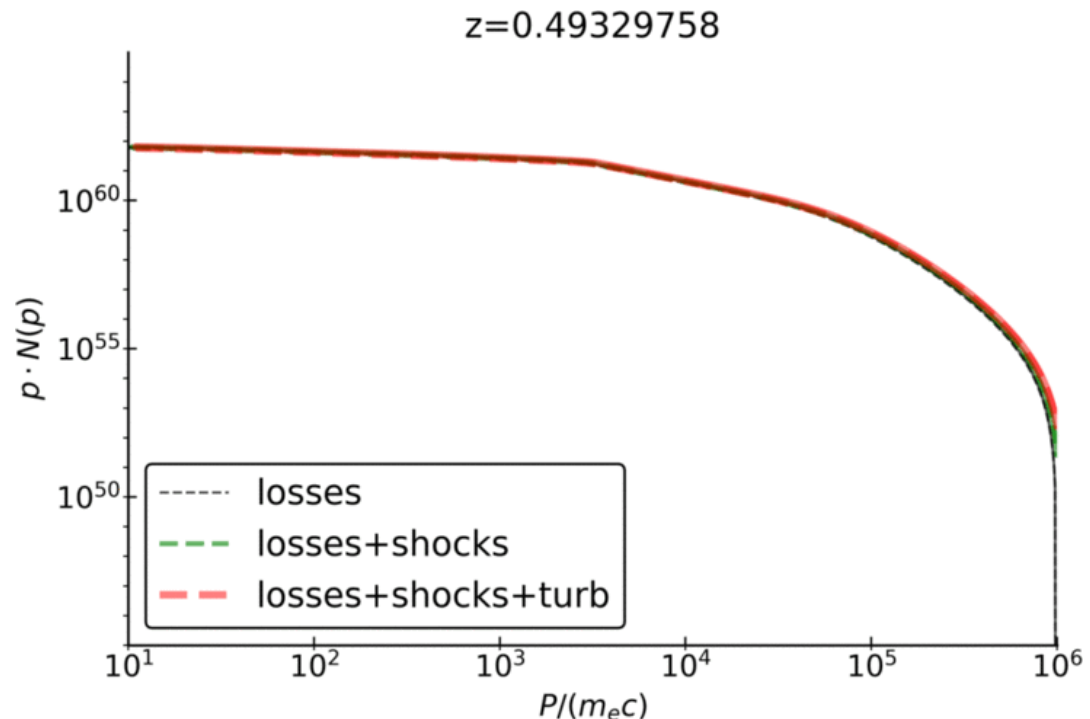
$$\frac{\partial N(E)}{\partial t} = D \nabla^2 N(E) + \frac{\partial}{\partial E} [b(E)N(E)] + Q(E)$$

numerical evolution from $z=0.5$ to $z=0.0$ of 2 populations of relativistic electrons injected by a radio galaxy.

only losses

losses+Fermi I accel.

losses+Fermi I & II accel.



RADIO HALOS AND TURBULENT RE-ACCELERATION

[Fermi II acceleration on electrons](#) in the ICM is a good explanation for Radio Halos (Brunetti+01, Takizawa+04, Cassano & Brunetti 06):

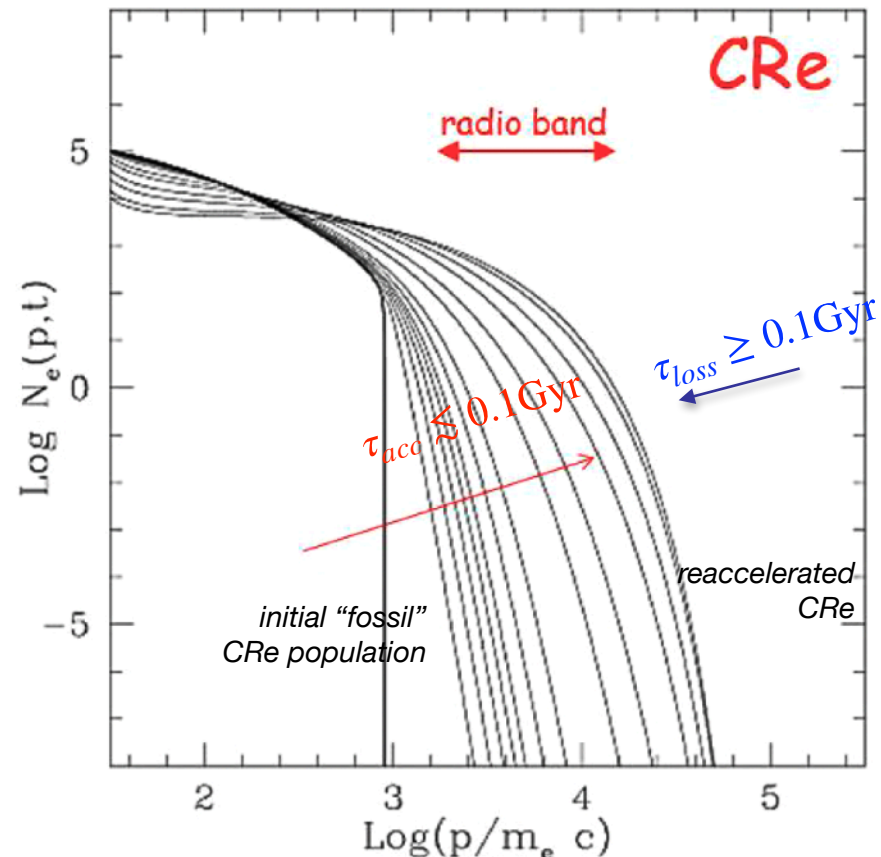
- a small fraction ($\eta_t \ll 1$) of turbulent kinetic power is channelled into CR acceleration:

$$\eta_t P_{kin} \propto \eta_t \rho \sigma_v^3$$

- but we need “fossil” mildly relativistic electrons ($\gamma \sim 10^2 - 10^3$) to be present in the ICM, as a result of previous injections (radio galaxies, shocks...)

It so, there is an approximate balance between radiative energy losses and the slow acceleration by turbulent Fermi II.

