Magnetically driven matter power spectrum and clumping factor during recombination

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1 Baryon density spectrum

$$\mathrm{Sp}(\boldsymbol{B}) \sim k^{-5/3} \tag{1}$$

$$\mathrm{Sp}(\boldsymbol{J}\times\boldsymbol{B}) \sim k^{1/3} \tag{2}$$

$$\mathrm{Sp}(\rho) \sim \mathrm{Sp}(\boldsymbol{\nabla}\cdot\boldsymbol{u}) \sim \mathrm{Sp}(\boldsymbol{\nabla}\cdot(\boldsymbol{J}\times\boldsymbol{B})) \sim k^{7/3} \tag{3}$$

2 Dissipation wave number

$$k_{\mathrm{d}} \sim (\boldsymbol{\omega}_{\mathrm{rms}}/\nu)^{1/2} \tag{4}$$

$$\boldsymbol{\omega} = \boldsymbol{\nabla}\times\boldsymbol{u} \tag{5}$$

$$k_{\mathrm{mag}} \sim (\boldsymbol{J}_{\mathrm{rms}}/\eta)^{1/2} \tag{6}$$

$$\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B} \tag{7}$$

3 Introduction

During the radiation dominated era of the universe, the viscosity of the plasma is mainly due to radiation. At a redshift of around 10^8, the mean free path of the photons grows with time and leads to a sharp increase in the radiative viscosity. At the same time, the magnetic diffusivity remains comparatively small, and therefore the magnetic field can arguably retain an extended inertial range for a long time. As the mean free path continues to increase, the photons begin to stream freely and start to damp the velocity at all length scales rather than making the plasma behave viscously. During this time, the magnetic field still remains almost completely unaffected.

As the universe expands further, the radiation energy density decreases and becomes eventually smaller than the thermal energy of the baryons. Both can become even smaller than the magnetic energy density. This means that, if the magnetic field is strong enough, it can now affect the plasma motions. The radiative effects on the plasma are at that time no longer viscous, but the plasma motions are uniformly damped at all length scales. The magnetic field can then have a dramatic effect on the plasma, especially at the small resistive length scales.

The resulting Lorentz force can induce strong nonuniformities in the plasma at resistive length scales. In ref. [**?**], these effects are described as clumping and quantifies this through a time dependent clumping factor

$$b(t) = \langle[\rho_{\mathrm{b}}(\boldsymbol{x},t) - \overline{\rho}_{\mathrm{b}}(t)]^2\rangle/\overline{\rho}_{\mathrm{b}}(t)^2, \tag{8}$$

where ρ_{b} is the baryon density. The authors find values up to $b = 1.6$. Such a large clumping factor can affect the recombination history of the universe. It would lower the sound horizon and lead to a smaller Hubble parameter around the time of recombination. The difference between the Hubble parameters during recommendation and the contemporary low redshift universe is then not really a "tension," but an indicator of how strong the magnetic effect was.

4 Terminal velocity approximation

neglecting $\mathrm{D}\boldsymbol{u}/\mathrm{D}t$ and solve for $\alpha\boldsymbol{u}$; valid when $uk/\alpha \ll 1$ (definitions of a Reynolds number); $E_{\mathrm{K}}(k,t)$ becomes smaller when α is increased.

5 Blue spectra for the Lorentz force

Here, we explain qualitatively why the spectrum of $\boldsymbol{J}\times\boldsymbol{B}$ does not behave like that of a product of two functions with white noise. We also show that for $\boldsymbol{U}\times\boldsymbol{B}$, it is actually true that the spectrum is white as long as $\boldsymbol{U}$ is independent of $\boldsymbol{B}$. Even for

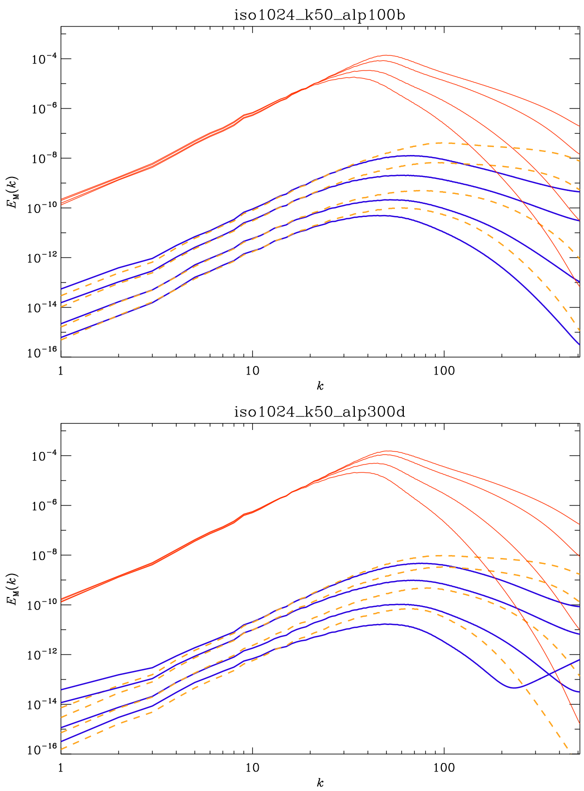

Figure 1: No terminal velocity approximation, $\alpha = 100$ and 300. Times $t = 5$, 20, 100, and ≈ 300. 1. amplaa=5e-2, kpeak_aa=50, eta=1e-6, nu=5e-6. 2. amplaa=5e-2, kpeak_aa=50.

