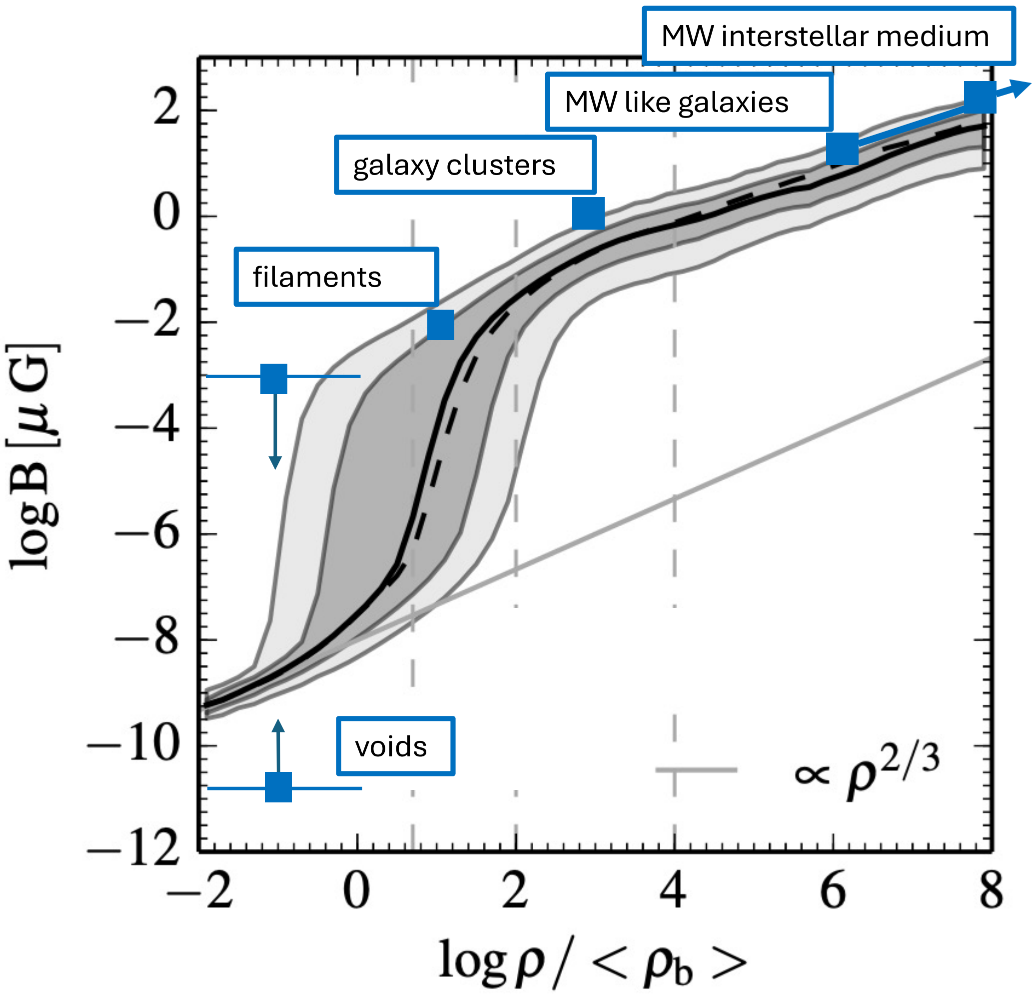
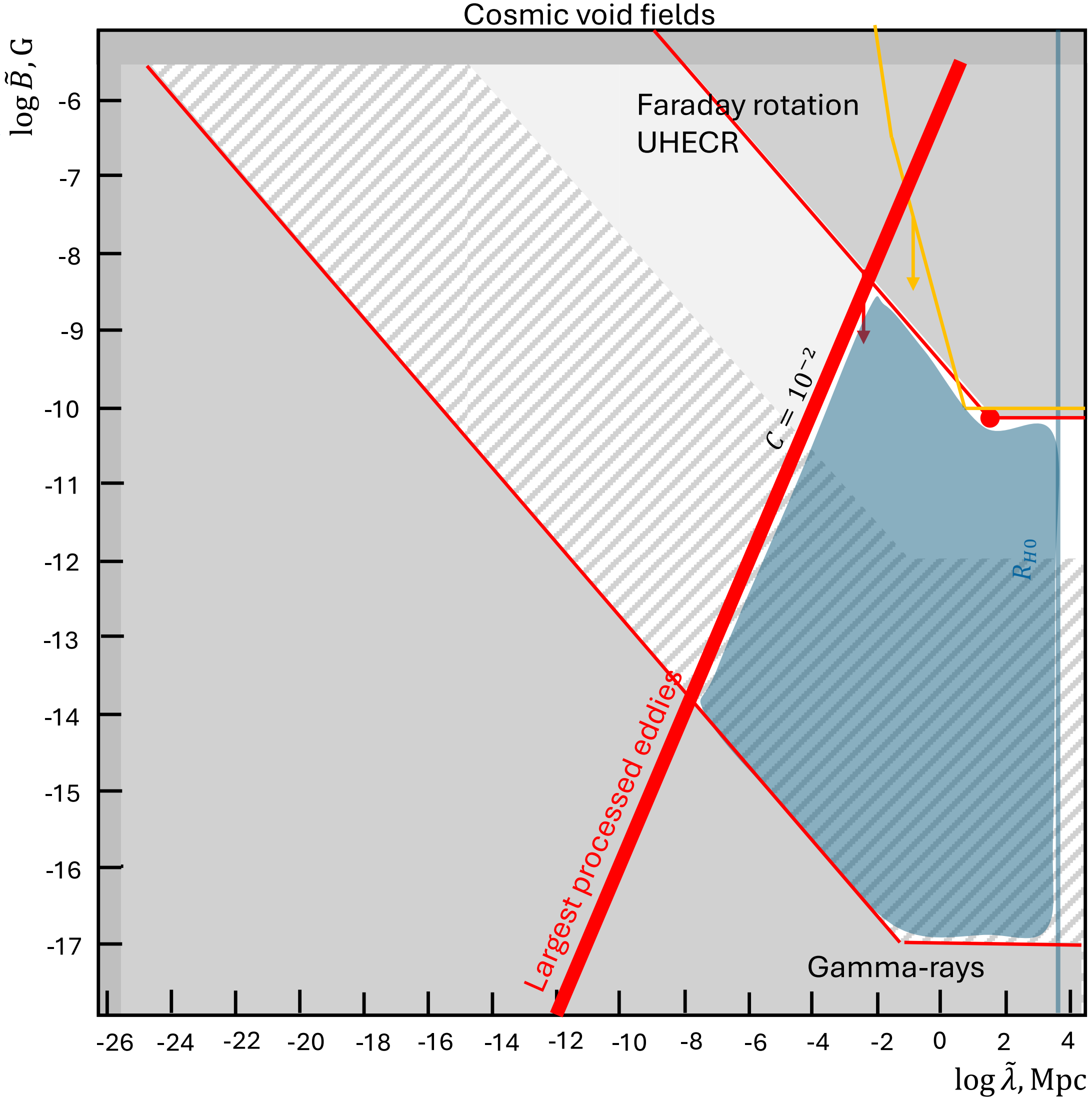
Summary lecture 4





Cosmological magnetic field observations
Lecture 5
Magnetic field measurements across cosmological epochs

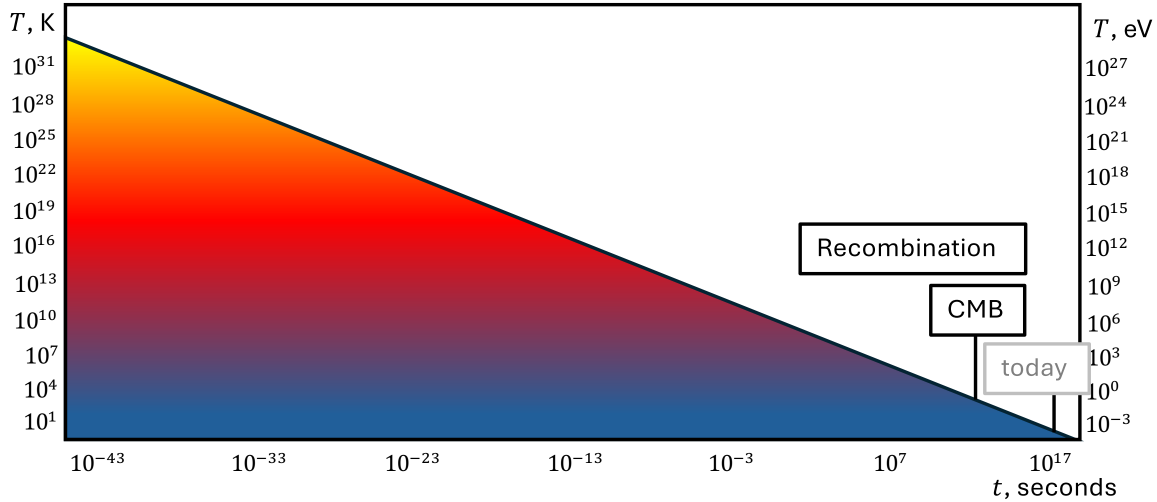
Measurement / limits from Recombination epoch
- Physics of recombination in presence of magnetic fields
- Hubble tension problem
- Global fit of Cosmic Microwave Background, Supernova Type Ia, Baryon Acoustic Oscillations data with account of magnetic field

Measurement / limits from gravitational wave data
- Gravitational wave production by magnetic fields
- Stochastic gravitational wave background
- Nano-Hz gravitational wave background
- Cosmological interpretation of gravitational wave backgorund detection

General discussion
- Is there a convincing evidence for cosmological magnetic field?
- Where would this field come from?

Recombination

The Universe at temperatures $T \ll 1$ MeV is a mixture of blackbody photon gas, neutrinos and dark matter (that are not interacting with the rest of matter) and non-relativistic gas of protons, atomic nuclei and electrons.