Simulated evolution of a CRe spectrum under Fermi II reacceleration



RADIO HALOS AND TURBULENT RE-ACCELERATION

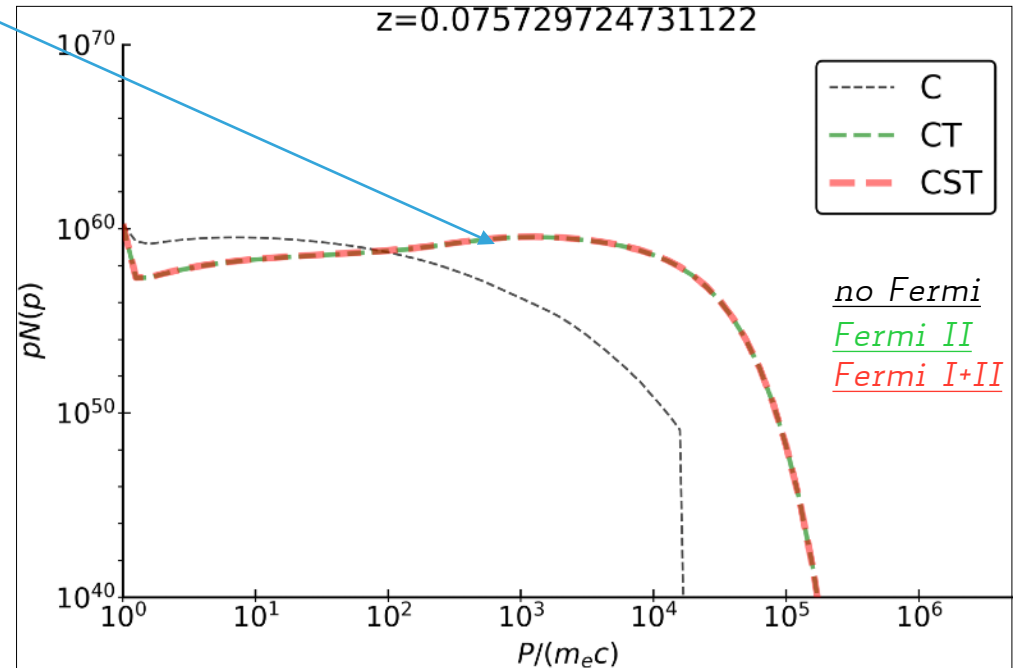
LOFAR – LBA : 50MHz



5Mpc

JVLA : 1400MHz

- *First cosmological* simulation of radio halo formation!
- Diffusion-Loss equation solved numerically for $\sim 10^5$ passive “tracers” particles advected in the simulation
- Equations integrated from $z=2$ to $z=0$, Fermi II and I acceleration
- Extended \sim Mpc radio emission naturally emerge via Fermi II re-acceleration and with $\sim \mu\text{G}$ magnetic fields

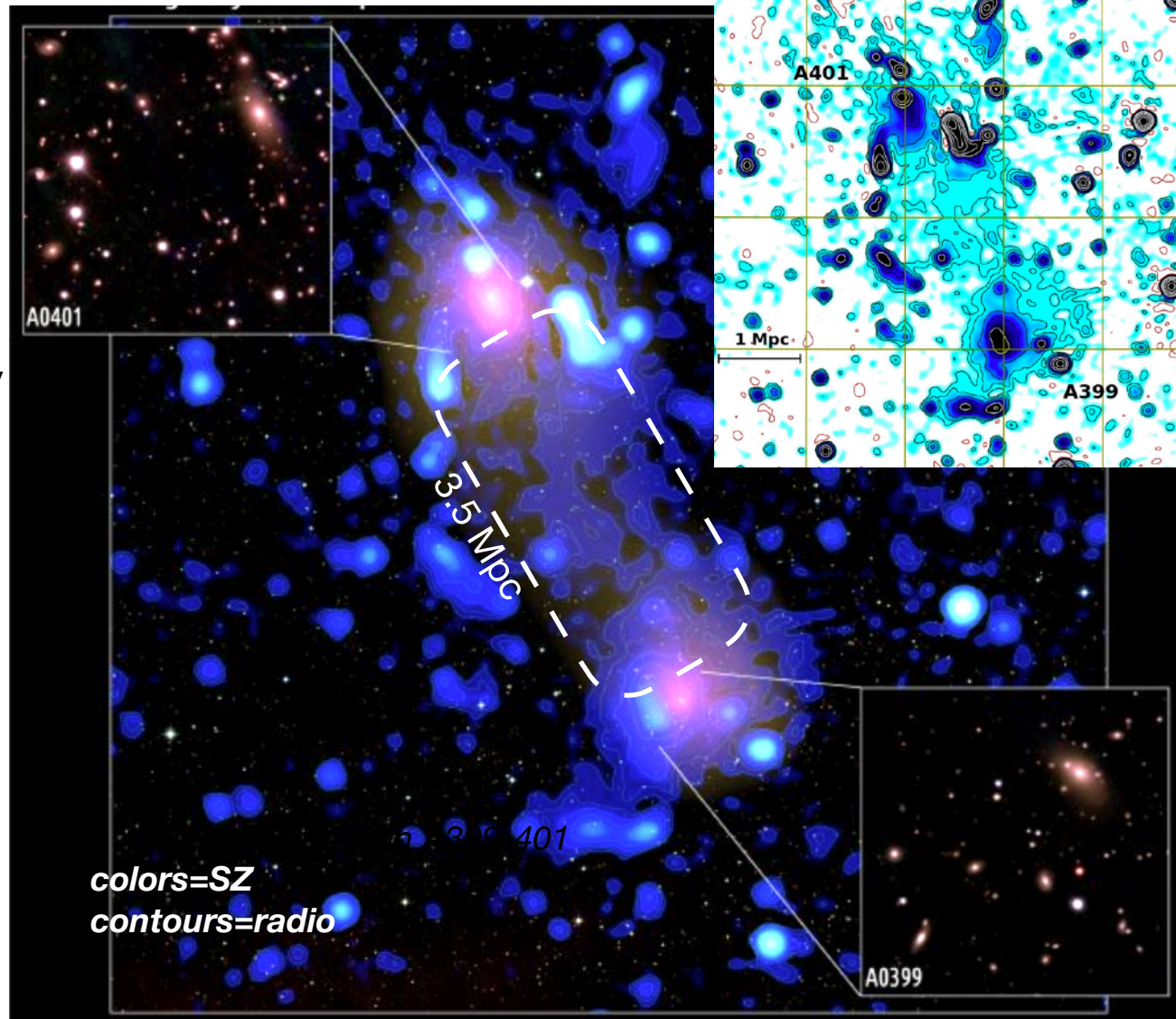


INTRACLUSTER “BRIDGES”

- LOFAR has discovered a few $\sim 3\text{Mpc}$ long “radio bridges” in between *pre-merging* clusters of galaxies.

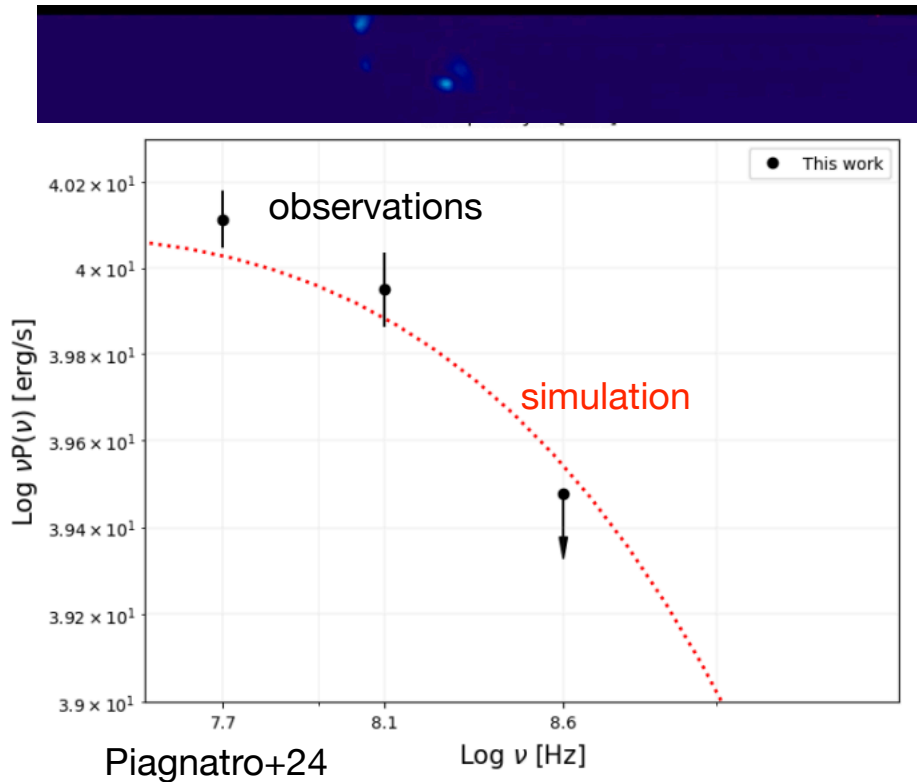
What is its origin?

