Test field procedure

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Equations

The MHD equation for the magnetic vector potential $\boldsymbol{A}$ and the velocity $\boldsymbol{U}$ for an isothermal equation of state with constant speed of sound, $c_{\rm s}$, and external forcing functions $\boldsymbol{\mathcal{F}}_{\rm ext}$ and $\boldsymbol{\mathcal{E}}_{\rm ext}$ can be written in the form

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J} + \boldsymbol{\mathcal{E}}_{\rm ext} - \boldsymbol{\nabla}\Phi, \tag{1}$$

$$\frac{{\rm D}\boldsymbol{U}}{{\rm D}t} = \boldsymbol{J} \times \boldsymbol{B}/\rho - \nu\boldsymbol{Q} + \boldsymbol{\mathcal{F}}_{\rm ext} - \boldsymbol{\nabla}h, \tag{2}$$

$$\frac{{\rm D}h}{{\rm D}t} = -c_{\rm s}^2 \boldsymbol{\nabla}\cdot\boldsymbol{U}, \tag{3}$$

where ${\rm D}/{\rm D}t = \partial/\partial t + \boldsymbol{U}\cdot\boldsymbol{\nabla}\boldsymbol{U}$ is the advective derivative, $h = c_{\rm s}^2 \ln\rho$ is the pseudo enthalpy, ρ is the density, ϕ is the electric potential, $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$ is the magnetic field, $\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B}/\mu_0$ is the current density, and $\boldsymbol{Q} = \boldsymbol{\nabla}\times\boldsymbol{W} - \frac{4}{3}\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{U} + \mathsf{S}\cdot\boldsymbol{\nabla}\ln\rho$ is a modified curl of the vorticity $\boldsymbol{W} = \boldsymbol{\nabla}\times\boldsymbol{U}$, where $\mathsf{S}_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{U}$ is the traceless rate of strain tensor.

In addition to the equation for $\boldsymbol{A}$ we consider the equations for the mean field, $\overline{\boldsymbol{A}}$, and for the fluctuation, $\boldsymbol{a} = \boldsymbol{A} = \overline{\boldsymbol{A}}$, i.e.

$$\frac{\partial \overline{\boldsymbol{A}}}{\partial t} = \overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{u}\times\boldsymbol{b}} - \eta\overline{\boldsymbol{J}} - \boldsymbol{\nabla}\overline{\Phi} \tag{4}$$

$$\frac{\partial \boldsymbol{a}}{\partial t} = \overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u}\times\overline{\boldsymbol{B}} + \boldsymbol{u}\times\boldsymbol{b} - \overline{\boldsymbol{u}\times\boldsymbol{b}} - \eta\boldsymbol{j} + \boldsymbol{\mathcal{E}}_{\rm ext} - \boldsymbol{\nabla}\phi. \tag{5}$$

In the following we consider the evolution of the fluctuating field, $\boldsymbol{b}^{pq} = \boldsymbol{\nabla}\times\boldsymbol{a}^{pq}$, that results from arbitrary test fields, $\overline{\boldsymbol{B}}^{pq}$,

$$\frac{\partial \boldsymbol{a}^{pq}}{\partial t} = \overline{\boldsymbol{U}} \times \boldsymbol{b}^{pq} + \boldsymbol{u}\times\overline{\boldsymbol{B}}^{pq} + \boldsymbol{u}\times\boldsymbol{b}^{pq} - \overline{\boldsymbol{u}\times\boldsymbol{b}^{pq}} - \eta\boldsymbol{j}^{pq} + \boldsymbol{\mathcal{E}}_{\rm ext} - \boldsymbol{\nabla}\phi^{pq}. \tag{6}$$

Under the assumption of SOCA the terms that are nonlinear in the fluctuations are neglected, i.e.

$$\frac{\partial \boldsymbol{a}^{pq}}{\partial t} = \overline{\boldsymbol{U}} \times \boldsymbol{b}^{pq} + \boldsymbol{u}\times\overline{\boldsymbol{B}}^{pq} - \eta\boldsymbol{j}^{pq} + \boldsymbol{\mathcal{E}}_{\rm ext} - \boldsymbol{\nabla}\phi^{pq} \quad \text{(SOCA)}. \tag{7}$$

In either of the two cases the turbulent transport coefficients would be calculated from $\overline{\boldsymbol{\mathcal{E}}}^{pq} = \overline{\boldsymbol{u} \times \boldsymbol{b}^{pq}}$.

If the mean flow contains linear shear, it is advantageous to rewrite $\overline{\boldsymbol{U}} = Sx\hat{\boldsymbol{y}} + \overline{\boldsymbol{U}}^1$ so that[1]

$$\frac{\mathcal{D}\boldsymbol{a}^{pq}}{\mathcal{D}t} = -Sa_y^{pq}\hat{\boldsymbol{x}} + \overline{\boldsymbol{U}}^1 \times \boldsymbol{b}^{pq} + \boldsymbol{u} \times \overline{\boldsymbol{B}}^{pq} + \boldsymbol{u} \times \boldsymbol{b}^{pq} - \overline{\boldsymbol{u} \times \boldsymbol{b}^{pq}} - \eta \boldsymbol{j}^{pq} + \boldsymbol{\mathcal{E}}_{\rm ext} - ... \quad (8)$$

where $\mathcal{D}/\mathcal{D}t = \partial/\partial t + Sx\partial/\partial y$ is the advective derivative with respect to the linear shear flow and the dots indicate additional gradient terms that can be removed by a gauge transformation.

We now assume two-dimensional (horizontal) averages, i.e.

$$\overline{\boldsymbol{B}}(z,t) = \frac{1}{L_x L_y} \int \boldsymbol{B}(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}x. \quad (9)$$

We assume that the emf is given by

$$\overline{\boldsymbol{\mathcal{E}}}_i^{pq} = \overline{\boldsymbol{\mathcal{E}}}_i^0 + \alpha_{ij}\overline{B}_j^{pq} + \eta_{ij3}\overline{B}_{j,3}^{pq}. \quad (10)$$

In the absence of an imposed mean field there can still be an emf, $\overline{\boldsymbol{\mathcal{E}}}_i^0$, due to large scale dynamo action.

