

Some comments on the BRRK manuscript

1.

Concerning Section 1

Do we not consider *either* rotation *or* shear?

In the case of rotation (1) with $-2\mathbf{\Omega} \times \mathbf{U}$ added on the right-hand side and (2) apply.

In the case of shear (5) with $\mathbf{\Omega} = \mathbf{0}$ (and minus in front of $SU_x \hat{\mathbf{y}}$) and (6) apply.

If indeed simultaneously rotation *and* shear are considered I would first expect $2\mathbf{\Omega} \times (\mathbf{U} + \mathbf{U}^S)$ instead of $2\mathbf{\Omega} \times \mathbf{U}$ in (5). Of course, $\mathbf{\Omega} \times \mathbf{U}^S$ can be understood as a gradient and subsumed into the centrifugal term. Is this the reason why $2\mathbf{\Omega} \times \mathbf{U}^S$ does not occur?

It is not convincing to justify the neglect of the centrifugal force by “we shall not be interested in” as it surely influences the density profile and hence the flow. Instead we should be honest in saying that incompatibility with periodic boundary conditions is the main reason and giving some hints why in the considered parameter range the influence of the centrifugal force is small.

What means “weakly compressible”? Should we give a few examples of Mach numbers in the sections with the results?

Equation (7): \overline{B}^{pq} instead of \overline{B} ?

We should somewhere say that \mathbf{u} means $\mathbf{U} - \overline{\mathbf{U}}$.

2.

I find it difficult to see the clear logical line in Sections 1 and 2.

We have to explain three [or four] issues:

(a) the equations describing the turbulence

(first part of the actual Section 1)

(b) the structure of the mean electromotive force under our assumptions on the turbulence

(actually at several places in Section 2, see also section X below)

(c) the test field method

(actually a part of Section 2)

[(d) shear-current dynamo]

I would like to present the reader the logical units (b) and (c) in a less “distributed” form. Is perhaps the order (b) - (a) - (c) better?

3.

When restricting our considerations anyway on $\alpha_{ij} = 0$ it is without interest whether or not we put $\overline{B}_3 = 0$.

4.

I see no justification of (14) and (15). If e.g. (14) were true we had $\eta_{11}^* = \eta_{22}^*$, that is $\epsilon = 0$.

Do you have good reasons for the validity of (14) and (15), and did you introduce ϵ only in view of fluctuations of η_{11}^* and η_{22}^* which occur in the simulations?

Fig. 5 exhibits indeed rather small values of ϵ .

See section X below.

5.

It seems reasonable to discuss the results for the case of rotation in terms of η_t and δ .

In the case of shear we may surely put $\eta_{12}^* = \delta + \mu$ and $\eta_{21}^* = -\delta + \mu$. However, what is then the meaning of δ and μ ? Should we in this case not simply speak about the η_{ij}^* only? In the condition for the shear-current dynamo we have primarily η_{21}^* and not $\delta - \mu$!

By the way, we never discussed $\eta_{11}^* - \eta_{22}^*$.

6.

Concerning (19) and (20)

I think that the wave number which is here denoted by k_1 may be not too large but otherwise arbitrary. It is not the one related to the cell size in the simulations. Another notation, simply k ?

In these equations η is obviously put equal to zero, or η_t has to be interpreted as $\eta_t + \eta$.

As mentioned above we should use η_{12} and η_{21} instead of δ and μ .

In that sense I found

$$\begin{pmatrix} \lambda + (\eta_{11} + \eta)k^2 & S + \eta_{12}k^2 \\ \eta_{21}k^2 & \lambda + (\eta_{22} + \eta)k^2 \end{pmatrix} \begin{pmatrix} \bar{A}_x \\ \bar{A}_y \end{pmatrix} = 0 \quad (1)$$

and

$$\frac{\lambda_{1,2}}{(\eta_t + \eta)k^2} = -1 \pm \frac{\sqrt{(S/k^2 + \eta_{12})\eta_{21} + \epsilon^2}}{\eta_t + \eta}. \quad (2)$$

7.

Concerning Section 3.2

The result on the impossibility of exponentially growing solutions goes in two respects beyond the (analytical) results of Rüdiger and Kichatinov and of Rädler and Stepanov: it is not restricted to SOCA, and it considers (weakly) compressible turbulence. Of course, the situation with magnetically driven turbulence remains to be investigated.

Stand $\tilde{\eta}_{12}$ and $\tilde{\eta}_{21}$ in Fig. 4 for η_{12}^* and η_{21}^* ?

X.