- **Fermi I / DSA?** Unlikely because there are no detected shocks, but many are needed to cover the long bridge size
- **Fermi II ?** It seems more likely given the steep emission spectrum $I(\nu) \propto \nu^{-1.3}$ and morphology



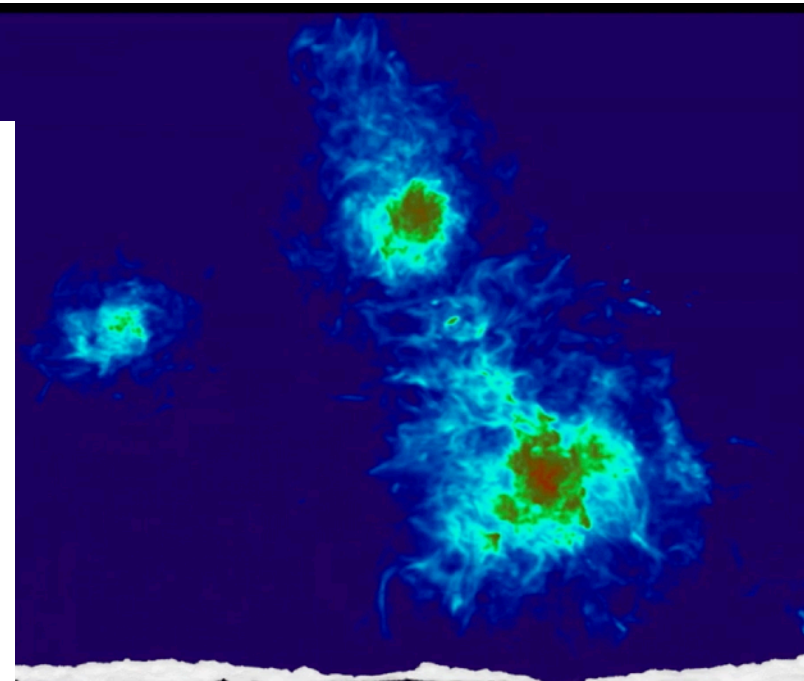
INTRACLUSTER “BRIDGES”

- ▶ Best model: Adiabatic Stochastic Acceleration (Brunetti & Lazarian 2016)
- ▶ With simulations we interpret it as a $\sim 1\text{Gyr}$ “transient” epoch with large kinetic power in solenoidal turbulent motions, with $B \geq 0.5\mu\text{G}$ (Balboni+23)
- ▶ $P_{\text{radio}} \approx \eta P_{\text{kin}}$ with $P_{\text{kin}} \sim 10^{45}\text{erg/s}$ and $\eta \sim 10^{-5}$ (dissipation on CR and B-fields)



Piagnatro+24

Log ν [Hz]



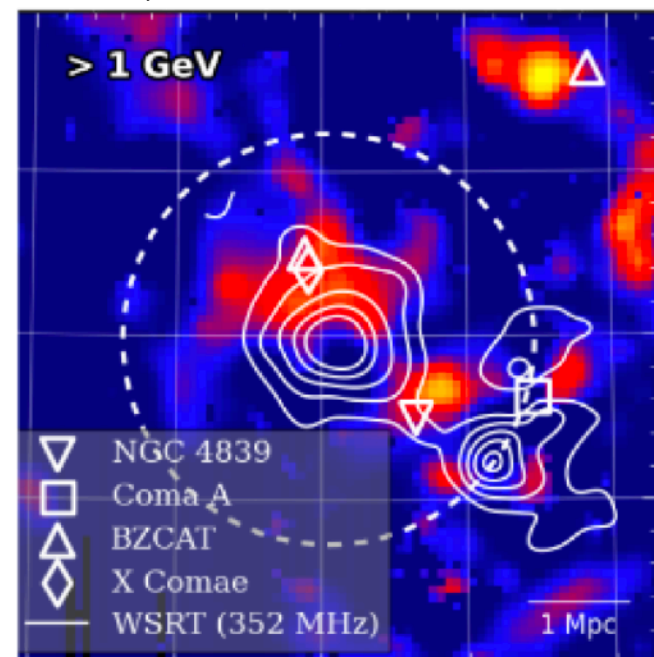
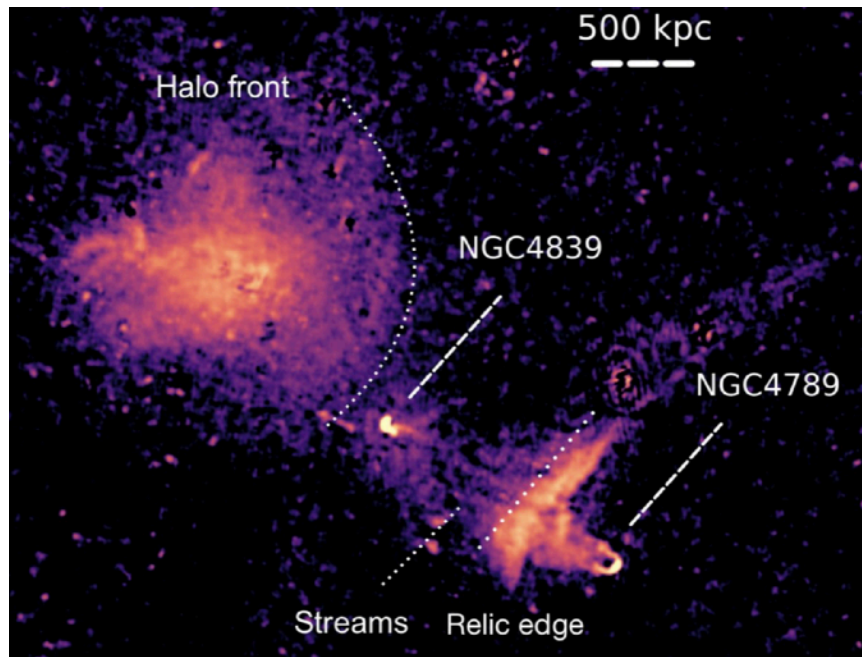
observation of the A399-A401
bridge:

turbulence has boosted the radio emission, while the X-ray emission remains very low.

WHERE ARE THE COSMIC RAY PROTONS?

- The mystery of “missing” cosmic ray **protons** (from γ -rays) is exacerbated by the fact that instead we have evidence of plenty of cosmic ray **electrons** (from **radio** observations).
- Why are cosmic ray electron accelerated, and not cosmic ray protons? Does this pose any problem to the DSA model of cluster shocks?

Coma cluster in radio (LOFAR, Bonafede+20) & in γ -rays (FERMi, Ackermann+13)



(the only γ -ray emission is due to AGN)