Define four test fields as follows:

$$\overline{\boldsymbol{B}}^{11} = \begin{pmatrix} \cos kz \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{21} = \begin{pmatrix} \sin kz \\ 0 \\ 0 \end{pmatrix}, \quad (11)$$

$$\overline{\boldsymbol{B}}^{12} = \begin{pmatrix} 0 \\ \cos kz \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{22} = \begin{pmatrix} 0 \\ \sin kz \\ 0 \end{pmatrix}. \quad (12)$$

with

$$\overline{\boldsymbol{J}}^{11} = \begin{pmatrix} 0 \\ -k\sin kz \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{J}}^{21} = \begin{pmatrix} 0 \\ k\cos kz \\ 0 \end{pmatrix}, \quad (13)$$

$$\overline{\boldsymbol{J}}^{12} = \begin{pmatrix} k\sin kz \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{J}}^{22} = \begin{pmatrix} -k\cos kz \\ 0 \\ 0 \end{pmatrix}. \quad (14)$$

So, for each of the four combinations of i and j $(= p)$ the set of two coefficient, α_{ij} and η_{ij3}, is obtained as

$$\begin{pmatrix} \alpha_{ij} \\ \eta_{ij3}k \end{pmatrix} = \mathsf{M}^{-1} \begin{pmatrix} \overline{\boldsymbol{\mathcal{E}}}_i^{1j} \\ \overline{\boldsymbol{\mathcal{E}}}_i^{2j} \end{pmatrix}, \quad (15)$$

where the matrices

$$\mathsf{M} = \begin{pmatrix} \cos kz & -\sin kz \\ \sin kz & \cos kz \end{pmatrix} \quad \mathsf{M}^{-1} = \begin{pmatrix} \cos kz & \sin kz \\ -\sin kz & \cos kz \end{pmatrix} \quad (16)$$

[1]Note that $(\overline{\boldsymbol{U}}^0 \times \boldsymbol{\nabla} \times \boldsymbol{a})_i = \overline{U}_j^0(a_{j,i} - a_{i,j})$ and $\overline{U}_j^0 a_{j,i} = -\overline{U}_{j,i}^0 a_j$ + gradient term.

are the same for each value of p and each of the two components $i = 1, 2$ of $\overline{\boldsymbol{\mathcal{E}}}_i^{pq}$. Finally, $\tilde{\eta}$ is calculated using equation (55). Note that $\det \mathsf{M} = 1$. The η_{ij3} are related to η_{ij}^* in the formulation

$$\overline{\boldsymbol{\mathcal{E}}}_i^{pq} = \alpha_{ij} \overline{B}_j^{pq} - \eta_{ij}^* \overline{J}_j^{pq} \tag{17}$$

via

$$\begin{pmatrix} \eta_{11}^* & \eta_{12}^* \\ \eta_{21}^* & \eta_{22}^* \end{pmatrix} = \begin{pmatrix} \eta_{123} & -\eta_{113} \\ \eta_{223} & -\eta_{213} \end{pmatrix}. \tag{18}$$

At first instance, we might only need the test fields $\overline{\boldsymbol{B}}^{11}$ and $\overline{\boldsymbol{B}}^{21}$, so we get

$$\begin{pmatrix} \alpha_{i1} \\ \eta_{i13} k \end{pmatrix} = \mathsf{M}^{-1} \begin{pmatrix} \overline{\boldsymbol{\mathcal{E}}}_i^{11} \\ \overline{\boldsymbol{\mathcal{E}}}_i^{21} \end{pmatrix}, \tag{19}$$

so we get only α_{11} and α_{11}, as well as $\eta_{113} = -\eta_{12}^*$ and $\eta_{213} = -\eta_{22}^*$. This can be useful in simple cases (e.g. isotropy).

Roberts flow

Use the Roberts flow as a test flow. Defined here as

$$\boldsymbol{U}(x, y) = k_{\rm f}^{-1} \boldsymbol{\nabla} \times (\varphi \boldsymbol{z}) + k_{\rm f}^{-2} \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\varphi \boldsymbol{z}), \tag{20}$$

with the stream function $\varphi = \sqrt{2}\, U_0 \cos k_x x \cos k_y y$, where $k_x = k_y = \pi/a$. This flow is fully helical with $\boldsymbol{W} = k_{\rm f} \boldsymbol{U}$, were $k_{\rm f}^2 = k_x^2 + k_y^2$ and $\boldsymbol{W} = \boldsymbol{\nabla} \times \boldsymbol{U}$. The flow is normalized such that $\langle \boldsymbol{U}^2 \rangle = U_0^2$. In components form, the velocity is given by

$$\boldsymbol{U} = U_0 \begin{pmatrix} -\cos kx \, \sin ky \\ +\sin kx \, \cos ky \\ \sqrt{2} \, \cos kx \, \cos ky \end{pmatrix} \tag{21}$$

Test field procedure

We calculate

$$\frac{\partial \boldsymbol{b}^{(p,q)}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{u} \times \overline{\boldsymbol{B}}^{(p,q)} \right) + \eta \nabla^2 \boldsymbol{b}^{(p,q)} + \boldsymbol{G} \tag{22}$$

and $\overline{\boldsymbol{\mathcal{E}}}^{(p,q)} \equiv \overline{\boldsymbol{u} \times \boldsymbol{b}^{(p,q)}}$ for each of the 9 test field $\overline{\boldsymbol{B}}^{(p,q)}$,

$$\overline{\boldsymbol{B}}^{(1,0)} = \begin{pmatrix} \sin z \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(2,0)} = \begin{pmatrix} 0 \\ \sin z \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(3,0)} = \begin{pmatrix} 0 \\ 0 \\ \sin z \end{pmatrix}, \tag{23}$$

$$\overline{\boldsymbol{B}}^{(1,1)} = \begin{pmatrix} \cos x \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(2,1)} = \begin{pmatrix} 0 \\ \cos x \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(3,1)} = \begin{pmatrix} 0 \\ 0 \\ \cos x \end{pmatrix}, \tag{24}$$