Assume first

$$\mathcal{E}_i = \mathcal{E}_i^{(0)} + \alpha_{ij}\bar{B}_j + \eta_{ijk}\partial\bar{B}_j/\partial x_k. \quad (3)$$

Assume that there is no small-scale dynamo so that $\mathcal{E}^{(0)} = \mathbf{0}$ (or admit only such small-scale dynamos for which $\mathcal{E}^{(0)} = \mathbf{0}$). Assume that all averaged quantities are independent of x and y . Then all (first-order) derivatives of $\bar{\mathbf{B}}$ can be expressed by $\bar{\mathbf{J}} = \nabla \times \bar{\mathbf{B}} = (-\partial\bar{B}_y/\partial z, \partial\bar{B}_x/\partial z, 0)$, and \bar{B}_z is independent of z . Then (3) may be replaced by

$$\mathcal{E}_i = \alpha_{ij}\bar{B}_j - \eta_{ij}\bar{J}_j. \quad (4)$$

(Since $\bar{J}_3 = 0$ we may put $\eta_{i3} = 0$ but we will do so only later.) Assume homogeneous background turbulence. Then α_{ij} and η_{ij} are independent of z . Assume in addition that the background turbulence is isotropic and non-helical. (The actual turbulence under the influence of rotation has of course helical features although the mean kinetic helicity density remains zero.) Then we have $\alpha_{ij} = 0$. (I do believe that this conclusion is correct but I see at the moment no rigorous proof.)

Consider first the case of rotation about an axis aligned with $\mathbf{e} = (0, 0, 1)$ but exclude any shear. Then the actual turbulence is axisymmetric about the rotation axis, and therefore η_{ij} must have the structure

$$\eta_{ij} = \eta_0\delta_{ij} + \delta\epsilon_{ijk}e_k + \delta'e_ie_j \quad (5)$$

with three coefficients η_0 , δ and δ' . Considering $\bar{J}_3 = 0$ we put now $\eta_{i3} = 0$ and obtain so

$$\eta_{ij} = \eta_0(\delta_{ij} - e_ie_j) + \delta\epsilon_{ijk}e_k. \quad (6)$$

Thus we have

$$\mathcal{E} = -\eta_0 \bar{\mathbf{J}} + \delta \mathbf{e} \times \bar{\mathbf{J}}. \quad (7)$$

Note that $\mathcal{E}_z = 0$.

Consider next the case of shear but without rotation. The turbulence is then no longer axisymmetric. Apart from δ_{ij} the only construction element for η_{ij} is $U_{kl} = \partial U_k^S / \partial x_l$. Without further specification of U_{kl} we have

$$\begin{aligned} \eta_{ij} = & \eta_0 \delta_{ij} + \kappa_1 U_{ij} + \kappa_2 U_{ji} + \kappa_3 \delta_{ij} U_{kk} \\ & + \kappa'_1 U_{ik} U_{jk} + \kappa'_2 U_{ki} U_{kj} + \kappa'_3 U_{ik} U_{kj} + \kappa'_4 U_{jk} U_{ki} + \kappa'_5 U_{ij} U_{kk} + \kappa'_6 U_{ji} U_{kk} + \dots, \end{aligned} \quad (8)$$

where \dots stands for terms of third and higher order in U_{kl} . Our assumption on \mathbf{U}^S implies $U_{kl} = S \delta_{k2} \delta_{l1}$. This leads to

$$\eta_{ij} = \eta_0 (\delta_{ij} - \delta_{i3}) + \mu_{12} \delta_{i1} \delta_{j2} + \mu_{21} \delta_{i2} \delta_{j1} + \mu_{11} \delta_{i1} \delta_{j1} + \mu_{22} \delta_{i2} \delta_{j2}, \quad (9)$$

or

$$\eta_{11} = \eta_0 + \mu_{11}, \quad \eta_{12} = \mu_{12}, \quad \eta_{21} = \mu_{21}, \quad \eta_{22} = \eta_0 + \mu_{22}. \quad (10)$$

The η_0 term in (8) has been modified such that $\eta_{i3} = 0$. The μ_{12} and μ_{21} are odd functions of S vanishing like S as $S \rightarrow 0$, the μ_{11} and μ_{22} even functions of S vanishing like S^2 as $S \rightarrow 0$.

Here we have again $\mathcal{E}_z = 0$.

(Does μ_{11} and μ_{22} coincide? At the moment I see no reason for that. The analytical (SOCA) calculations by Rädler and Stepanov are restricted to linearity in U_{kl} and say therefore nothing about μ_{11} and μ_{22} . The smallness of ϵ exhibited in Fig. 5 could be a consequence of the fact that μ_{11} and μ_{22} are proportional to S^2 for small S .)