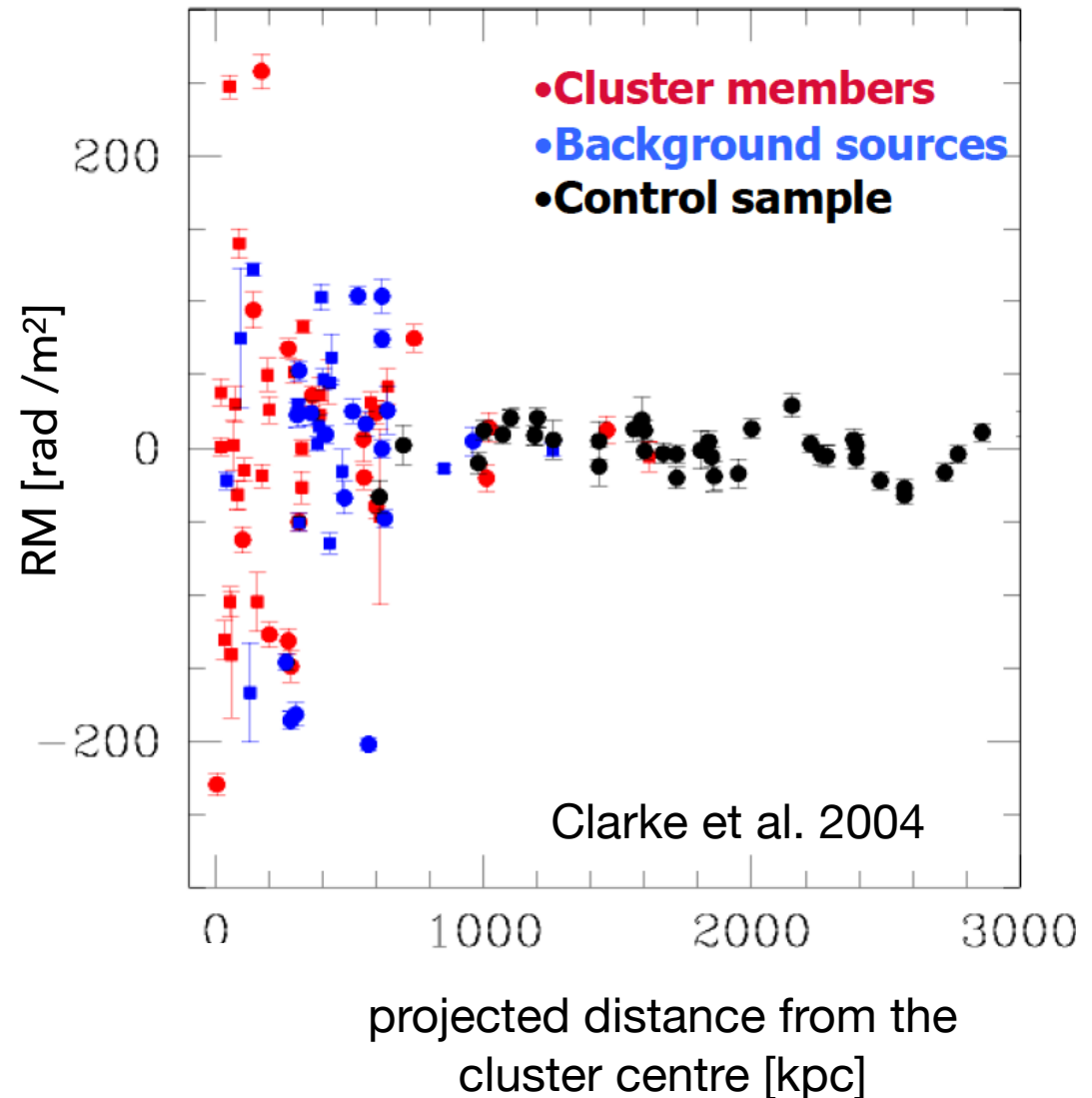
Plausible proposed solutions (but nothing sure):

- a) **weak shocks in the ICM are efficient in accelerating electrons and not protons** (need Particle in Cell simulations to investigate this)
- b) most of emissions require seed **cosmic rays from AGNs, and AGN jets are mostly leptonic**

GALAXY CLUSTERS : FARADAY ROTATION

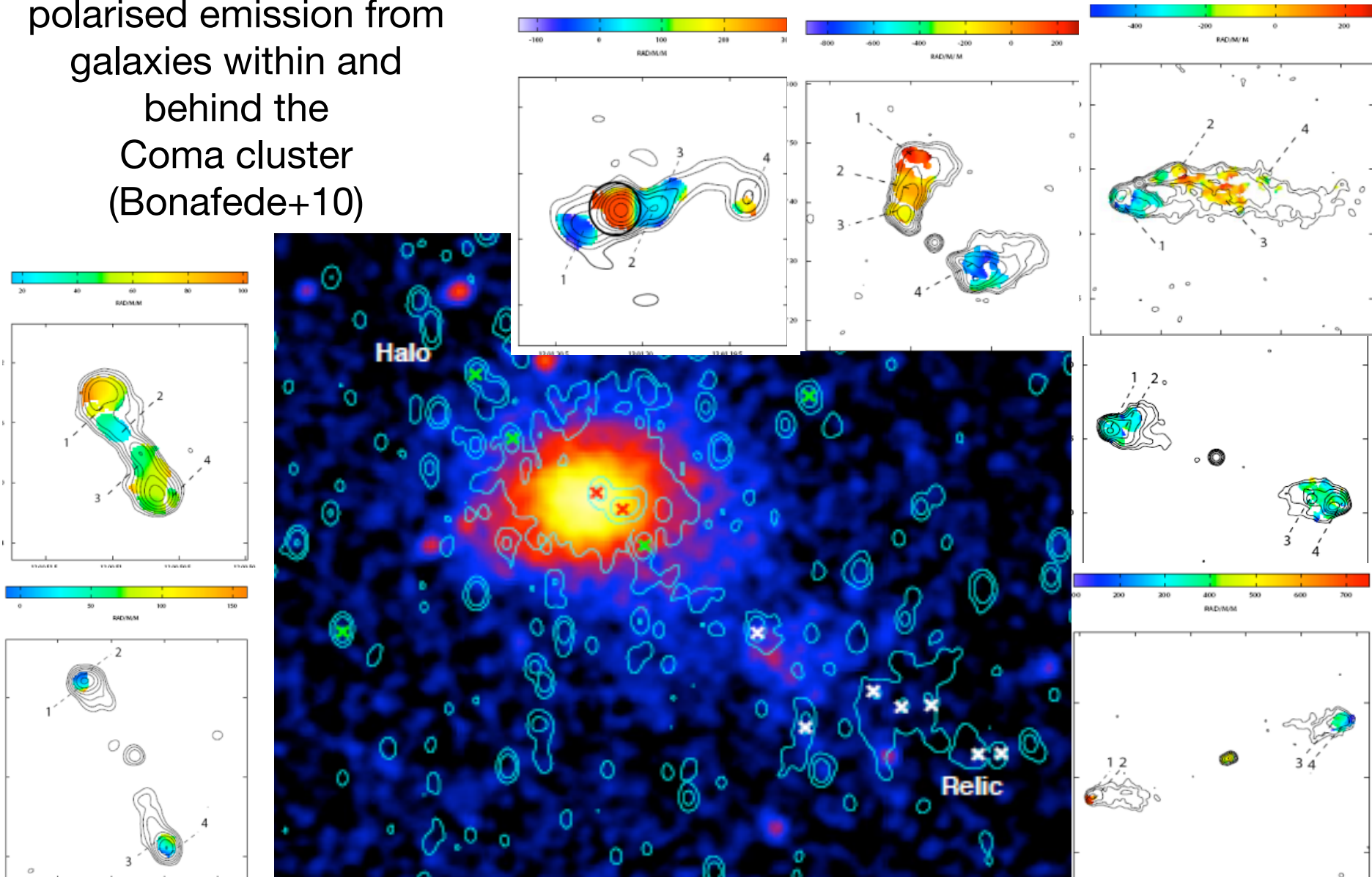
Sources observed
through the cluster
and cluster sources:

there is a **strong**
trend for sources
projected onto
clusters to have RM
much larger than
galaxies in the field