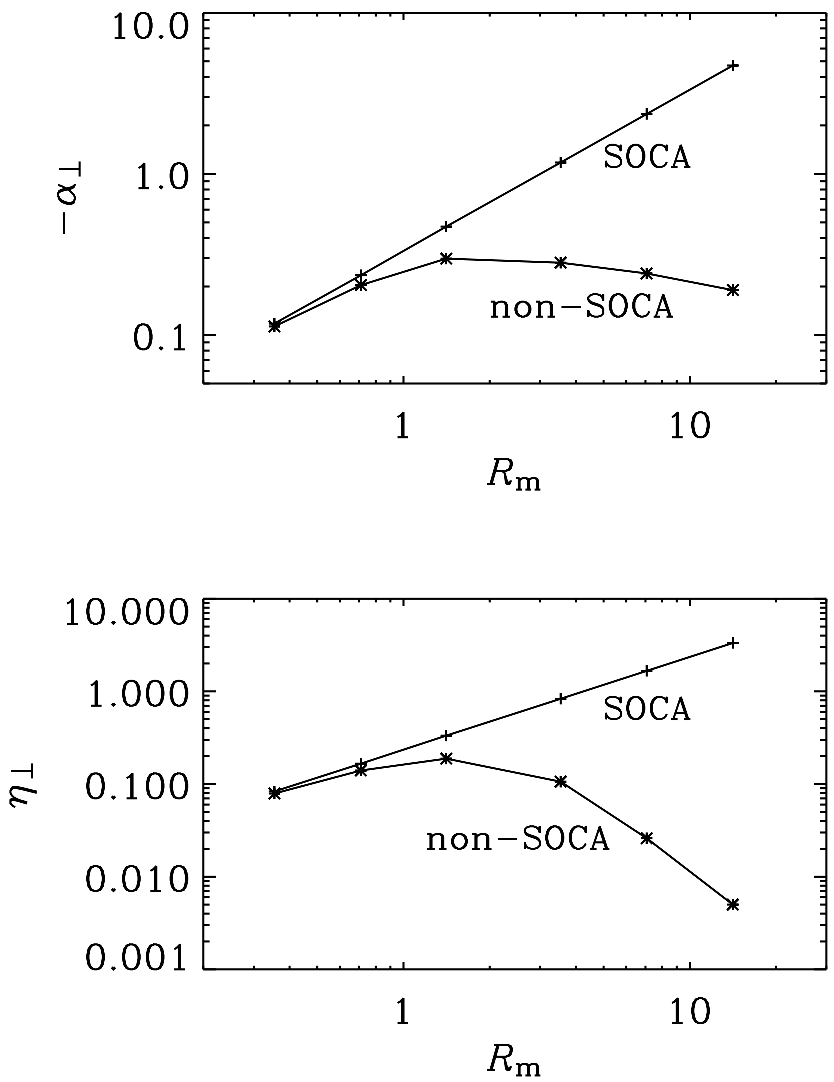

Figure 1: Roberts flow

$$\overline{\boldsymbol{B}}^{(1,3)} = \begin{pmatrix} \cos z \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(2,3)} = \begin{pmatrix} 0 \\ \cos z \\ 0 \end{pmatrix}, \quad \overline{\boldsymbol{B}}^{(3,3)} = \begin{pmatrix} 0 \\ 0 \\ \cos z \end{pmatrix}, \qquad (25)$$

where we have assumed that $\overline{\boldsymbol{B}}$ is an average over the y direction, so there is no dependence on y. For each of these test fields there is only a single nonvanishing (and constant) component of the derivative matrix $\overline{B}_{j,k}$. This gives the components of

$$\eta_{ijk} = \left(\overline{\mathcal{E}}_i^{(p,q)} - \alpha_{ij}\overline{B}_j^{(p,q)}\right)\Big/\,\overline{B}_{j,k}^{(p,q)} \quad \text{(second step, } q=4,5,...,9). \tag{26}$$

Results for Roberts flow

Consider dimensional quantities using $2a=2\pi$, so $k_1=1$, $k_\mathrm{f}=\sqrt{2}$, $U_0=1$, and $\eta=0.05$, so $R_\mathrm{m}=U_0/(\eta k_\mathrm{f})=14.14$. We first adopt horizontal xy averages, and find in that case

$$\alpha_\perp=-4.714, \quad \eta_\perp=3.333, \tag{27}$$

independent of z.

Table 1:

	SOCA		non-SOCA	
R_m	$\alpha_\perp$	$\eta_\perp$	$\alpha_\perp$	$\eta_\perp$
14.14	−4.708	3.329	−0.190	0.005
7.07	−2.354	1.664	−0.241	0.026
3.54	−1.177	0.832	−0.281	0.106
1.41	−0.471	0.333	−0.298	0.188
0.71	−0.235	0.166	−0.193	0.133

Next, we consider only y averages, and consider α_{ij} and η_{ijk} as functions of x and z.

$$\alpha_{ij}(x,z)=\begin{pmatrix}-4.714 & 0 & 0\\ 0 & -4.714(1+\cos 2x) & 0\\ 0 & -3.333\sin 2x & 0\end{pmatrix} \tag{28}$$

$$\eta_{ijz}(x,z)=\begin{pmatrix}0 & 3.333(1+\cos 2x) & 2.357\sin 2x\\ -3.333(1+\cos 2x) & 0 & 0\\ 2.357\sin 2x & 0 & 0\end{pmatrix} \tag{29}$$

For x-derivatives, we need $\sin x$ and $\cos x$ test fields.

$$\alpha_{ij}(x,z)=\begin{pmatrix}-2.828 & 0 & 0\\ 0 & -8.5(1+\cos 2x) & 2\sin 2x\\ 0 & 4\sin 2x & 0\end{pmatrix} \tag{30}$$

$$\eta_{ijz}(x,z)=\begin{pmatrix}0 & 0 & 0\\ 0 & 5.65\sin 2x & 3(1+\cos 2x)\\ 0 & 3(1+\cos 2x) & 0\end{pmatrix} \tag{31}$$

Isolating $\boldsymbol{\delta}$ effect, etc.

Split

$$\eta_{ijk}\overline{B}_{j,k} = \eta^{(\mathrm{A})}_{ijk}\overline{B}^{(\mathrm{A})}_{j,k} + \kappa_{ijk}\overline{B}^{(\mathrm{S})}_{j,k}. \tag{32}$$

where

$$\overline{B}^{(\mathrm{A})}_{j,k} = -\tfrac{1}{2}\epsilon_{jkl}\overline{J}_l \tag{33}$$

so

$$\eta^{(\mathrm{A})}_{ijk}\overline{B}^{(\mathrm{A})}_{j,k} = -\tfrac{1}{2}\eta^{(\mathrm{A})}_{ijk}\epsilon_{jkl}\overline{J}_l \equiv -\tilde{\eta}_{il}\overline{J}_l \tag{34}$$

where

$$\tilde{\eta}_{il} = \tfrac{1}{2}\eta^{(\mathrm{A})}_{ijk}\epsilon_{jkl} \tag{35}$$

$$\tilde{\eta}_{il} = \eta_{il} + \epsilon_{ilm}\delta_m \tag{36}$$

where $\overline{\boldsymbol{\mathcal{E}}} = ... + \boldsymbol{\delta}\times\overline{\boldsymbol{J}}$ gives the contribution to the $\boldsymbol{\delta}$ effect, representing the antisymmetric part of $\tilde{\eta}_{il}$. To calculate $\boldsymbol{\delta}$ we write

$$\delta_m = -\tfrac{1}{2}\epsilon_{mil}\tilde{\eta}_{il} \tag{37}$$

so

$$\delta_m = \tfrac{1}{4}\epsilon_{mil}\eta_{ijk}\epsilon_{jkl} = \tfrac{1}{4}(\eta_{kmk} - \eta_{jjm}) \tag{38}$$