As soon as the temperature drops below the ionisation energy of hydrogen $E_I \sim 10$ eV, electrons and protons can recombine into hydrogen atoms. Densities of protons n_p, free elecrons, n_e, and hydrogen atoms, n_H, in thermal equilibrium are derived from Maxwell distribution,

$$n_i = g\left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_i}{T}\right)$$

so that

$$\frac{n_H}{n_p n_e} = \frac{1-X}{X n_p} = \frac{g_H}{g_e g_p}\left(\frac{m_H}{m_e m_p}\right)^{\frac{3}{2}}\left(\frac{T}{2\pi}\right)^{-\frac{3}{2}} \exp\left(\frac{m_p + m_e - m_H}{T}\right) \simeq \left(\frac{m_e T}{2\pi}\right)^{-\frac{3}{2}} \exp\left(\frac{E_I}{T}\right)$$

(Saha equation for, X, the ionisation fraction: $X = n_p/(n_p + n_H)$).

The number density of protons and atomic nuclei is much smaller than that of photons, $\eta_b = (n_p + n_H)/n_\gamma \simeq 10^{-10}$, $n_\gamma = (2\zeta(3)/\pi^2)T^3$. Using $n_p = X\eta_b n_\gamma = X\eta_b(2\zeta(3)/\pi^2)T^3$, one finds the ionisation fraction as a function of temperature

$$\frac{1-X}{X^2} = \eta_b \frac{\zeta(3)\, 2^{\frac{5}{2}}}{\pi^{1/2}}\left(\frac{T}{m_e}\right)^{\frac{3}{2}} \exp\left(\frac{E_I}{T}\right)$$

The ionisation fraction is $X \simeq 0.5$ when the temperature is $T \simeq 0.3$ eV.

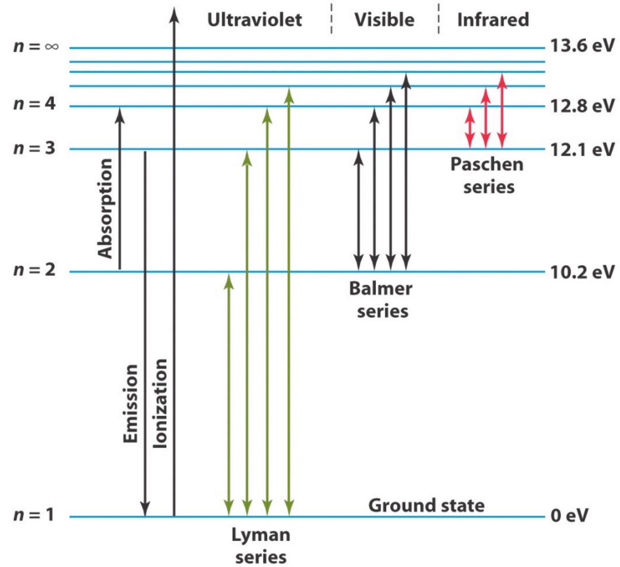
Recombination

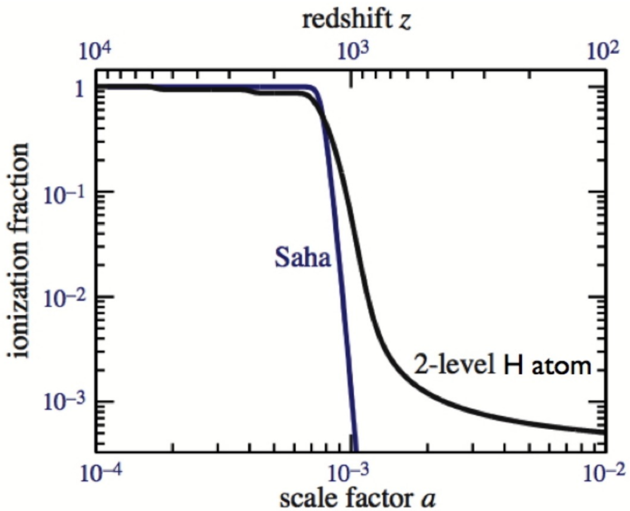
Recombination of electron and positron results in decrease of $X$and also in emission of a photon. If the energy of the photon is $E_\gamma = E_I$, the photon released at formation of the hydrogen atom would ultimately be absorbed by another hydrogen atom and would re-ionise it, increasing X. As a result, no change of X would happen.

An alternative possibility is that electron settles at an energy level $n > 1$, and than transits to the ground state releasing another photon. The energy of the two released photons is not sufficient for re-ionisation of another atom and the decrease of X is preserved.

Other than photoelectric emission / absorption, photon gas is coupled to matter through Compton scattering process, $\gamma + e^- \to \gamma + e^-$. Once the abundance of free electrons drops, this process becomes inefficient and photons "decouple" from matter.

Overall, kinetic equations for coupled photon, proton, electron and hydrogen gases needs to be solved to establish the details of Recombination dynamics.





Recombination in the presence of magnetic field

Apart from photon gas, proton+electron gas is coupled to magnetic field. Lorentz force induces plasma motions, guided by the Euler equation with the Lotentz force term

$$\rho_b \partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} + c_s^2 \nabla\rho_b = -\frac{\vec{B}\times(\nabla\times\vec{B})}{4\pi} - \alpha\rho_b\vec{v}$$

where $\alpha\vec{v}$ is the "photon drag" term due to the Compton scattering of photons with mean free path $l_C = (\sigma_T n_e)^{-1}$. This expression is valid on scales much smaller than the mean free path of photons. The coefficient $\alpha = (4\rho_\gamma/3\rho_b) l_C$. c_s is the baryonic speed of sound, $c_s^2 = p_b/\rho_b$.

Magnetic field is dynamically important on distance scales on which the first term o the r.h.s. is comparable to the second:

$$\frac{B^2}{4\pi\rho_b L} \sim \alpha v$$

On these scales, the velocity $v \sim v_{A,b}^2/\alpha L$ is established ($v_{A,b} = B/\sqrt{4\pi\rho_b}$ is the Alfven velocity).

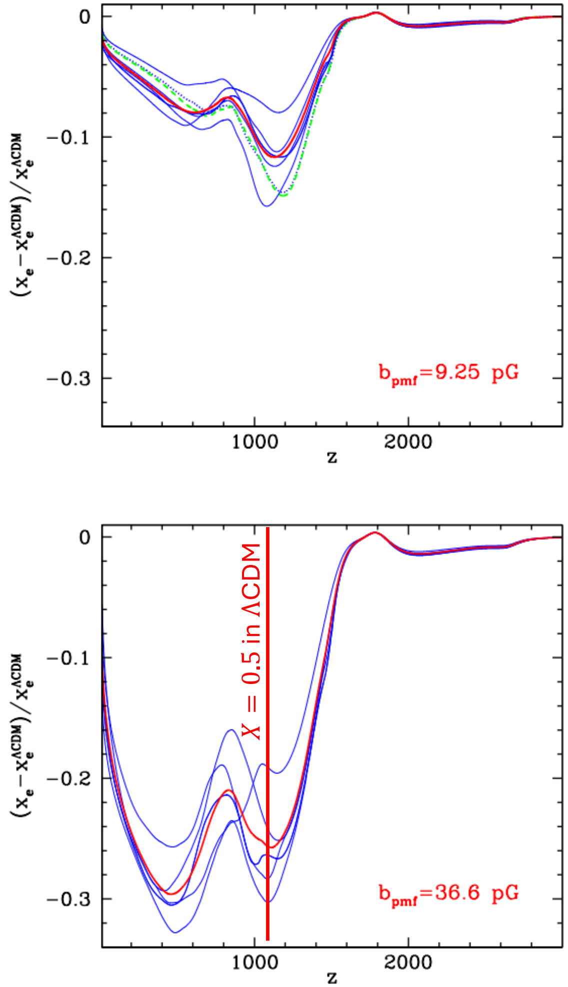
Plasma motions lead to formation of inhomogeneities in the baryonic gas. Using the last term on the l.h.s. of Euler equation, one can estimate the level of the density fluctuations

$$\frac{c_s^2}{L}\frac{\delta\rho_b}{\rho_b} \sim \alpha v \sim \frac{v_{A,b}^2}{L}, \qquad \frac{\delta\rho_b}{\rho_b} \sim \frac{v_{A,b}^2}{c_s^2}$$

If magnetic field is strong enough,

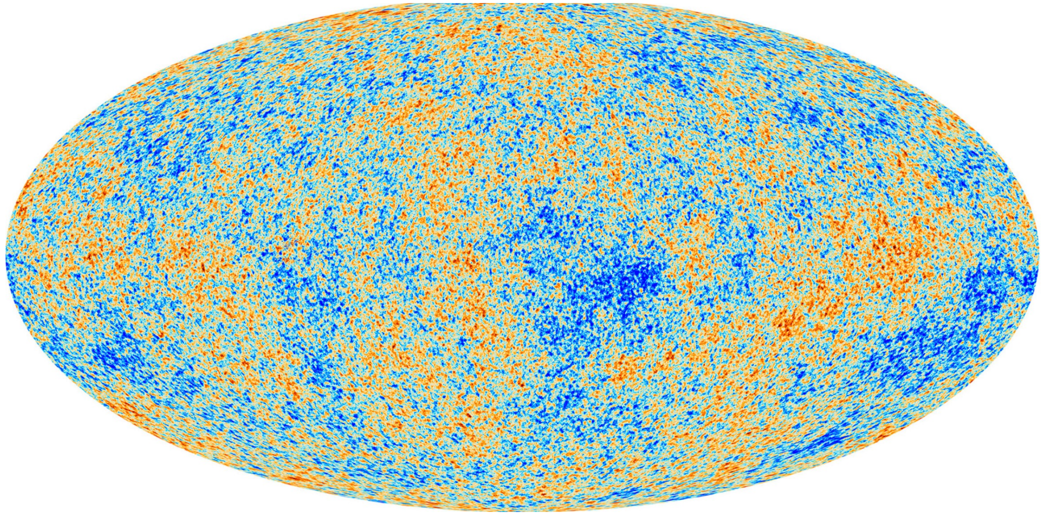
$$\frac{B^2}{4\pi\rho_b} \gg \frac{T^2}{m_p^2}, \qquad B \gg \sqrt{4\pi\, m_p \eta_b n_\gamma}\frac{T}{m_p} \sim \sqrt{\frac{8\zeta(3)}{\pi}}\frac{\eta_b^{\frac{1}{2}} T^{\frac{5}{2}}}{m_p^{\frac{1}{2}}}$$

fluctuations of the baryonic density are of the order of one. In this case, ionisation fraction is decreased in over-dense regions and recombination completes earlier.