GALAXY CLUSTERS : FARADAY ROTATION

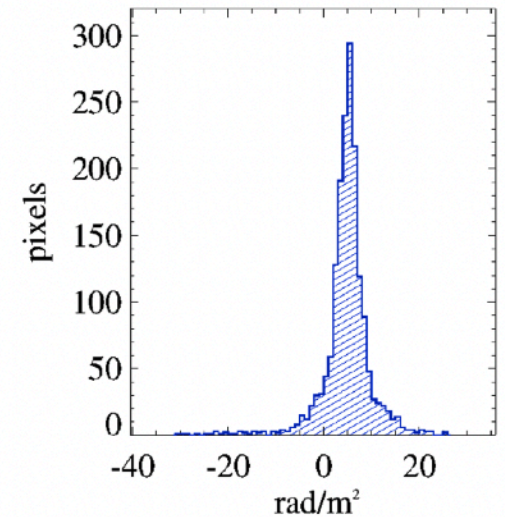
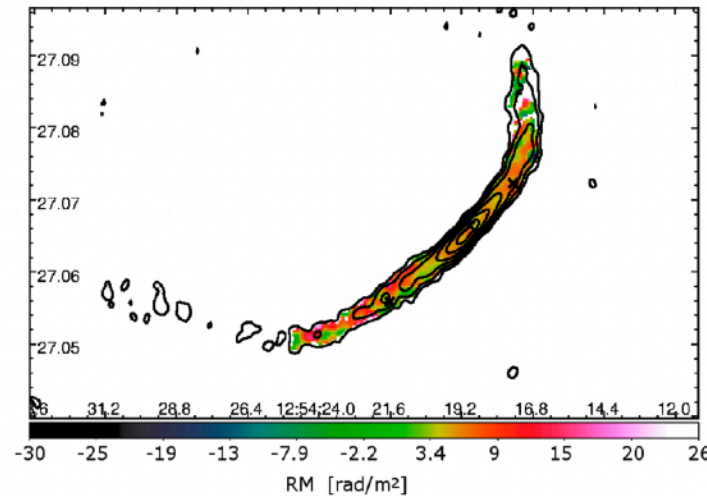
Example:
polarised emission from
galaxies within and
behind the
Coma cluster
(Bonafede+10)



GALAXY CLUSTERS : FARADAY ROTATION

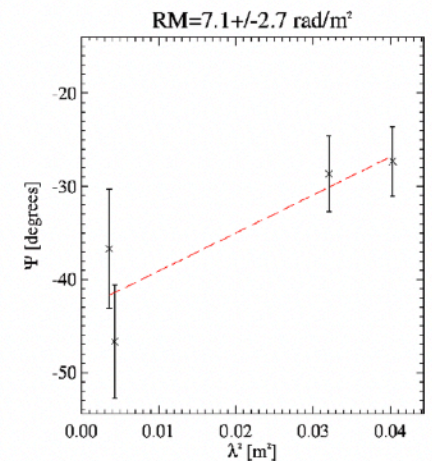
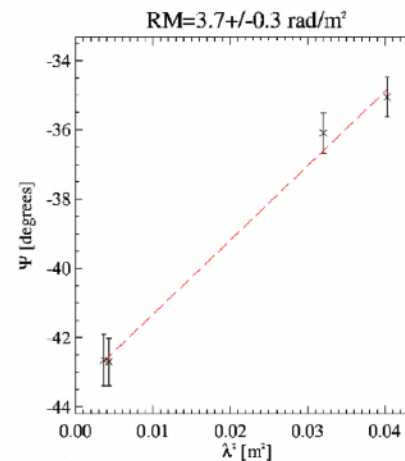
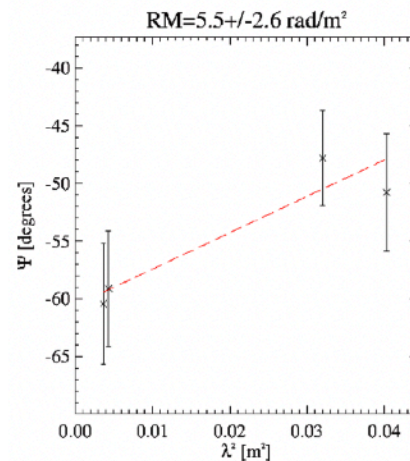
Example:
polarised emission from
galaxies within and
beyond the
Coma cluster
(Bonafede+10)

distribution of RM
for this source



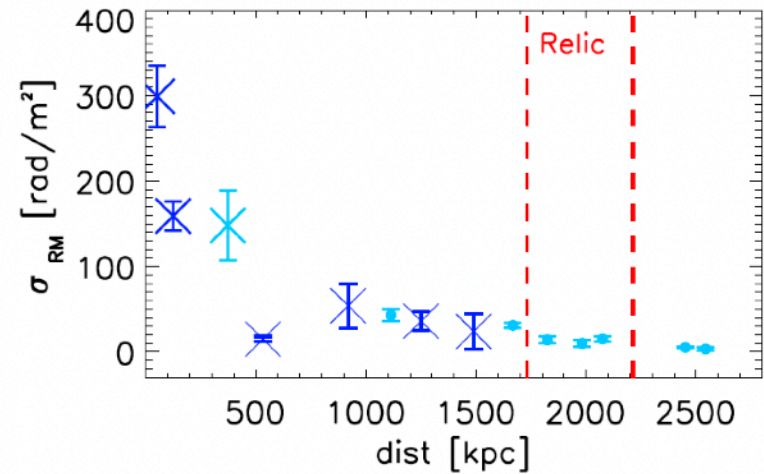
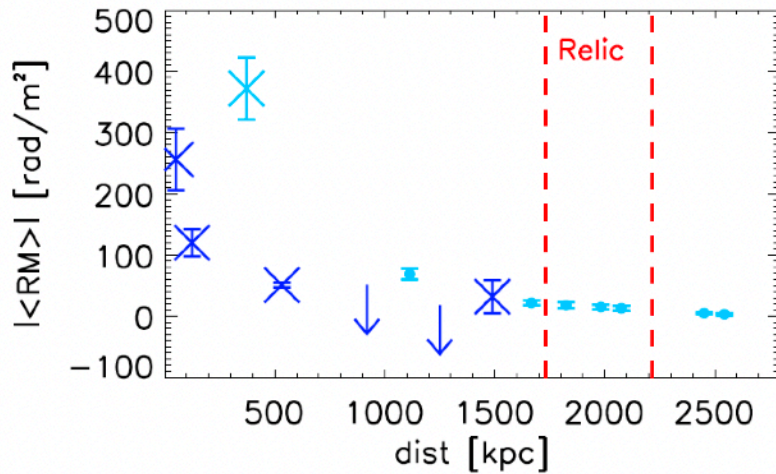
Polarisation angle
vs λ^2 relation for 3
different pixels.

the slope of this is
the Rotation
Measure



GALAXY CLUSTERS : FARADAY ROTATION

Profile of average RM (left) and dispersion of RM (right) for 14 polarised sources as a function of the distance from the centre of Coma