$$\delta_1 = \tfrac{1}{4}(\eta_{k1k} - \eta_{jj1}) = \tfrac{1}{4}(\eta_{111} + \eta_{313} - \eta_{111} - \eta_{331}) = \tfrac{1}{4}(\eta_{313} - \eta_{221} - \eta_{331}) \tag{39}$$

$$\delta_2 = \tfrac{1}{4}(\eta_{k2k} - \eta_{jj2}) = \tfrac{1}{4}(\eta_{121} + \eta_{323} - \eta_{112} - \eta_{332}) = \tfrac{1}{4}(\eta_{121} + \eta_{323}) \tag{40}$$

$$\delta_3 = \tfrac{1}{4}(\eta_{k3k} - \eta_{jj3}) = \tfrac{1}{4}(\eta_{131} + \eta_{333} - \eta_{113} - \eta_{333}) = \tfrac{1}{4}(\eta_{131} - \eta_{113} - \eta_{223}) \tag{41}$$

Symmetric part of η_{il} as in $\overline{\boldsymbol{\mathcal{E}}}_i = ... - \eta_{ij}\overline{J}_j$ is

$$\eta_{il} = \tfrac{1}{4}(\eta_{ijk}\epsilon_{jkl} + \eta_{ljk}\epsilon_{jki}) \tag{42}$$

$$\eta_{11} = \tfrac{1}{2}\eta_{1jk}\epsilon_{jk1} = \tfrac{1}{2}(\eta_{123} - \eta_{132}) = +\tfrac{1}{2}\eta_{123} \tag{43}$$

$$\eta_{22} = \tfrac{1}{2}\eta_{2jk}\epsilon_{jk2} = \tfrac{1}{2}(\eta_{231} - \eta_{213}) \tag{44}$$

$$\eta_{33} = \tfrac{1}{2}\eta_{3jk}\epsilon_{jk3} = \tfrac{1}{2}(\eta_{312} - \eta_{321}) = -\tfrac{1}{2}\eta_{321} \tag{45}$$

$$\eta_{12} = \tfrac{1}{4}(\eta_{131} + \eta_{113} - \eta_{223}) \tag{46}$$

$$\eta_{23} = -\tfrac{1}{4}(\eta_{313} + \eta_{221} - \eta_{331}) \tag{47}$$

$$\eta_{13} = \tfrac{1}{4}(\eta_{323} - \eta_{121}) \tag{48}$$

Antisymmetric part of α tensor

$$\gamma_j = \tfrac{1}{2}\epsilon_{ijk}\alpha_{ik} \tag{49}$$

$$\gamma_1 = \tfrac{1}{2}(\alpha_{32} - \alpha_{23}) \tag{50}$$

$$\gamma_2 = \tfrac{1}{2}(\alpha_{13} - \alpha_{31}) \tag{51}$$

$$\gamma_3 = \tfrac{1}{2}(\alpha_{21} - \alpha_{12}) \tag{52}$$

If there is only z-dependence

If there is only z-dependence, only $2 \times 2 \times 2$ components will be relevant, i.e. α_{ij} and η_{ij3} for $i, j = 1, 2$. Furthermore, because there are only z derivatives,

$$\overline{B}^{(A)}_{j,k} = \overline{B}^{(S)}_{j,k}, \tag{53}$$

so the information contained in κ_{ijk} must be the same as in $\tilde{il}$, and so we can write

$$\overline{\boldsymbol{\mathcal{E}}}_i = \alpha_{ij}\overline{B}_j - \tilde{\eta}_{ij}\overline{J}_j, \quad i, j = 1, 2, \tag{54}$$

where $\overline{\boldsymbol{J}} = \boldsymbol{\nabla} \times \overline{\boldsymbol{B}}$ is the mean current density, and

$$\tilde{\eta}_{il} = \eta_{ijk}\epsilon_{jkl} \tag{55}$$

is the resistivity tensor operating only on the mean current density. Express these as

$$\delta_3 = -\tfrac{1}{2}(\eta_{113} + \eta_{223}) \tag{56}$$

$$\eta_{11} = +\eta_{123} \tag{57}$$

$$\eta_{22} = -\eta_{213} \tag{58}$$

$$\eta^{S}_{12} = \eta^{S}_{21} = \tfrac{1}{2}(\eta_{113} - \eta_{223}) \tag{59}$$

and for the α tensor

$$\gamma_3 = \tfrac{1}{2}(\alpha_{21} - \alpha_{12}) \tag{60}$$

$$\alpha^{S}_{11} = \alpha_{11} \tag{61}$$

$$\alpha^{S}_{22} = \alpha_{22} \tag{62}$$

$$\alpha^{S}_{12} = \alpha^{S}_{21} = \tfrac{1}{2}(\alpha_{12} + \alpha_{21}) \tag{63}$$

In the 1-D case it may be advantageous to stick to the original α_{ij} and η_{ij3} tensors, so we note therefore

$$\tilde{\eta}_{11} = +\tfrac{1}{2}\eta_{123} \tag{64}$$

$$\tilde{\eta}_{22} = -\tfrac{1}{2}\eta_{213} \tag{65}$$

$$\tilde{\eta}_{12} = -\tfrac{1}{2}\eta_{113} \tag{66}$$

$$\tilde{\eta}_{21} = +\tfrac{1}{2}\eta_{223} \tag{67}$$

Isotropic turbulence

$\alpha_{ij}(k=1)$

-0.150	-0.001	0.001
-0.001	-0.151	0.000
0.001	0.000	-0.147

$\eta_{ijz}(k=1)$

0.000	-0.032	0.000
0.032	0.000	0.000
0.000	0.000	0.000

$\alpha_{ij}(k=2)$

-0.144	-0.001	0.001
-0.001	-0.145	0.000
0.001	0.000	-0.131

$\eta_{ijz}(k=2)$

-0.000	-0.061	0.000
0.060	-0.000	0.000
0.000	0.001	0.000

$\alpha_{ij}(k=3)$

-0.136	0.000	0.001
0.001	-0.133	0.000
0.001	-0.000	-0.108

$\eta_{ijz}(k=3)$

0.001	-0.083	0.000
0.083	0.001	-0.000
0.001	0.000	0.001

nosoca32c31 (SOCA): α_{ij}

-4.708	-0.000
0.000	-4.708

η_{ij}

3.329	0.000
-0.000	3.329

α_{ij}

-0.190	-0.000
0.000	-0.190

(nosoca32c32_4proc, non-SOCA):