Abel et al., https://arxiv.org/abs/2503.09599

Cosmic Microwave Background

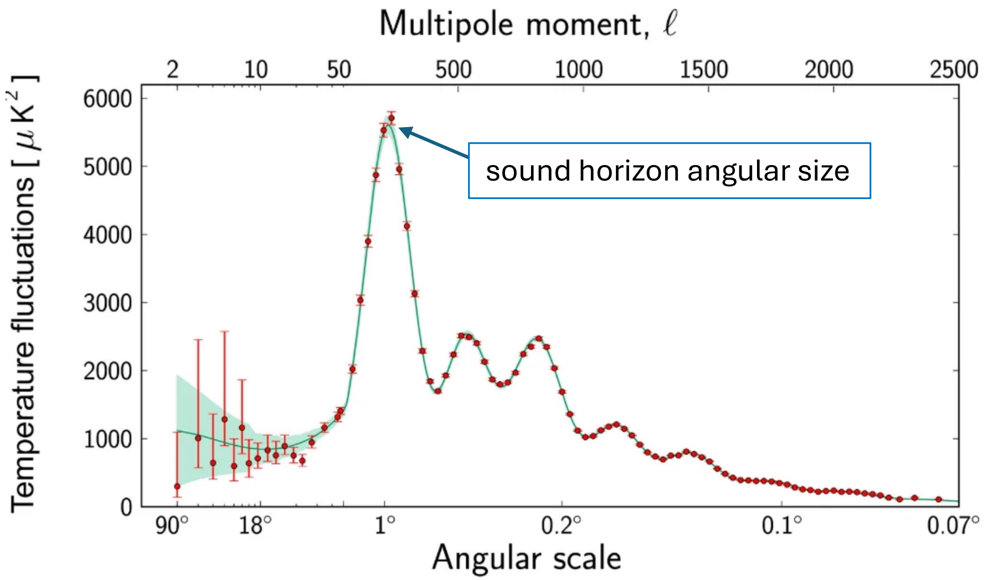


Photons remained coupled to baryonic gas as long as there were enough free electrons in the medium so that the rate of the Compton scattering reaction $\gamma + e^- \to \gamma + e^-$, $R_C = \sigma_T n_e$ remained higher than the expansion rate H.

Fluctuations of the baryon-photon gas density had the form of sound waves propagating with the speed $c_s^2 = p/\rho = w = 1/3$. The longest wavelength of such perturbations at the moment of photon decoupling was $\lambda_s = c_s t_H$.

This wavelength (the size of the “sound horozon” at the ”last scattering” of photons) is imprinted in the angular power spectrum of CMB fluctuations.

Earlier completion of recombination in presence of magnetic field would decrease the size of the sound horizon and shift the peak of the CMB angular power spectrum.



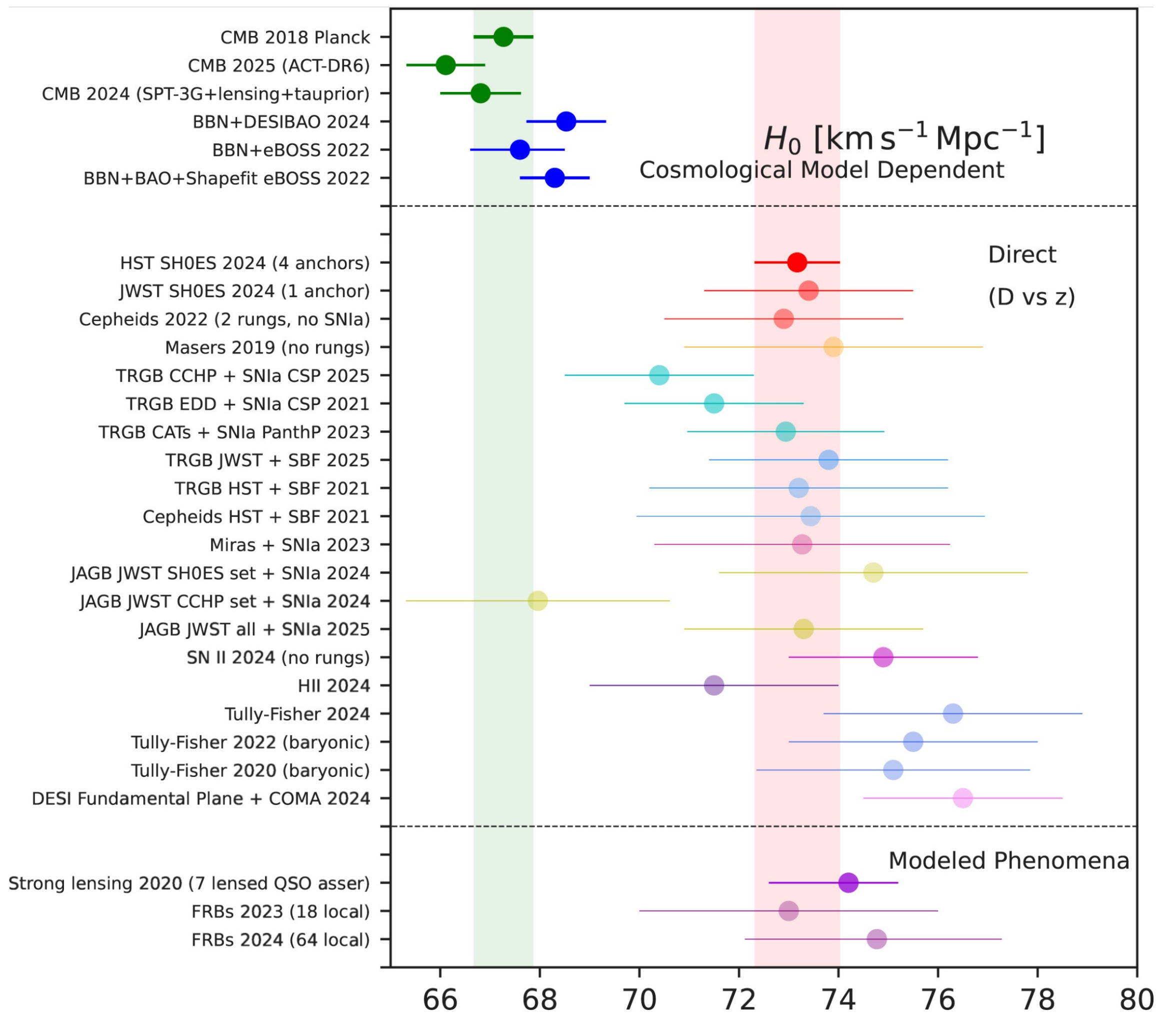
”Hubble tension” problem

The expansion rate of the present-day Universe, H_0, can be measured using a variety of techniques that can be divided onto two broad types:

- Measurements based on the data at small z
- Measurements based on the data at large z

Small-z measurements include e.g. observations of Type Ia supernova in the Universe up to redshifts $z \sim 1$.

These different types of measurements are inconsistent at $> 5\sigma$ significance level.



https://arxiv.org/abs/2504.01669

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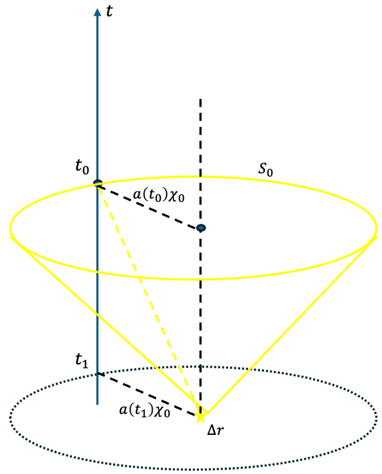
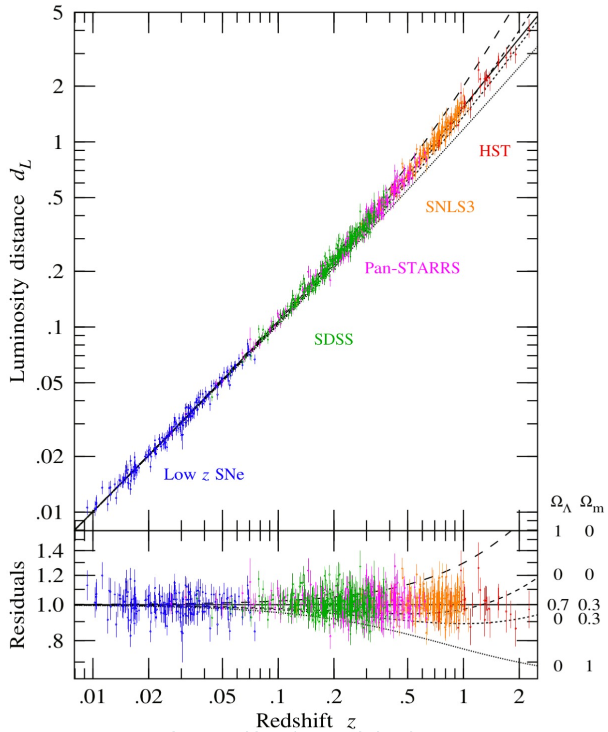




https://arxiv.org/abs/1710.00845

The Hubble parameter in SN Ia observations is derived from measurements of luminosity distance-redshift relation:

$$D_L = (1+z)D_c \simeq \frac{1}{H_0}\left(z + \frac{1}{2}(1-q_0)z^2\right)$$

where

$$D_c = \int_a^1 \frac{da'}{H(a')a'^2} = \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')^2\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}$$

The main assumption of the measurement is that there exist ”standard” or “standartizeable candles” for which the luminosity distances are measurable. Supernova Type Ia are considered to be such sources.