X-ray emission

Magnetic field

simulated RM

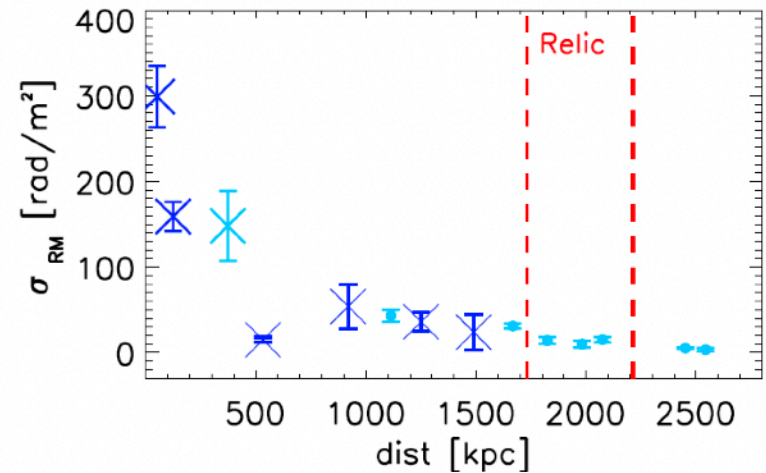
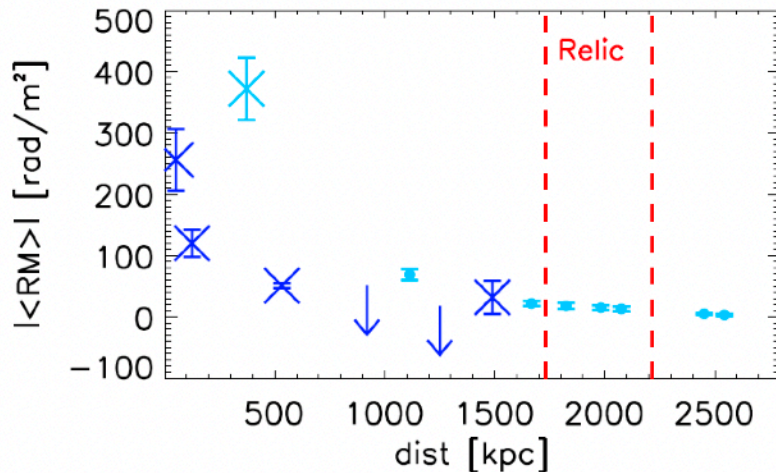
best fit β -model

random realisation of
 $P_B(k) \propto k^{n_B}$ with $B \propto B_0 n^\eta$

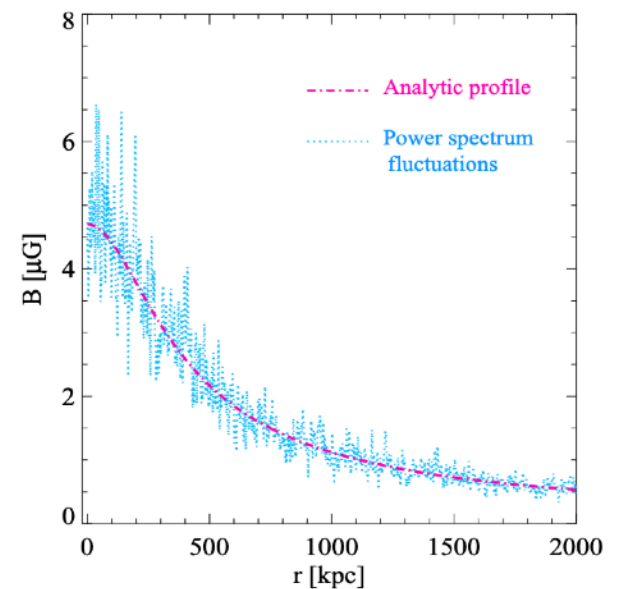
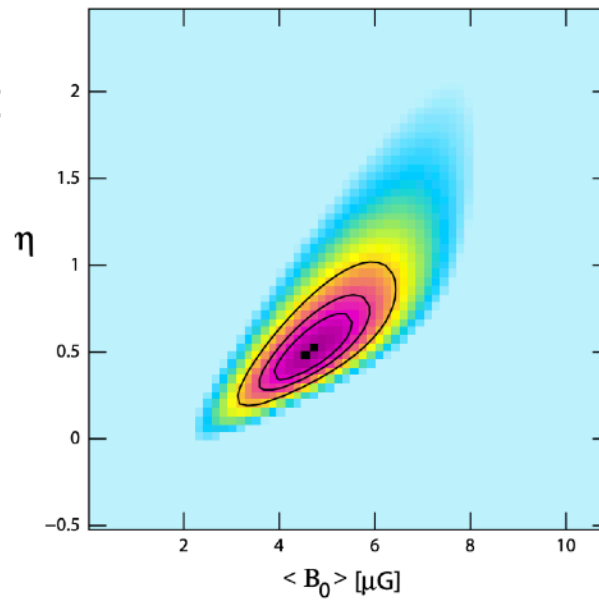
$\text{RM} \propto n B_{\parallel}$

GALAXY CLUSTERS : FARADAY ROTATION

Profile of average RM (left) and dispersion of RM (right) for 14 polarised sources as a function of the distance from the centre of Coma

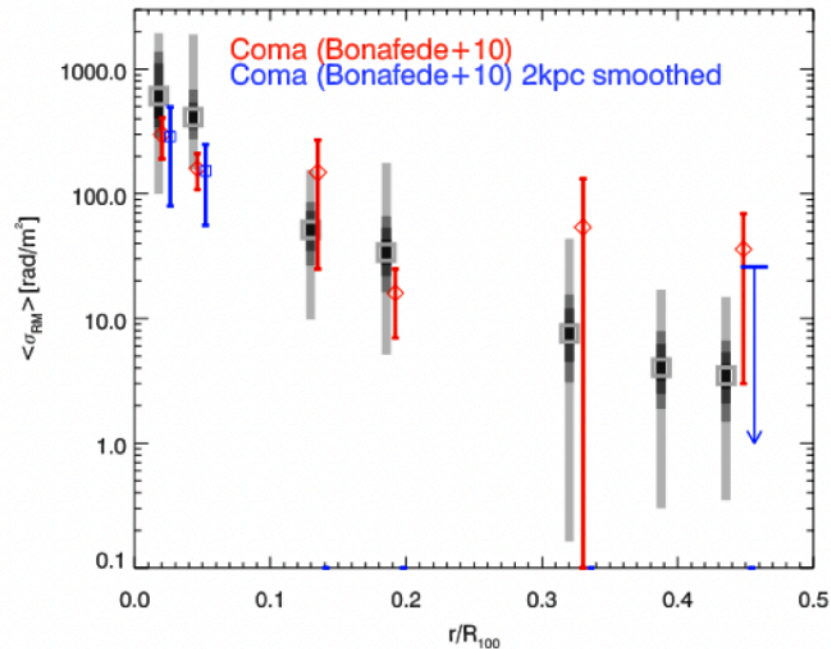
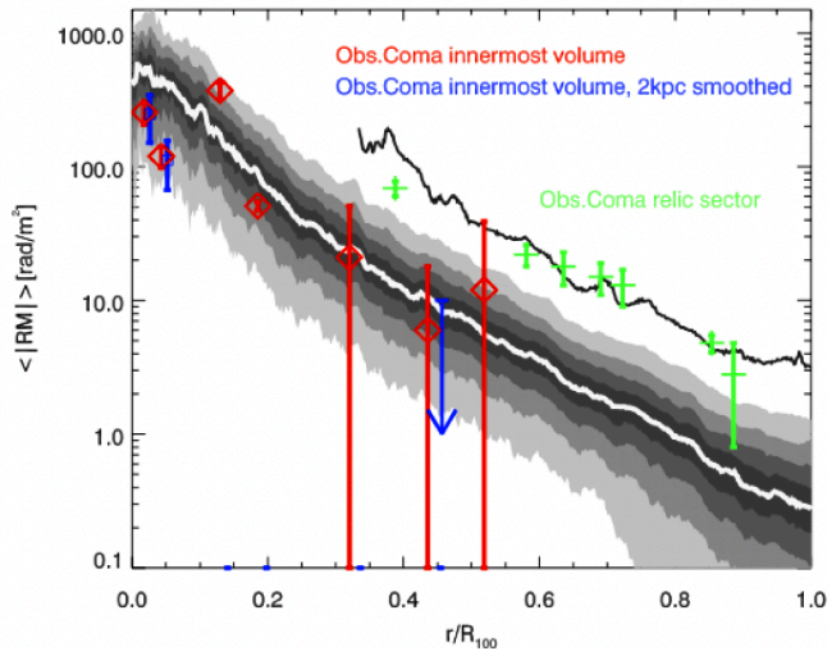