η_{ij}

-0.005	0.000
-0.000	-0.005

Helical homogeneous turbulence

etat	alpha	alp0	eta	urms	brms	tmax	run
0.0108	-0.0283	-0.0157	5e-3	0.119	0.128	337	impNOSOCA32a2 (started)
diverges.....		-0.0128	1e-3	0.141	0.151	601	impNOSOCA64a2
0.0078	-0.0261	-0.0091	1e-3	0.153	0.166	186	impNOSOCA128a2 (giga)
0.0088	-0.0410	-0.0157	5e-3	0.119	0.128	17514	imp32c
0.0145	-0.0647	-0.0128	2e-3	0.141	0.152	1235	imp64b
0.0156	-0.0711	-0.0089	1e-3	0.151	0.166	418	imp128b
0.0089	-0.0411	-0.0157	2e-3	0.119	0.128	8520	imp32b
0.0155	-0.0699	-0.0091	1e-3	0.150	0.165	487	imp64a (restarted)
0.0149	-0.0697	-0.00550	5e-4	0.155	0.175	525	imp128a

Helical homogeneous turbulence

Use $\eta = \nu = 0.1$

etat	alpha	urms	force	tmax	run
7.52e-5	3.83e-4	3.88e-02	5.0e-2	527	kin16a
7.52e-7	3.83e-6	3.89e-03	5.0e-3	588	kin16a

Nonhelical homogeneous turbulence with rotation

Note nonlinearity in Omega dependence of delta and etat.

A negative value of force0 indicates magnetic forcing.

etat	2del-kap	Omega	urms	brms	force0	eta	tmax	run
0.00622	0.00003	0.0	0.0663	0.131	-0.01	5e-3	4960	brot32o
0.00595	0.00027	0.01	0.0663	0.131	-0.01	5e-3	7027	brot32h
0.00615	0.00046	0.02	0.0663	0.131	-0.01	5e-3	5052	brot32g
0.00578	0.00063	0.05	0.0662	0.131	-0.01	5e-3	5218	brot32f
0.00539	0.00069	0.1	0.066	0.131	-0.01	5e-3	4526	brot32a
0.0049	0.0006	0.2	0.065	0.130	-0.01	5e-3	1690	brot32b (giga2)
0.00408	0.00039	0.5	0.060	0.134	-0.01	5e-3	4843	brot32c
0.00383	0.00018	1.0	0.0524	0.139	-0.01	5e-3	4909	brot32d
0.00363	0.00006	2.0	0.0432	0.145	-0.01	5e-3	4967	brot32e
		0.01			-0.01	5e-3		brotNOSOCA32h
		0.02			-0.01	5e-3		brotNOSOCA32g
		0.05			-0.01	5e-3		brotNOSOCA32f
0.00489	0.00090	0.1	0.0659	0.131	-0.01	5e-3	3411	brotNOSOCA32b
0.00417	0.00092	0.2	0.0695	0.131	-0.01	5e-3	8063	brotNOSOCA32a (stopped)
0.00381	0.00053	0.5	0.0600	0.134	-0.01	5e-3	3563	brotNOSOCA32c
0.00354	0.00024	1.0	0.0524	0.139	-0.01	5e-3	3541	brotNOSOCA32d
0.00340	0.00012	2.0	0.0432	0.145	-0.01	5e-3	3723	brotNOSOCA32e

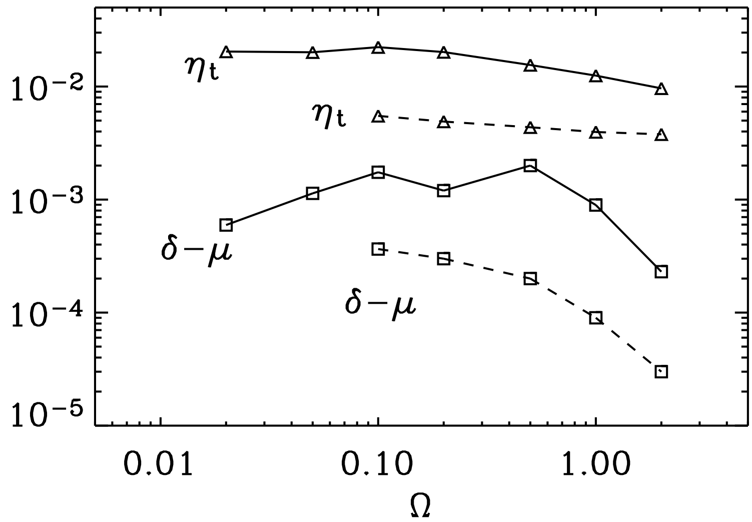

Figure 2: Dependence of η_t and $\delta-\kappa$ on Ω. Solid line is for kinetic forcing, and dashed line is for magnetic forcing.

0.0229	0.00078	0.01	0.149	0.000	+0.05	5e-3	3553	rot32f
0.0228	0.00133	0.02	0.149	0.000	+0.05	5e-3	3407	rot32g
0.0220	0.00245	0.05	0.149	0.000	+0.05	5e-3	3542	rot32h
0.0214	0.00354	0.1	0.149	0.000	+0.05	5e-3	10279	rot32c
0.02157	0.00470	0.2	0.150	0.000	+0.05	5e-3	12722	rot32a
0.0175	0.0040	0.5	0.150	0.000	+0.05	5e-3	13006	rot32b
0.01416	0.00177	1.0	0.152	0.000	+0.05	5e-3	4056	rot32d
0.00964	0.00046	2.0	0.151	0.000	+0.05	5e-3	4203	rot32e
0.01839	-.00005	0.0	0.149	0.000	+0.05	5e-3	12236	rotNOSOCA32o
		0.01	0.149	0.000	+0.05	5e-3		rotNOSOCA32g
		0.02	0.149	0.000	+0.05	5e-3		rotNOSOCA32g
		0.05	0.149	0.000	+0.05	5e-3		rotNOSOCA32f
0.01605	0.00174	0.1	0.149	0.000	+0.05	5e-3	3039	rotNOSOCA32c
0.01668	0.00371	0.2	0.149	0.000	+0.05	5e-3	3106	rotNOSOCA32a
0.0147	0.0077	0.5	0.150	0.000	+0.05	5e-3	4310	rotNOSOCA32b
0.0137	0.00401	1.0	0.152	0.000	+0.05	5e-3	3010	rotNOSOCA32d
0.0097	0.00101	2.0	0.150	0.000	+0.05	5e-3	3008	rotNOSOCA32e
0.0122	0.00048	0.1	0.100	0.159	-0.01	2e-3	538	brot64a
0.0094	0.00070	0.2	0.099	0.161	-0.01	2e-3	338	brot64b (lost data)
0.01003	0.00028	0.0	0.100	0.159	-0.01	2e-3	2433	brotNOSOCA64o (OK)
0.00866	0.00149	0.2	0.098	0.160	-0.01	2e-3	2508	brotNOSOCA64b