”Hubble tension” problem

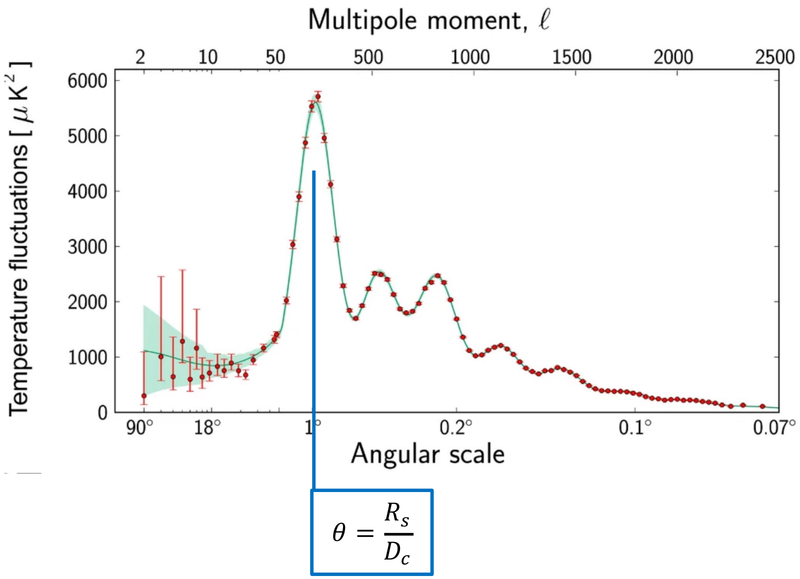
In CMB measurements, the Hubble parameter is derived from the measurement of the angular scale of CMB fluctuations, $\theta = R_s/D_c$, where $R_s = R_H/\sqrt{3}$ is the ”sound horizon” at the moment of decoupling of photons and

$$D_c = \int_a^1 \frac{da'}{H(a')a'^2} = \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')^2\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}$$

Is the comoving distance to the “last scattering surface”, of photons.

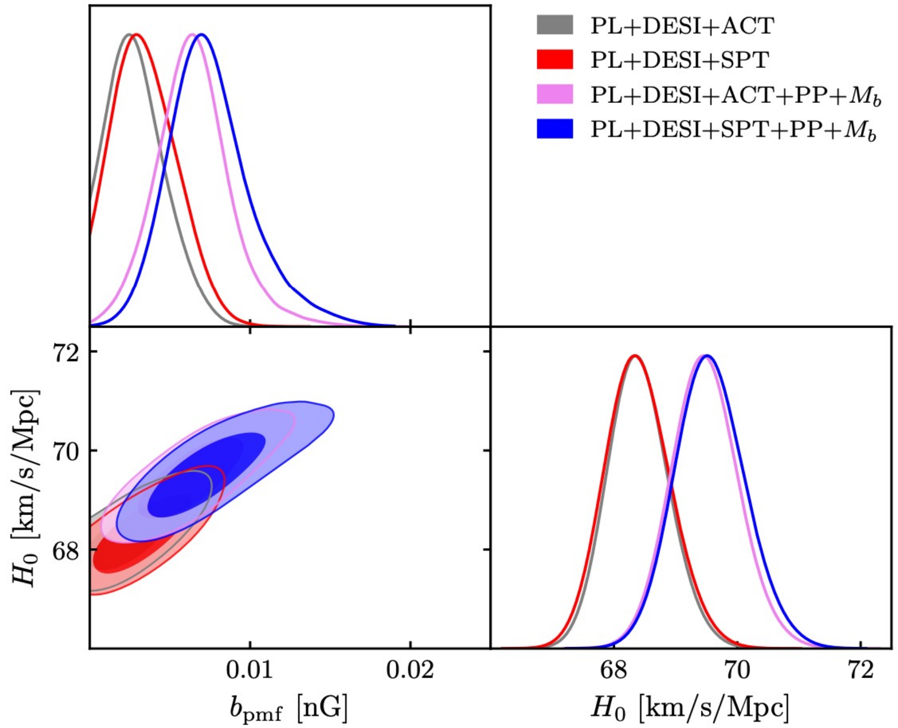
Increase in H_0 would lead to decrease of D_c and increase of θ (that is the measured quantity). Same θ can be found if R_s is smaller than derived from ΛCDM model.

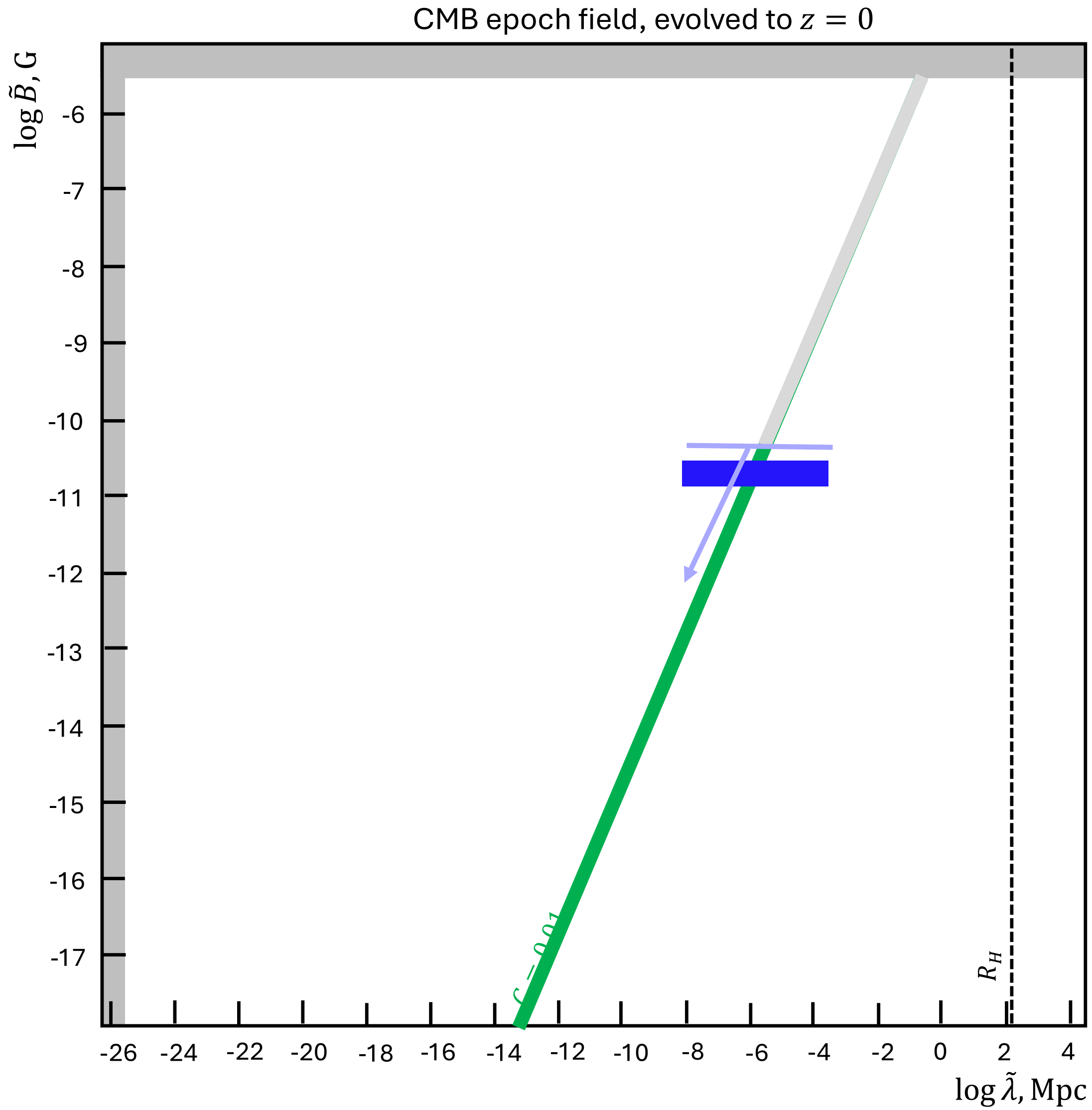
Earlier recombination in presence of magnetic field leads to smaller R_s.



$b\Lambda$CDM model

https://arxiv.org/abs/2503.09599





Global fit to the CMB+LSS data favours non-zero B and relaxes the Hubble tension

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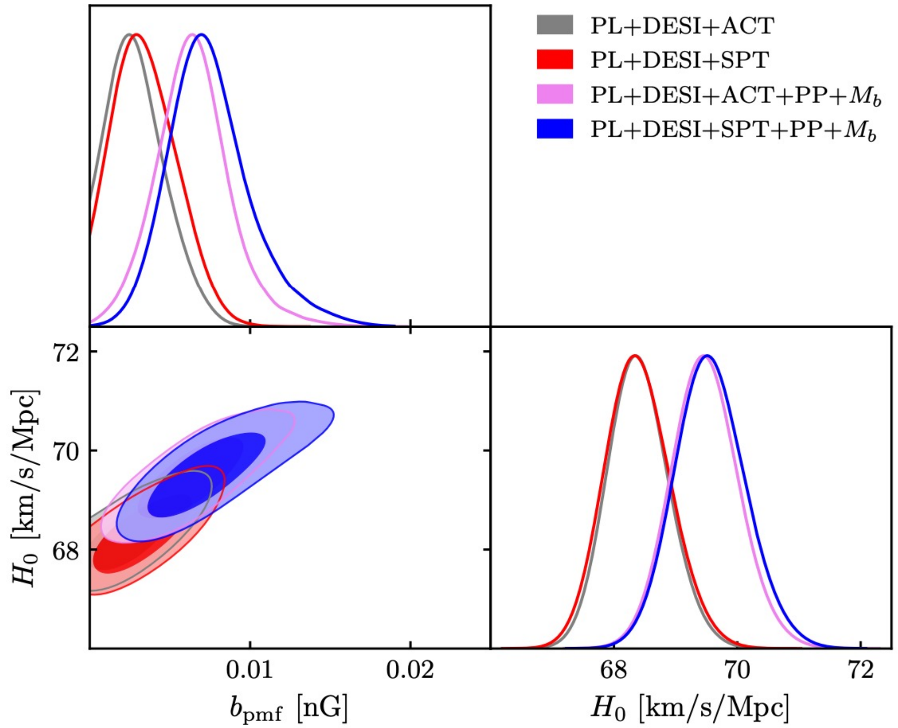