Likelihood for the best radial profile of B
 $B \propto B_0 n^\eta$
(assuming a Kolmogorov spectrum for B)

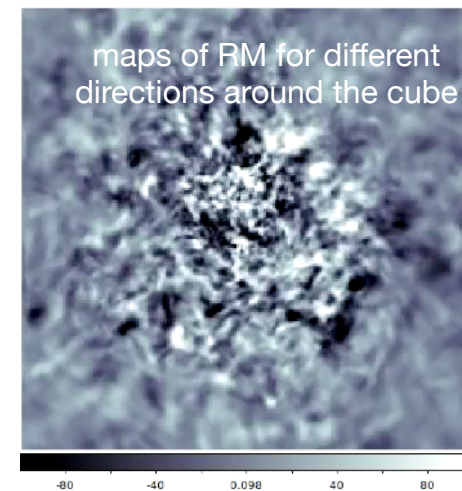
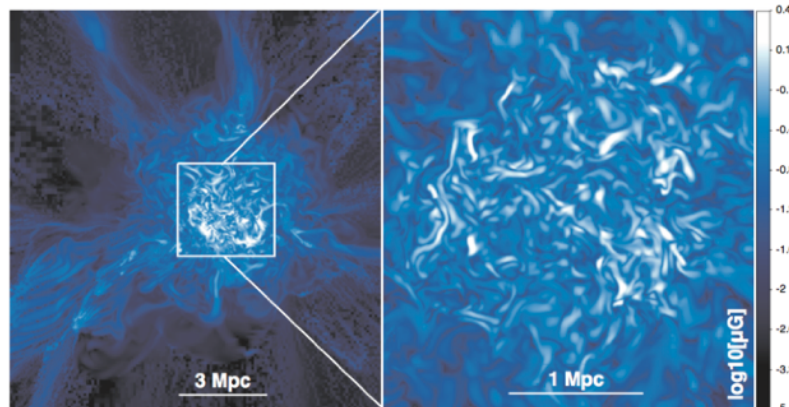


GALAXY CLUSTERS : FARADAY ROTATION

Simulated vs real RM and σ_{RM} in the Coma cluster



Modern MHD simulations of clusters of galaxies can overall reproduce the observed RM properties with $B \sim 1 - 5 \mu\text{G}$ fields



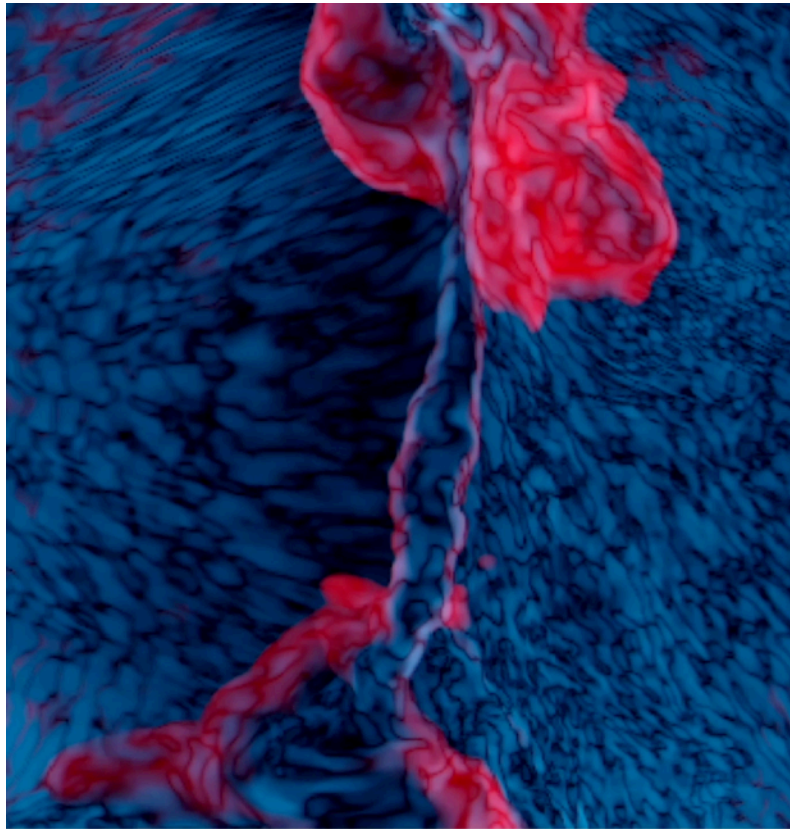
WHY NOT USING CLUSTERS OF GALAXIES TO STUDY PMFS?

Filaments: no or little dynamo,
memory of seed B_0 is **preserved**

$$\langle B_{Fila} \rangle \sim B_0 (n/n_0)^{2/3}$$

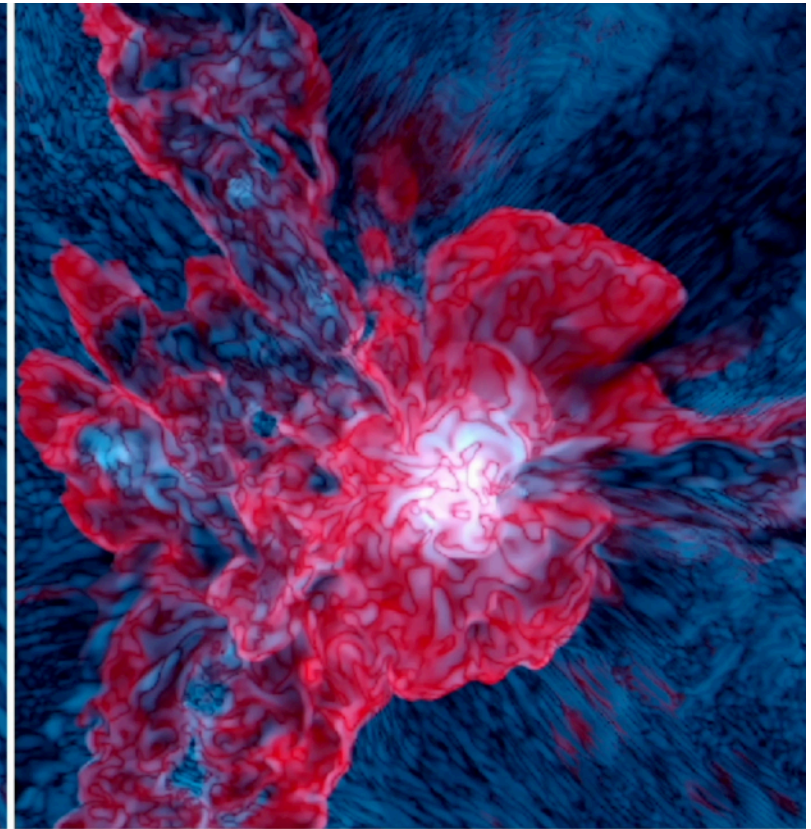
Clusters: dynamo amplification,
memory of seed B_0 is **lost**