0.00529	0.00064	2.0	0.071	0.192	-0.01	2e-3	2976	brotNOSOCA64e
0.0257	0.00046	0.0	0.193	0.000	+0.05	2e-3	1724	rotNOSOCA64o (STOPPED)
0.0304	0.00325	0.5	0.202	0.000	+0.05	2e-3	805	rot64a (STOPPED)
0.0270	0.00045	0.2	0.195	0.000	+0.05	2e-3	2434	rotNOSOCA64b (ANALYZE)
0.0327	0.01212	0.5	0.202	0.000	+0.05	2e-3	748	rotNOSOCA64c
0.02221	0.00159	1.0	0.255	0.000	+0.05	2e-3	1891	rotNOSOCA64d (stopped)
0.02244	0.00130	2.0	0.240	0.000	+0.05	2e-3	2248	rotNOSOCA64e
0.01542	0.00004	0.0	0.119	0.179	-0.01	1e-3	213	brot128o (workq)
0.0088	0.0003	0.5	0.112	0.187	-0.01	1e-3	198	brot128a (cant read)
0.0110	0.00126	0.0	0.116	0.177	-0.01	1e-3	308	brotNOSOCA128o (giga2, XX)
0.00781	0.00160	0.5	0.112	0.187	-0.01	1e-3	259	brotNOSOCA128a3 giga2)
0.0121	0.00035	0.2	0.11	0.18	-0.01	1e-3	400	brot128b (restarted XX)
0.03324	-.03000	0.0	0.215	0.000	+0.05	1e-3	276	rotNOSOCA128o (giga2)
0.0394	0.00076	0.5	0.229	0.000	+0.05	1e-3	406	rotNOSOCA128a2 (blew up!?)
		0.5		0.000	+0.05	1e-3		rotNOSOCA128a3 (giga2)
0.04529	0.00011	0.0	0.216	0.000	+0.05	1e-3	209	rot128o2 (workq, ANALYSE)
0.0354	0.00171	0.5	0.229	0.000	+0.05	1e-3	318	rot128c

New simulations, SOCA.

Omega	nu	eta	etatm	etaxym	etayxm	urms	force	tmax	run
5e-2	5e-1	5e-1	1.17e-05	2.36e-08	-1.65e-08	0.037	1e-1	462	SOCArot16c
2e-1	5e-1	5e-1	1.17e-05	4.94e-08	-6.54e-08	0.037	1e-1	270	SOCArot16d
1e0	5e-1	5e-1	1.17e-05	3.31e-07	-3.19e-07	0.037	1e-1	442	SOCArot16a
2e0	5e-1	5e-1	1.17e-05	5.09e-07	-6.26e-07	0.037	1e-1	121	SOCArot16b

In all cases $\tau_c = 0.045$ and $St = 0.0083$.

Nonhelical homogeneous turbulence with shear

There is no rotation here (except in 2 cases).

etat	etaxy	etayx	Shear	urms	brms	force0	eta	tmax	SOCA	run
0.0134	0.0090	-.0028	-0.20	0.141	0.000	0.05	5e-3	53	N	kinshear32a Om=.5
0.0142	0.0139	0.0014	-0.20	0.150	0.000	0.05	5e-3	380	Y	kinshear32b
0.0209	0.0098	-.0040	-0.20	0.200	0.000	0.05	1e-3	12	N	kinshear64a Om=.5
0.00008	.0098	-.0040	-0.20	0.200	0.000	0.05	1e-3	887	Y	kinshear16a

Runs with $S = -0.2$, $f_0 = 0.05$.

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1e-1	1e-1	7.98e-05	2.50e-06	3.19e-06	0.038	5e-2	330	Y	kinshear16a
5e-2	5e-2	3.42e-04	2.80e-05	2.55e-05	0.052	5e-2	567	Y	kinshear16d
2e-2	2e-2	2.54e-03	6.54e-04	3.80e-04	0.083	5e-2	887	Y	kinshear16b

Runs with $S = -0.2$, $f_0 = 0.05$.

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1e-1	2e-1	7.98e-05	2.50e-06	3.19e-06	0.038	5e-2	330	Y	kinshear16a

Runs with $S = -0.2$, $f_0 = 0.05$.

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1e-1	2e-1	5.29e-05	8.66e-07	1.60e-06	0.038	5e-2	458	N	NOSOCkinshear16f
1e-1	1e-1	7.98e-05	2.74e-06	3.29e-06	0.038	5e-2	632	N	NOSOCkinshear16a
1e-1	5e-2	1.07e-04	5.43e-06	3.74e-06	0.038	5e-2	458	N	NOSOCkinshear16g
1e-1	2e-2	1.33e-04	1.04e-05	3.14e-06	0.038	5e-2	445	N	NOSOCkinshear16e
1e-1	1e-2	1.46e-04	1.37e-05	2.23e-06	0.038	5e-2	917	N	NOSOCkinshear16b
1e-1	5e-3	1.52e-04	1.52e-05	1.43e-06	0.038	5e-2	458	N	NOSOCkinshear16c
1e-1	2e-3	1.57e-04	1.91e-05	1.11e-06	0.038	5e-2	458	N	NOSOCkinshear16d
1e-1	1e-3	1.69e-04	1.83e-05	-3.05e-06	0.038	5e-2	110	N	NOSOCkinshear16h

Magnetic forcing, runs with $S = -0.2$, $f_0 = 0.05$.

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1e-1	2e-1	1.09e-06	1.21e-07	4.87e-08	0.007	5e-2	369	N	NOSOCmagshear16a
1e-1	1e-1	1.49e-05	2.75e-06	7.39e-07	0.016	5e-2	464	N	NOSOCmagshear16a
1e-1	5e-2	1.35e-04	3.77e-05	6.20e-06	0.035	5e-2	449	N	NOSOCmagshear16a

Magnetic forcing, runs with $S = -0.2$, $f_0 = 0.02$.