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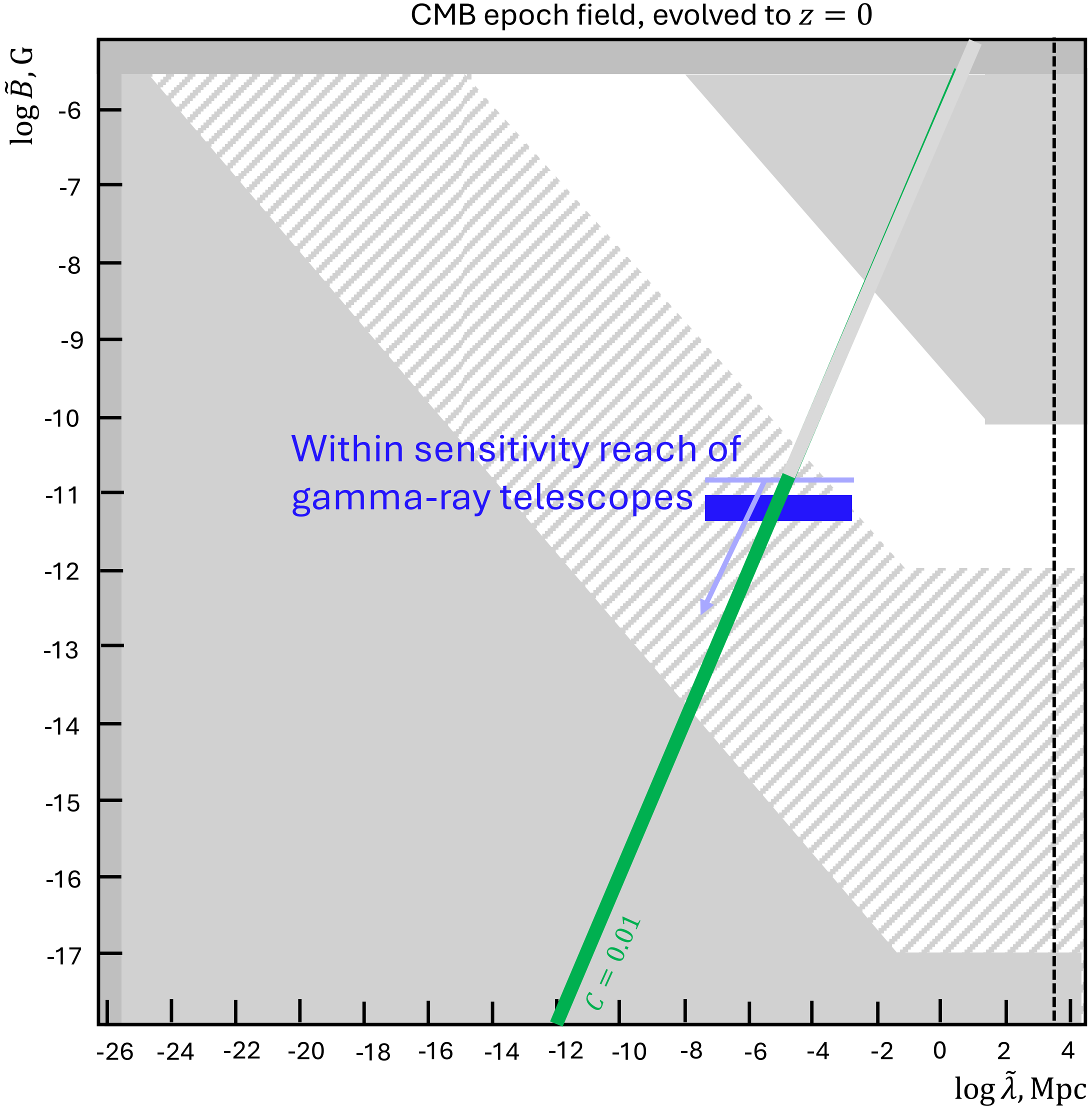


Gravitational wave signature of magnetic field

Wave equation for gravitational waves

$$\partial_{\tilde{t}}^2 \tilde{h} + k^2 \tilde{h} = \frac{16\pi G_N}{a} \tilde{T}^{TT}$$

The comoving quantities $\tilde{h}_{ij} = a h_{ij}, \tilde{T}^{TT} = a^4 T^{TT}$ where TT stands for "transverse traceless". Suppose the source term is constant, and is active during a finite time,

$$\tilde{T}^{TT} = \begin{cases} \frac{B^2}{8\pi}, 0 < \tilde{t} < \tilde{\tau} \\ 0, \qquad \tilde{t} > \tilde{\tau} \end{cases}$$

Solutions with $\tilde{h}(0) = \partial_{\tilde{t}} \tilde{h}(0) = 0$: $\tilde{h} = \frac{2G_N B^2}{ak^2}(1 - \cos k\tilde{t})$

At the end of forcing

$$\partial_{\tilde{t}} \tilde{h}(\tilde{\tau}) = \frac{2G_N B^2}{ak} \sin k\tilde{\tau}$$

And the energy density of gravitational waves is

$$\rho_{gw} = \frac{\left(\partial_{\tilde{t}} \tilde{h}\right)^2}{32\pi G_N} = \frac{G_N B^4}{8\pi a^2 k^2} \sin^2 k\tilde{\tau}$$

Modes with $k \ll \tilde{\tau}^{-1}$ have

$$\tilde{\rho}_{gw} = \frac{\left(\partial_{\tilde{t}} \tilde{h}\right)^2}{32\pi G_N} = \frac{G_N \tilde{B}^4 \tilde{\tau}^2}{8\pi a^2} = \frac{8\pi G_N \tilde{\tau}^2 \tilde{\rho}_B^2}{a^2} = \frac{8\pi G_N}{a^2} \tilde{\tau}^2 \Omega_B^2 \tilde{\rho}^2 = 3\Omega_B^2 \tilde{H}^2 \tilde{\tau}^2$$

Taking $\tilde{\tau} \sim \tilde{L}_{lpe}$, one finds

$$\tilde{\rho}_{gw} \sim \Omega_B^2 \left(H L_{lpe}\right)^2$$

The energy density in gravitational waves may be sizeable part of the overall energy density if magnetic field forcing the gravitational radiation has $B \sim B_{eq}$ in which case $L_{lpe} \sim R_H$. Once generated, the gravitational waves evolve as radiation and their relative density compared to the photon density stays approximately constant. This implies that present-day $\Omega_{gw} \sim \Omega_{CMB} \sim 10^{-4}$ if $B \sim B_{eq}$

Gravitational wave signature of magnetic field

$$\tilde{\rho}_{gw} \sim \Omega_B^2 \left(HL_{lpe}\right)^2, k \sim 2\pi/L_{lpe}$$

at production. Once generated, gravitational waves evolve as radiation and their relative density compared to the photon density stays approximately constant. This implies that present-day $\Omega_{gw} \sim \Omega_{CMB} \sim 10^{-4}$ if $B \sim B_{eq}$ or

$$\Omega_{gw} \sim 10^{-4} \left(\frac{B}{B_{eq}}\right)^4, \qquad \lambda_{gw} \sim \tilde{L}_{lpe}$$

today. The frequency

$$f_{gw} = \frac{1}{\lambda_{gw}} \sim 10 \left[\frac{\tilde{L}_{lpe}}{1 \text{ pc}}\right]^{-1} \text{nHz}$$

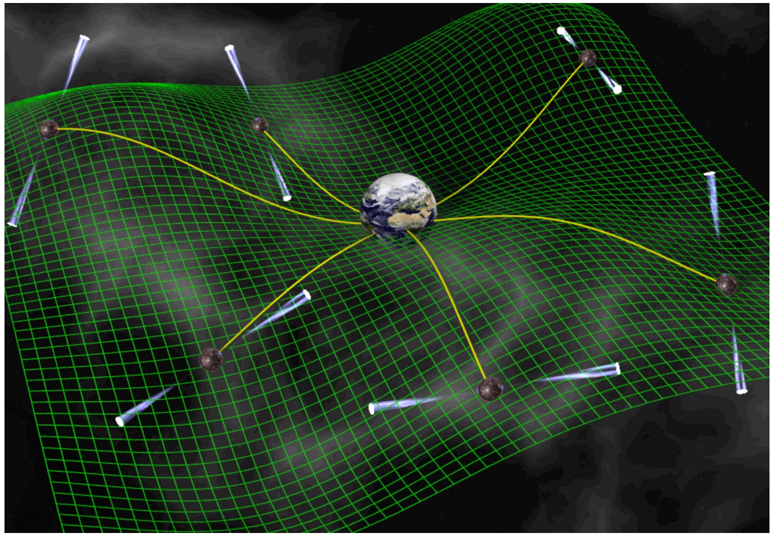
Gravitational wave detection: pulsar timing arrays

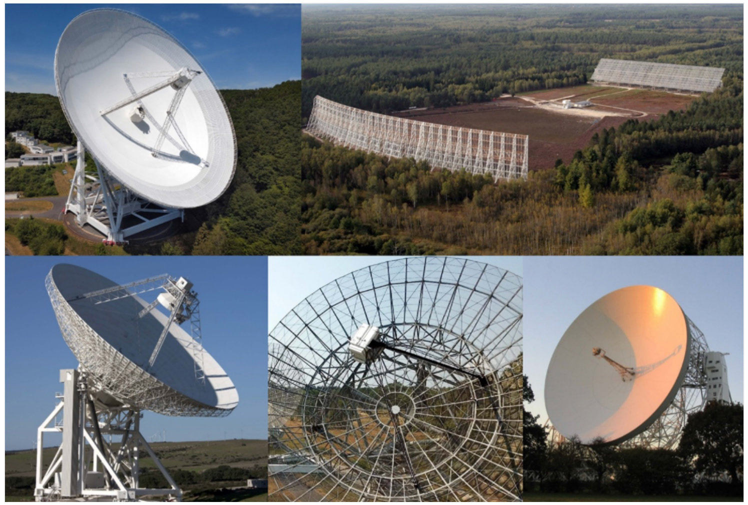
$$f_{gw} = \frac{1}{\lambda_{gw}} \sim 10 \left[\frac{\tilde{L}_{lpe}}{1\ \mathrm{pc}}\right]^{-1} \mathrm{nHz}$$

Gravitaitonal waves in this frequency range are detectable using “Pulsar Timing Arrays”: multiple radio telescope monitor a set of (millisecond) pulsars over many years, because they are known to be good clock standards (very regular and modelable pulse arriving time pattern). Gravitational waves passing through the lines-of-sight toward pulsars perturb the timing pattern of pulsars.