$$\langle B_{ICM} \rangle \sim (\eta \rho_{ICM} \sigma_v^2)^{0.5} \gg B_0$$



1.68e-12 6.18e-12 1.83e-11 5.09e-11 1.38e-10 3.72e-10

red=gas temperature,

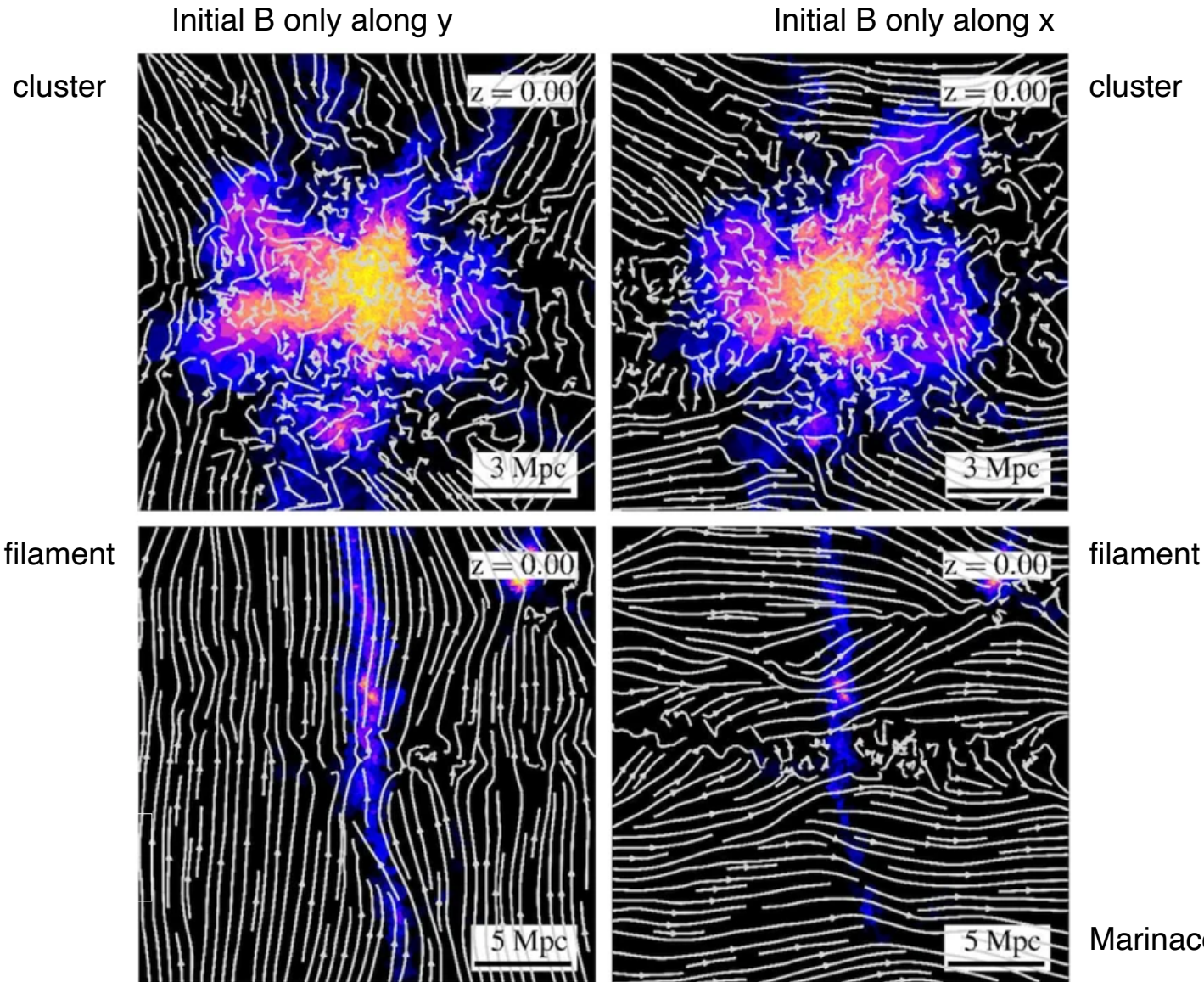


1.68e-12 6.18e-12 1.83e-11 5.09e-11 1.38e-10 3.72e-10

blue/yellow= B-field amplitude

more on dynamo tomorrow

WHY NOT USING CLUSTERS OF GALAXIES TO STUDY PMFS?



Marinacci + 2015

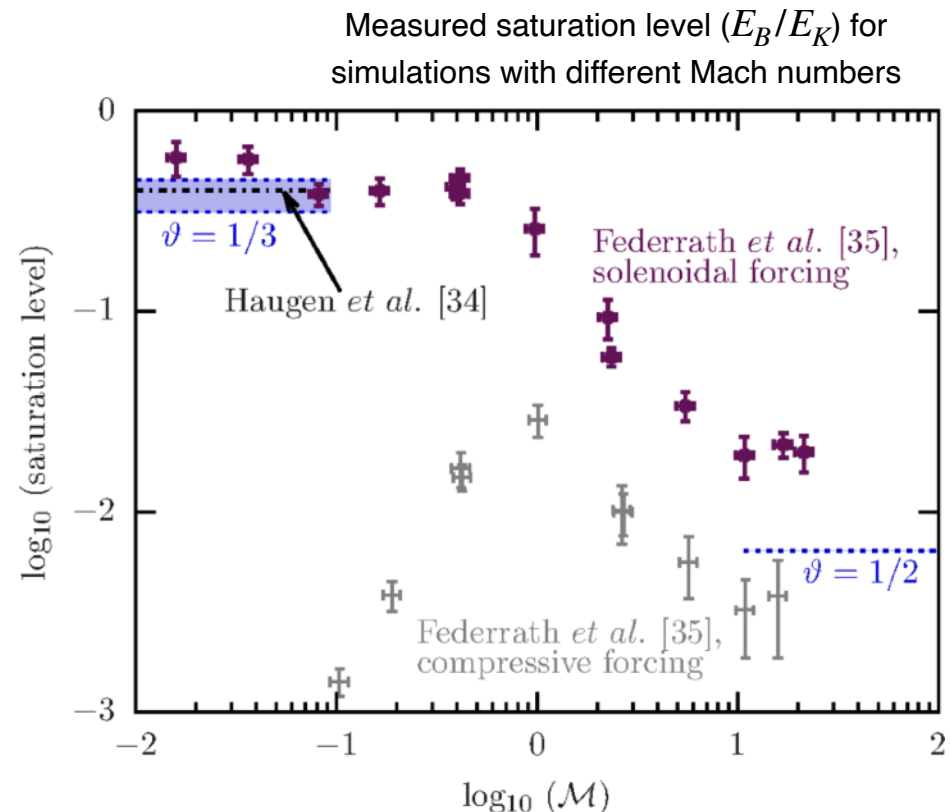
Initial memory of \vec{B}_{seed} is erased in clusters but not in filaments

more on dynamo tomorrow

WHY NOT USING CLUSTERS OF GALAXIES TO STUDY PMFS?

- Clusters are a nearly **closed systems** and B-field lines could be forced for many eddy turnover times; in filaments B-field lines can be forced only for ~ 1 eddy turnover time before being transported towards clusters
- Clusters have **more sources of astrophysical contaminations** (galaxies, AGN, jets, SNR..)
- The flow in the **ICM** is subsonic ($\mathcal{M} \leq 0.5$) and mostly solenoidal: **dynamo is very efficient and memory of seed fields is lost**
- The flow in **filaments** is trans/supersonic ($\mathcal{M} \sim 1 - 10$) and mostly compressive: **dynamo is not well developed and memory of seed fields is retained**

...However, the final answer is still rooted in numerics (what for a $\times 10^3$ higher resolution?) and in plasma properties in these very diluted environments (kinetic effects?)



SOME SUGGESTED READING

- *M. L. Norman* 2010 “*Simulating Galaxy Clusters*” <http://arxiv.org/abs/1005.1100>
- *Brunetti G. & Jones T.*, 2014, “*COSMIC RAYS IN GALAXY CLUSTERS AND THEIR NON-THERMAL EMISSION*” <https://arxiv.org/pdf/1401.7519>
- *Van Weeren R. et al.* 2019, “*Diffuse Radio Emission from Galaxy Clusters*” <https://arxiv.org/abs/1901.04496>