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1e-1	2e-2	6.07e-08	3.97e-09	9.51e-10	0.001	2e-2	289	N	NOSOCmagShear32f
1e-1	1e-2	5.41e-04	2.50e-04	2.80e-06	0.031	2e-2	448	N	NOSOCmagShear32b
1e-1	5e-3	1.49e-03	8.88e-04	-2.51e-05	0.048	2e-2	409	N	NOSOCmagShear32c

Magnetic forcing, runs with $S = -0.2$, no SOCA

nu	eta	etat	etaxy	etayx	urms	brms	force	tmax	run
1e-01	1e-01	2.59e-05	3.48e-06	1.12e-06	1.50e-02	1.80e-01	0.1	92.73	magnohel3
1e-01	1e-01	3.36e-04	3.40e-05	1.08e-05	5.60e-02	3.60e-01	0.1	30.27	magnohel3
1e-01	1e-01	2.98e-03	1.63e-04	1.81e-05	1.95e-01	6.86e-01	0.2	11.07	magnohel3
1e-01	1e-01	1.75e-02	5.59e-04	1.73e-04	5.80e-01	1.26e+00	0.4	4.19	magnohel
1e-01	1e-01	3.94e-02	2.07e-03	1.67e-03	1.25e+00	2.01e+00	0.8	0.61	magnohel
1e-01	1e-02	2.49e-02	4.12e-04	-6.39e-04	5.00e-01	1.22e+00	0.2	3.88	magnohel
1e-01	1e-03	2.95e-02	-1.35e-04	-2.40e-04	5.44e-01	1.44e+00	0.2	3.63	magnohel
1e-01	1e-04	2.95e-02	-2.30e-04	6.14e-05	5.48e-01	1.51e+00	0.2	3.62	magnohel

Kinetic forcing, runs with $S = -0.2$,

nu	eta	etat	etaxy	etayx	urms	force	tmax	SOCA	run
1.0e-01	1.0e-01	8.53e-05	2.83e-06	3.02e-06	3.60e-02	0.05	143.0	N	kinohel32D

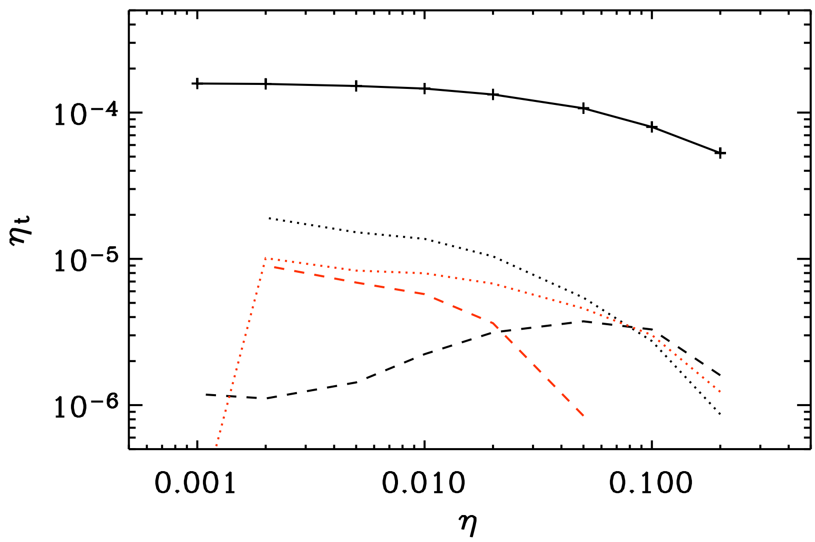

Figure 3: Dependence of $\eta_{\rm t}$ on η. Kinetic forcing.

New runs (September 2007)

Kinetic forcing, runs with $S = -0.1$.

Define $q_{ij} = \tilde{\eta}_{ij}/\tilde{\eta}_{\rm t}$ and $C_S = S/(\eta_{\rm t} k_1^2)$, so the growth rate becomes

$$\lambda = \eta_{\rm t} k_1^2 \left(-1 + \sqrt{q_{21}(q_{12} + C_S)}\right) \tag{68}$$

with $q_{21} \approx -0.1$, $q_{12} \approx +0.5$, $C_S \approx 30$, so that $\lambda/(\eta_{\rm t} k_1^2) \approx 2$.

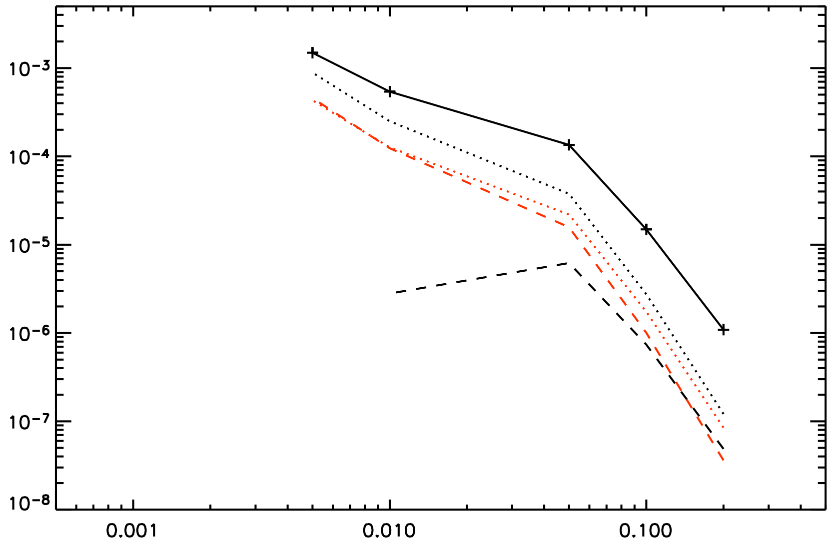

Figure 4: Dependence of $\eta_{\rm t}$ on η. Magnetic forcing.

Galloway–Proctor flow

$$\eta = 0.01 \quad \alpha_{ij} = \begin{pmatrix} -6.66 & -7.63 \\ 8.44 & 0.80 \end{pmatrix} \quad \eta_{ij} = \begin{pmatrix} -1.26 & -11.40 \\ 4.98 & -1.04 \end{pmatrix} \tag{69}$$

$$\eta = 0.02 \quad \alpha_{ij} = \begin{pmatrix} 1.58 & -0.53 \\ 0.44 & 0.01 \end{pmatrix} \quad \eta_{ij} = \begin{pmatrix} 1.27 & -0.35 \\ 0.48 & 1.24 \end{pmatrix} \tag{70}$$

$$\eta = 0.05 \quad \alpha_{ij} = \begin{pmatrix} 0.83 & -0.57 \\ 0.56 & 0.11 \end{pmatrix} \quad \eta_{ij} = \begin{pmatrix} 1.02 & -0.74 \\ 0.34 & 1.02 \end{pmatrix} \tag{71}$$

$Header: /var/cvs/brandenb/tex/mhd/alphaeta/notes.tex,v 1.63 2007/08/26 14:43:45 brandenb Exp $