Several PTA collaborations exist:

- NanoGRAV (North American Nanohertz Observatory for Gravitational Waves)
- EPTA (European Pulsar Timing Array)
- PPTA (Parkes Pulsar Timing Array)
- CPTA (Chinese pulsar timing array)
- MPTA (MeerKAT Pulsar Timing Array





EPTA

Gravitational wave detection: pulsar timing arrays

Consider a + polarized gravitational wave in z direction:

$$h_{ij} = \begin{pmatrix} h_+ & 0 & 0 \\ 0 & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$ds^2 = -dt^2 + \left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

Consider a pulsar at a distance D in x direction. Its signal propagates according to

$$dt = \sqrt{1 + h_+(t,x)}dx \simeq \left(1 + \frac{1}{2} h_+(t,x)\right) dx$$

Consider two subsequent pulses, at $t_{e1}, t_{e2} = t_{e1} + T$. Signal travel times

$$t_{r1} - t_{e1} = D + \frac{1}{2}\int_{t_{e1}}^{t_{e1}+D} dt\, h_+(t', t_{e1} + D - t')$$

$$t_{r2} - t_{e2} = D + \frac{1}{2}\int_{t_{e2}}^{t_{e2}+D} dt'\, h_+(t', t_{e1} + T + D - t')$$

Time between the reception of the two pulses

$$t_{r2} - t_{r1} = T + \Delta T$$

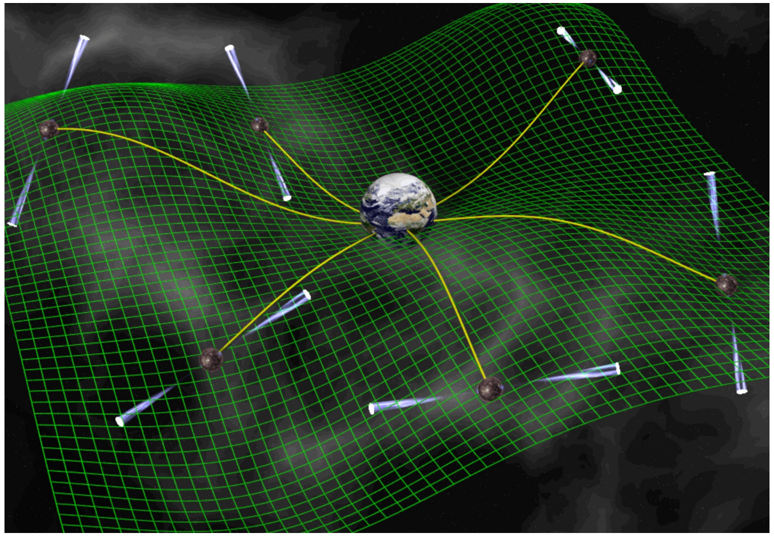
$$\Delta T = \int_{t_{e1}}^{t_{e1}+D} dt' \left[h_+\left(t' + T, x(t')\right) - h_+(t', x(t'))\right]$$

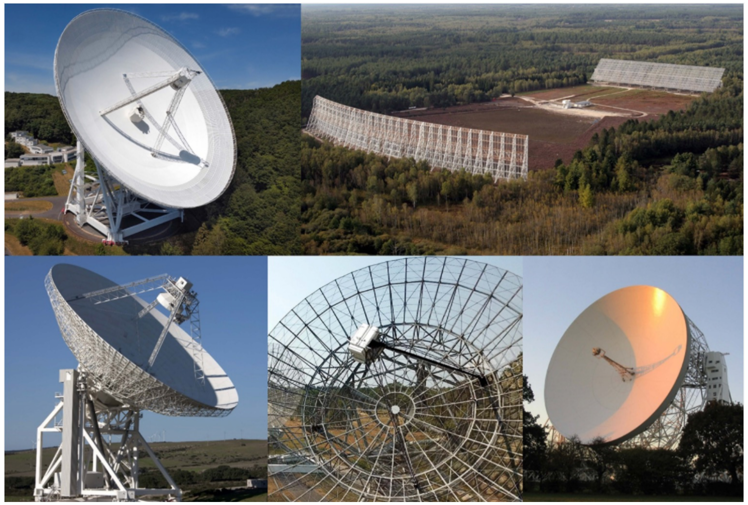
This corresponds to a relative shift of pulse arrival time by

$$z(t_{r1}) = \frac{\Delta T}{T} \simeq (h_+(t_{r1}, 0) - h_+(t_{e1}, D))$$

A general expression of arbitrary polarization and arbitrary wave $\vec{n}_w$ and pulrar $\vec{n}_p$ directions is

$$z(t_{r1}) = \frac{n_p^i n_p^j}{2(1 + \vec{n}_w \vec{n}_p)} \left(h_{ij}(t_{r1}, 0) - h_{ij}\left(t_{e1}, D\vec{n}_p\right)\right)$$





EPTA

Stochastic gravitational wave background

$$z(t_{r1}) = \frac{n_p^i n_p^j}{2(1+\vec{n}_w \vec{n}_p)} \left(h_{ij}(t_{r1},0) - h_{ij}\left(t_{e1}, D\vec{n}_p\right)\right)$$

Consider a set of pulsars in different directions and a set of ways going isortopically in all directions. Total metric perturbation

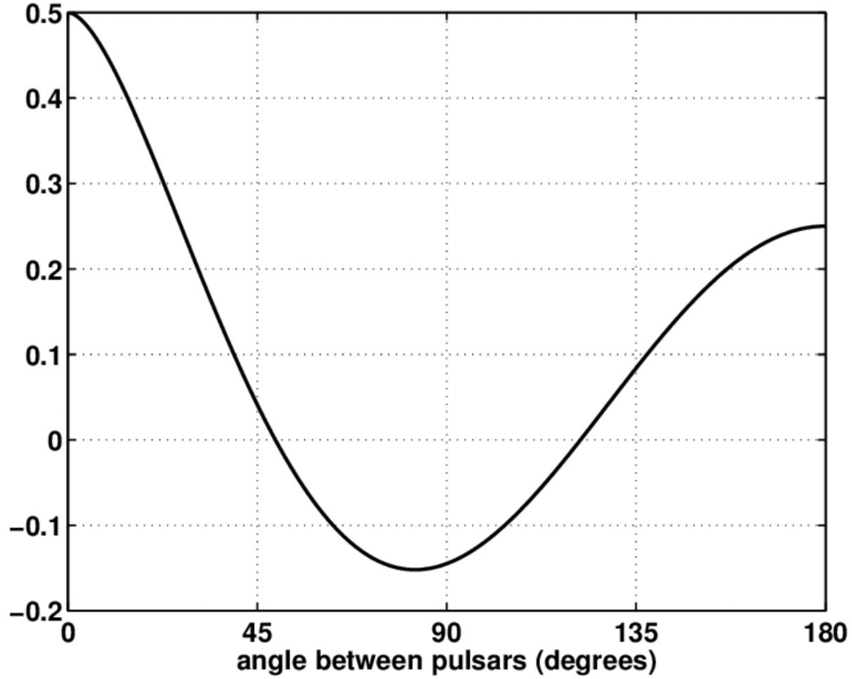
$$h_{ij} = \sum_{A=+,-} \int df \int d^2\vec{n}\, \hat{h}_A(f,\vec{n})\, e_{ij}^A(\vec{n}) e^{-i2\pi f(t-\vec{n}\vec{x})}$$

Considering two pulsars, p_a, p_b separated by an angle Θ on the sky, one can compute the time average correlator

$$\langle z_{p_a}(t), z_{p_b}(t)\rangle = \left(\frac{1-\cos\Theta}{2}\right)\log\left(\frac{1-\cos\Theta}{2}\right) - \left(\frac{1-\cos\Theta}{12}\right) + \frac{1}{3}$$

(the Hellings-Downs curve).

Observation of the Hellings-Downs correlation function is considered as a signature of the presence of Stochastic Gravitational Wave Background (SGWB) signal in the data (distinguishable this way from systematic effects and calibration errors).



Stochastic gravitational wave background detection

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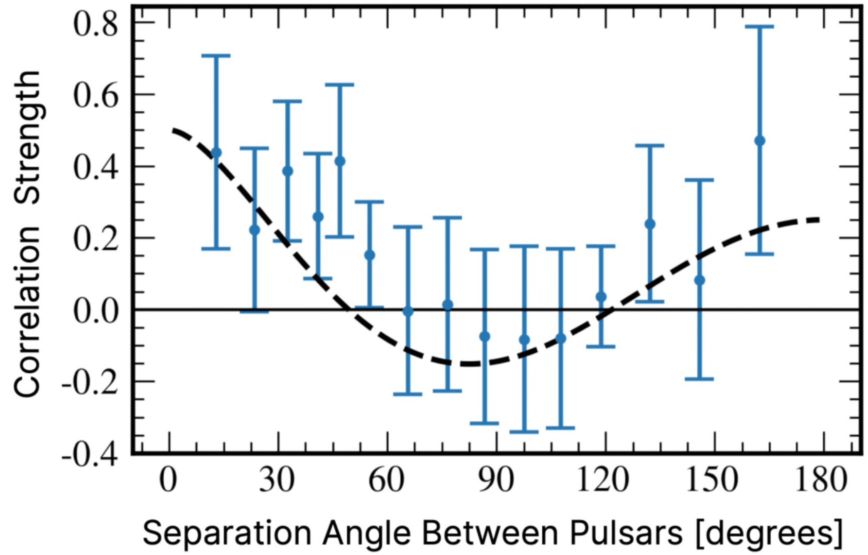
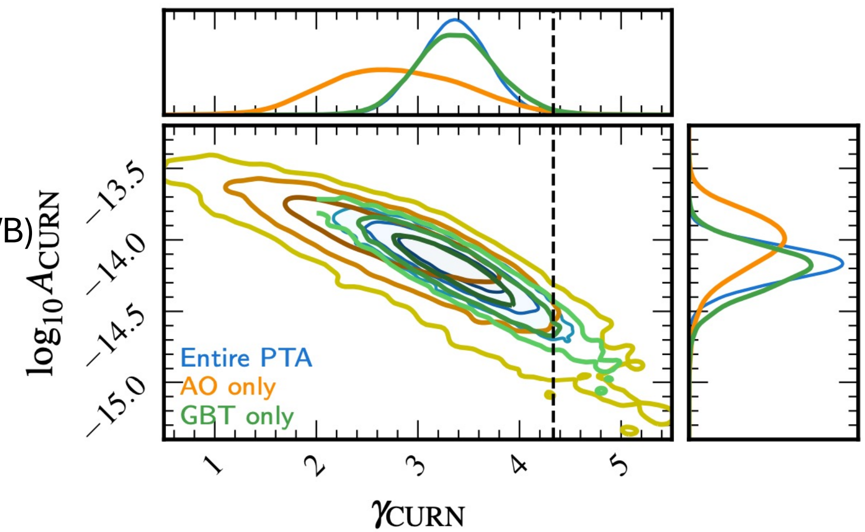