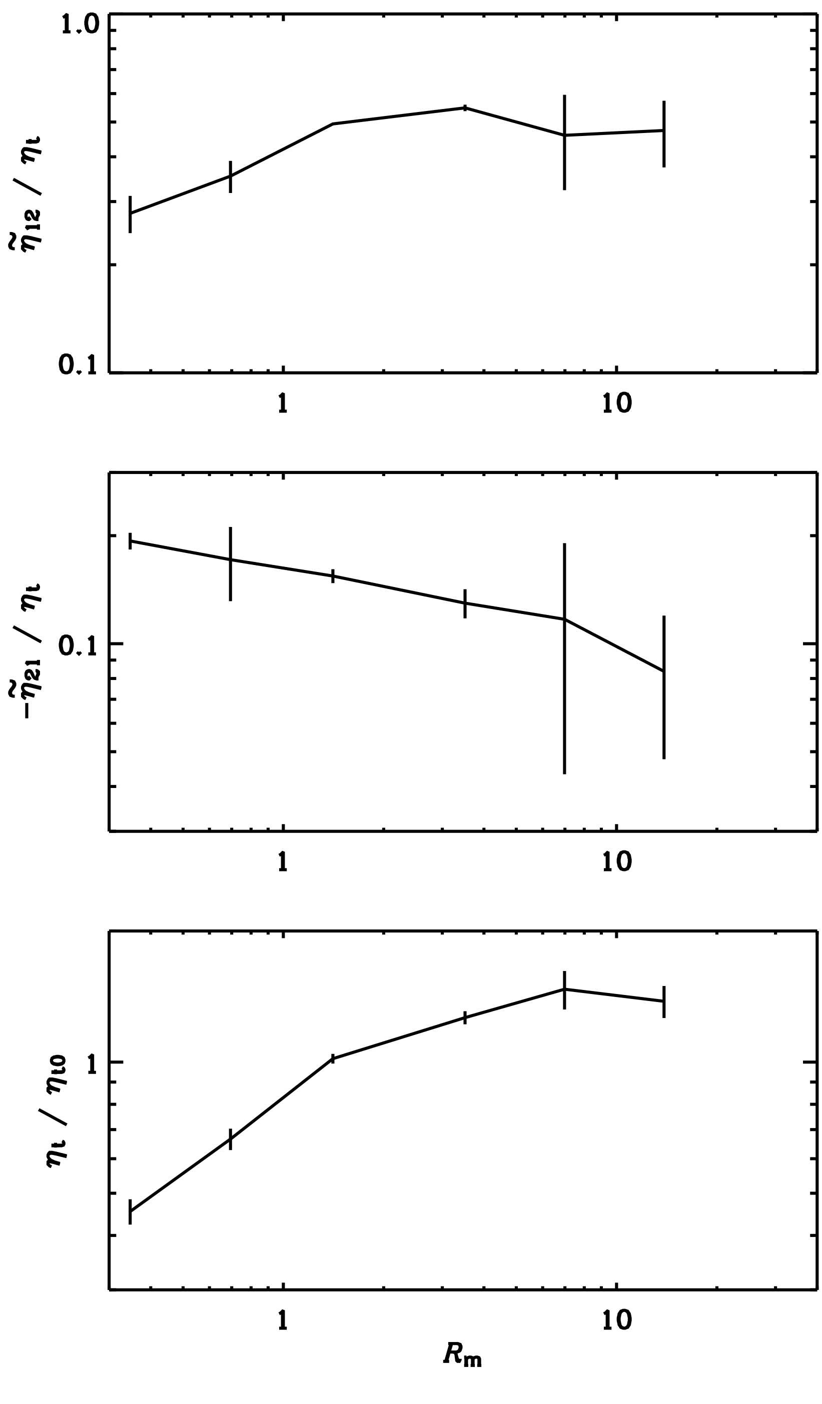

Figure 5: Roberts flow

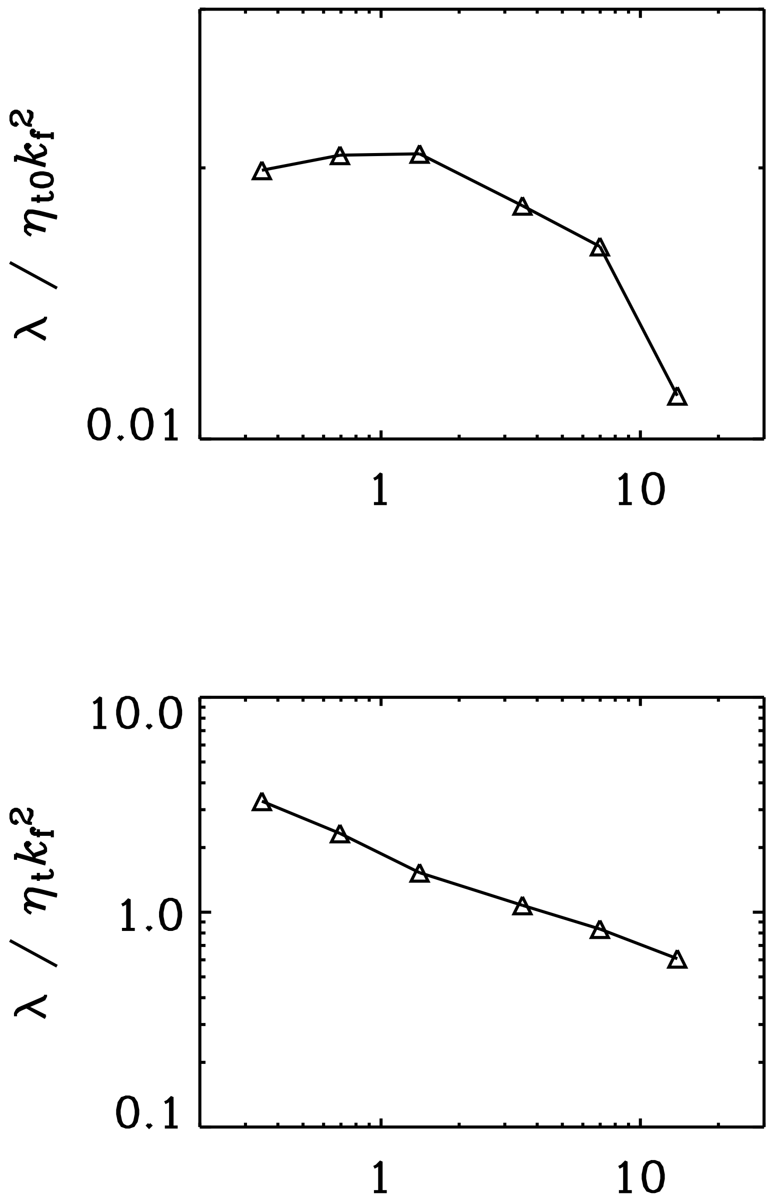

Figure 6: Roberts flow