NanoGRAV, https://arxiv.org/abs/2306.16213

Stochastic gravitational wave background detection

The energy density of the SGWB

$$\Omega_{SGWB} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \frac{2\pi^2 A_{SGWB}^2 f^2}{3H_0^2}\left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{2\alpha}$$

$$h_c(f) = A_{SGWB}\left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{\alpha}$$

For a population of inspiralling supermassive black holes, the theoretical expectation is $\alpha = -2/3$. The power spectrum is

$$S_{SGWB}(f) = \Gamma \frac{A_{\mathrm{SGWB}}^2}{12\pi^2}\left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{-\gamma} \mathrm{yr}^3, \gamma = 3 - 2\alpha$$

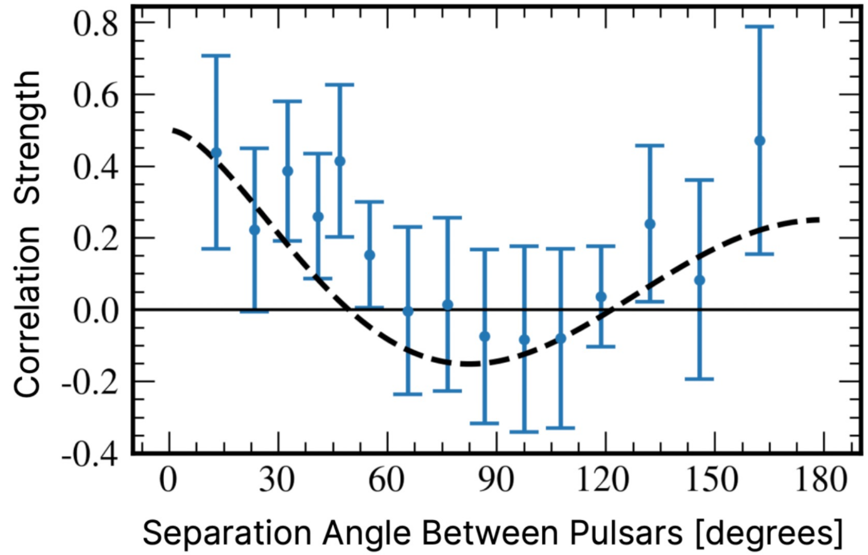
The measured values:

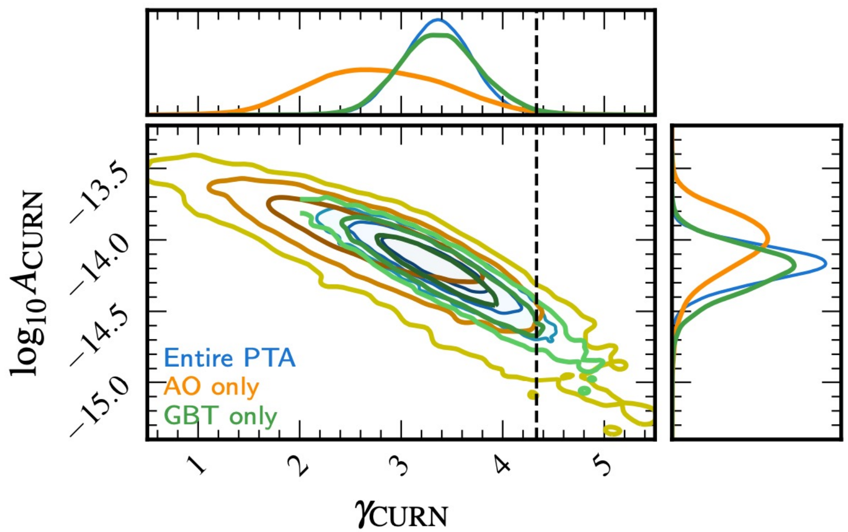
$$A_{SGWB} \simeq 10^{-14}, \gamma \simeq 3.5 \rightarrow \alpha = 1.25$$

So that

$$\Omega_{SGWB} \simeq 10^{-7}\left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{2.5}$$

Exercise 1 Consider the 1-σ contour of the gravitational wave signal measurement on the right and plot the spectrum of the gravitational wave signal measurement by PTAs in $f, \Omega_{SGWB}(f)$ representation.





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Gravitational wave signature of magnetic field?

The energy density of the SGWB

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$$h_c(f) = A_{SGWB} \left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{\alpha}$$

For a population of inspiralling supermassive black holes, the theoretical expectation is $\alpha = -2/3$. The power spectrum is

$$S_{SGWB}(f) = \Gamma \frac{\mathrm{A}_{\mathrm{SGWB}}^2}{12\pi^2} \left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{-\gamma} \mathrm{yr}^3, \gamma = 3 - 2\alpha$$

The measured values:

$$A_{SGWB} \simeq 10^{-14}, \gamma \simeq 3.5 \rightarrow \alpha = 1.25$$

So that

$$\Omega_{SGWB} \simeq 10^{-7} \left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{2.5}$$

Interpreting this as due to the presence of magnetic field,

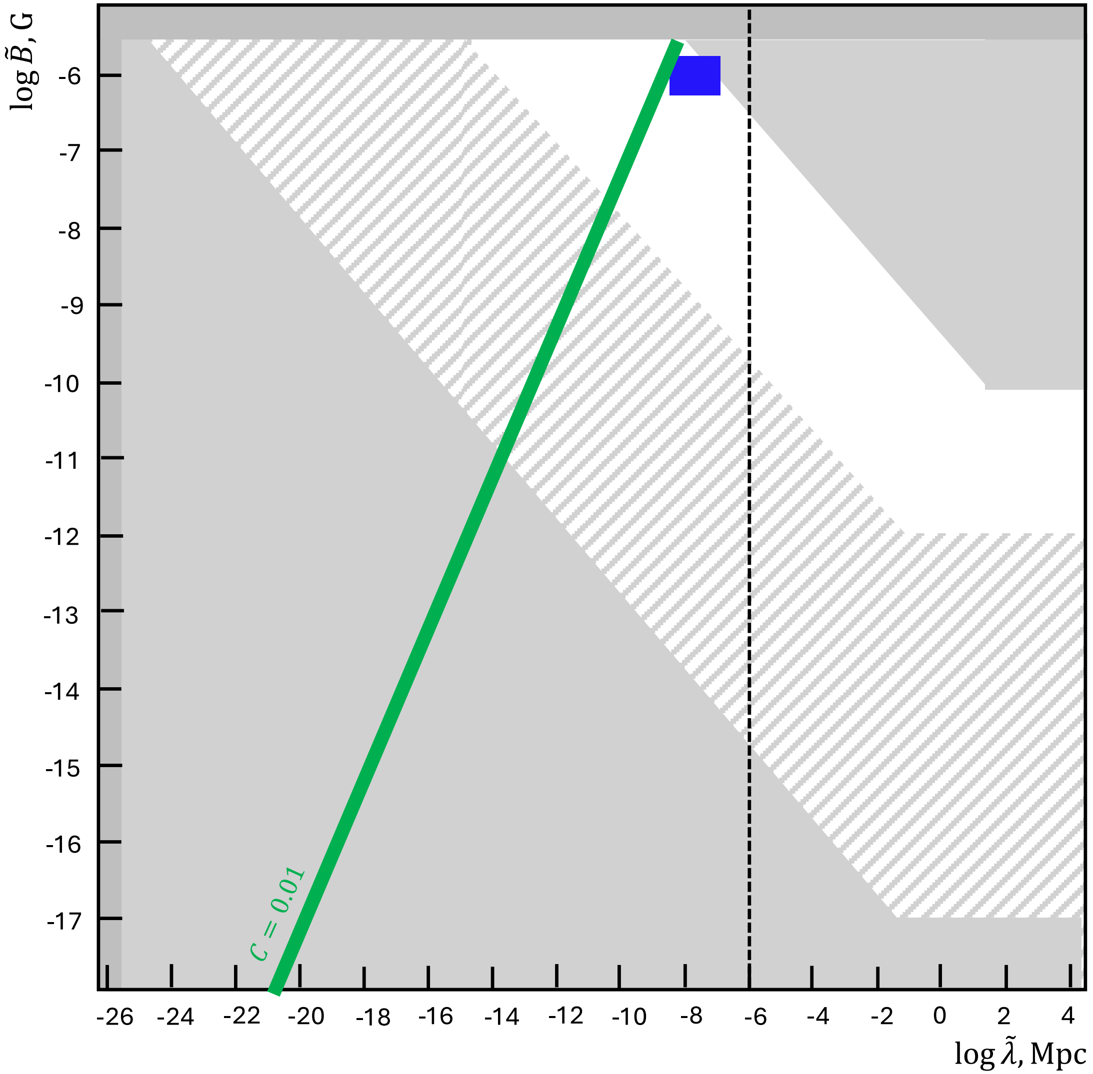
$$\Omega_{SGWB} \sim 10^{-7} \left(\frac{B}{0.2 B_{eq}}\right)^4$$

$$L_{lpe} \simeq 0.2\, C \left(\frac{B}{B_{eq}}\right) R_H$$

The field has to come from the QCD phase transition,

$$\tilde{R}_{H,QCD} \simeq 2 \left[\frac{g_*}{50}\right]^{-\frac{1}{6}} \left[\frac{T}{150\ \mathrm{MeV}}\right]^{-1} \mathrm{pc} \quad \text{@ QCD epoch}$$

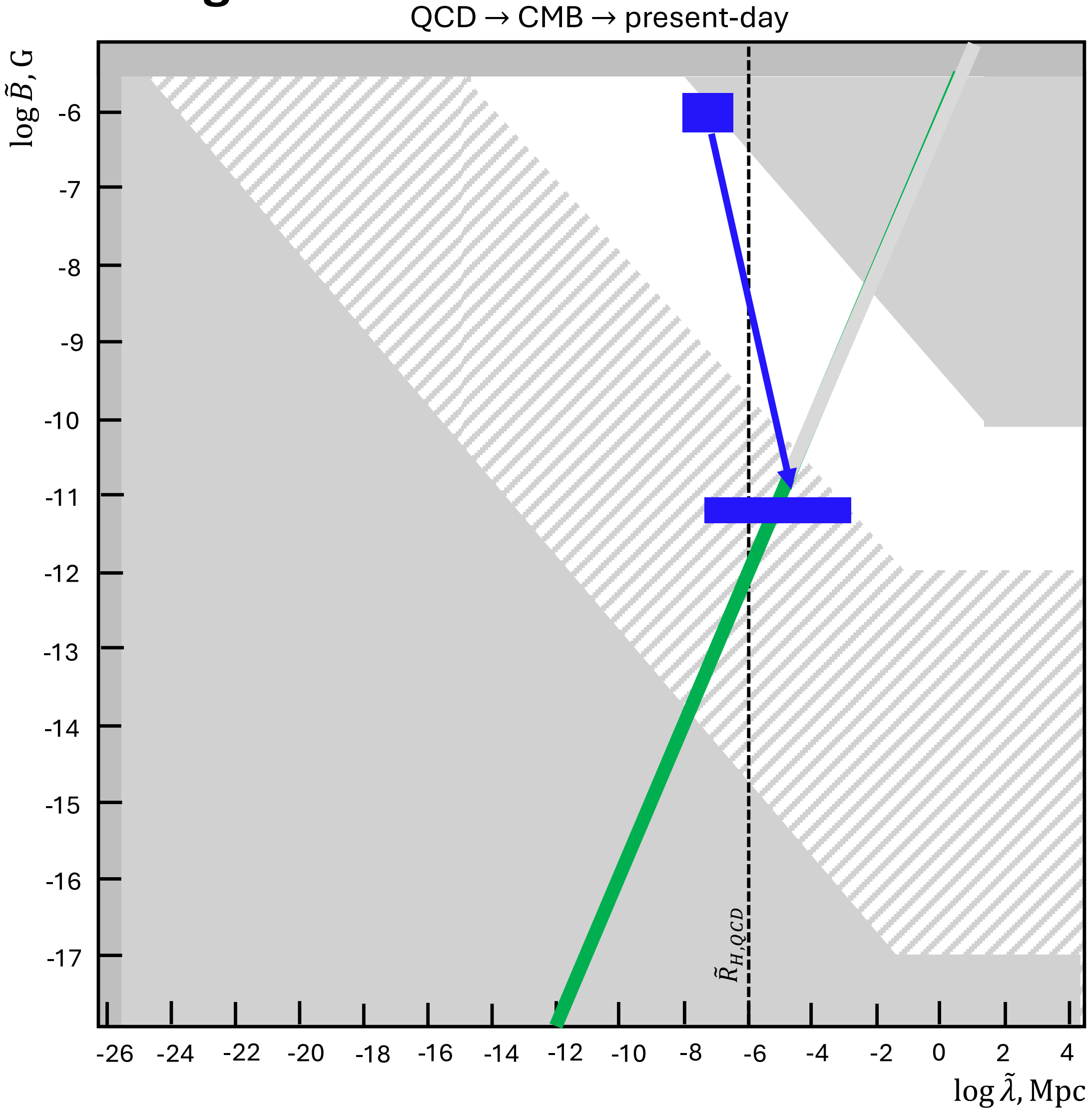
QCD epoch field, at production



https://arxiv.org/pdf/2009.14174

Evolutionary track of magnetic field?

The SGWB and CMB signatures of cosmological field are "not inconsistent" with each other. The evolutionary path connecting the two measurements is consistent with current understanding of evolution of **non-helical** magnetic field generated at the QCD phase transiiton.



https://arxiv.org/pdf/2009.14174

Magnetic field from Electroweak phase transition?

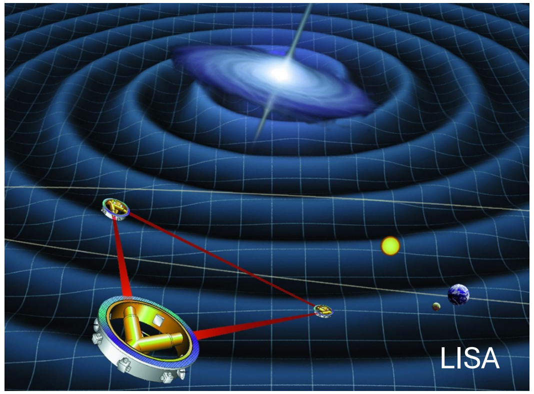
The horizon size is

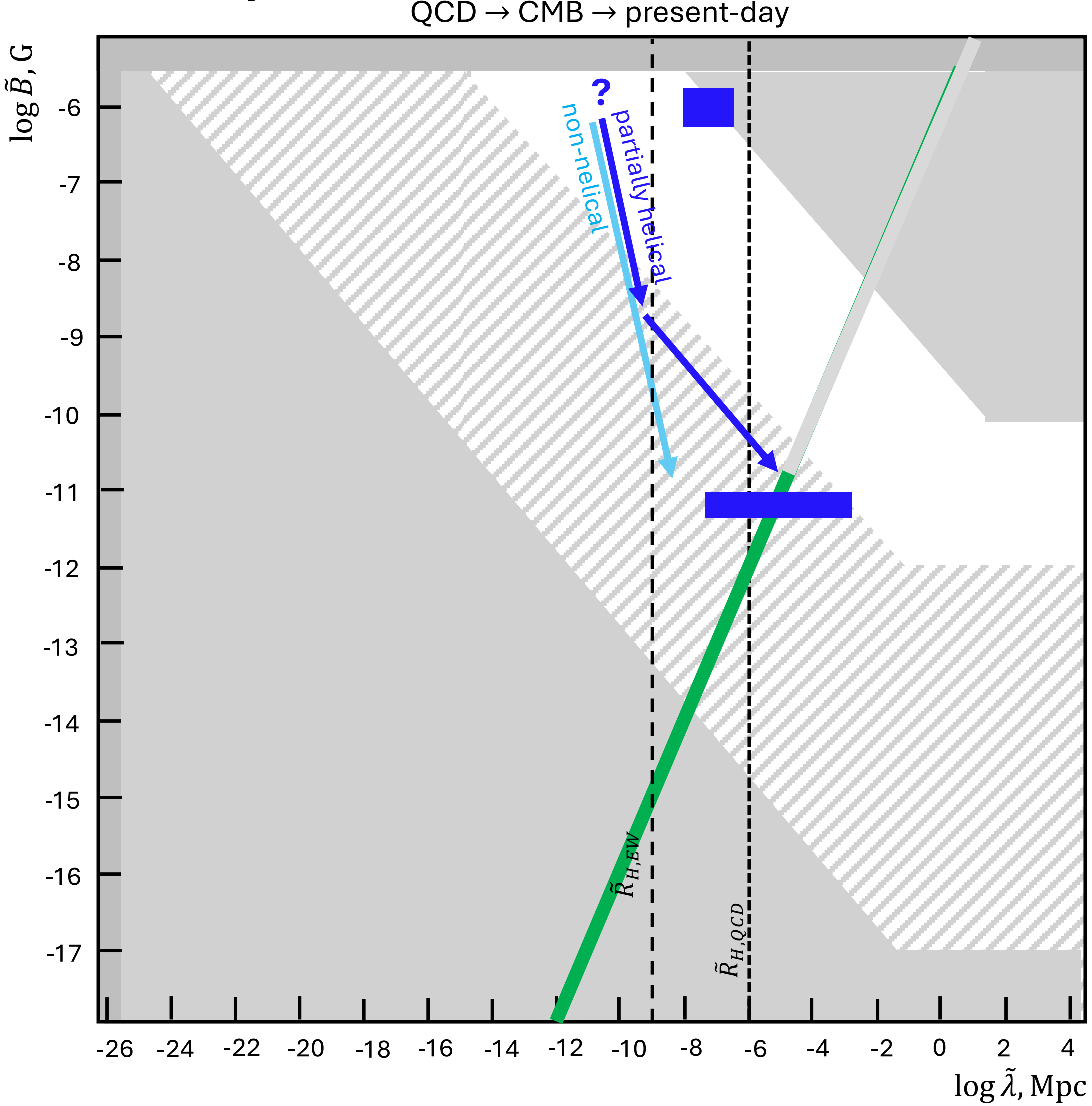
$$\tilde{R}_{H,EW} \simeq 10^{-3} \left[\frac{g_*}{10^2}\right]^{-\frac{1}{6}} \left[\frac{T}{170\ \text{GeV}}\right]^{-1} \text{pc} \quad \text{@ EW epoch}$$

Gravitational waves that would originate from this phase transition would have frequencies in the μHz range or higher.

Observations in μHz – mHz range will be possible with LISA (Large Space Interferometer Antenna).

A field originating from EW phase transition and leaving a relic consistent with the CMB signature would need to be partially helical (see lectures of A.Brandenburg for difference in evolution of helical and non-helical fields).





https://arxiv.org/pdf/2009.14174

Exercise 2. Express the sensitivities of different gravitational wave detectors in $f, \Omega_{SGWB}(f)$ representation. In cosmological context, these detectors are sensitive to the signal from different cosmological epochs (at different temperatures). Estimate the temperature ranges of sensitivity of each detector. Estimate the magnetic field strength that can be probed through the gravitational wave signature.