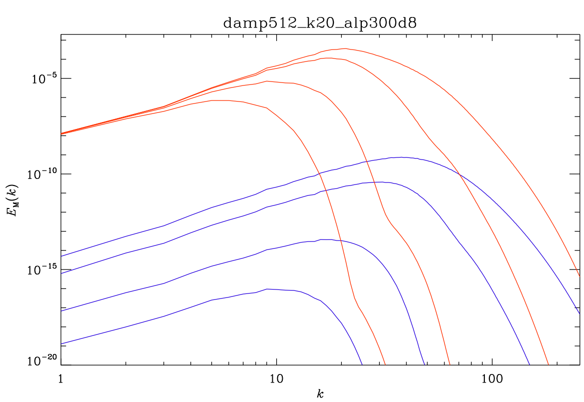

Figure 2: With terminal velocity approximation with $\alpha = 1000$, $\delta t = 3\times 10^{-3}$, and $\eta = 10^{-4}$. Times $t = 5$, 20, and ≈ 100.

$(\boldsymbol{J}\times\boldsymbol{B})\times\boldsymbol{B}$, the spectrum is white. Finally, we also demonstrate that the magnetic power in the inertial range directly affects the velocity subinertial range power.

5.1 Spectra

We define the spectrum Sp of a scalar or vectorial quantity, B or $\boldsymbol{B}$, respectively, such that its integral over the wavenumber k obeys $\int \mathrm{Sp}(\boldsymbol{B})\,\mathrm{d}k = \langle \boldsymbol{B}^2\rangle$. In 1-D, the spectrum of a δ-correlated function, which we call white noise, is flat, i.e., $\propto k^0$. In 3-D, owing to the integration over concentric shells in wavenumber space, which picks up an extra $4\pi k^2$ factor, a white noise spectrum increases quadratically with k, so it is $\propto k^2$.

For functions $\boldsymbol{B}$ with red spectra, where $\mathrm{Sp}(\boldsymbol{B}) \propto k^n$ with $n < 0$, the spectrum of $\boldsymbol{B}^2$ is also red, i.e., $\mathrm{Sp}(\boldsymbol{B}^2) \propto k^n$ with the same value of n. But this is not true for blue spectra, where $n > 0$ in 1-D and $n > 2$ in 3-D. In those cases, the result is always a white spectrum [**?**].

5.2 Lorentz force spectra

In view of the results of Ref. [**?**], we expect the spectrum of the product of two functions with blue subinertial range spectra to be always white. But this idea does not apply to the Lorentz force, $\boldsymbol{J}\times\boldsymbol{B}$, whose spectrum is blue, in spite of the fact that the spectra of both $\boldsymbol{J}^2$ and $\boldsymbol{B}^2$ are indeed white. The reason for this is connected with the fact that $\boldsymbol{J}$ and $\boldsymbol{B}$ are related to each other through a derivative. Indeed, one component to $\boldsymbol{J}\times\boldsymbol{B}$ is $-\boldsymbol{\nabla}(\boldsymbol{B}^2/2)$, so since the spectrum of $\boldsymbol{B}^2$ is white, the spectrum of its gradient involves an explicit k^2 factor, so it becomes blue.

This can easily be verified even in the one-dimensional case. Assume a white noise spectrum for a scalar function $A(x)$, so the spectrum $\mathrm{Sp}(A) \propto k^0$ is flat. Then, we have for its derivatives $B = \partial A/\partial x$ and $J = \partial B/\partial x$, as expected,

$$\mathrm{Sp}(B) \propto k^2, \quad \text{but} \quad \mathrm{Sp}(B^2) \propto k^0, \tag{9}$$

and

$$\mathrm{Sp}(J) \propto k^4, \quad \text{but} \quad \mathrm{Sp}(J^2) \propto k^0. \tag{10}$$

However, because $JB = (1/2)\,\partial(B^2)/\partial x$, we have

$$\mathrm{Sp}(JB) \propto k^2 \mathrm{Sp}(B^2) \propto k^2. \tag{11}$$

This is demonstrated in Figure 3.

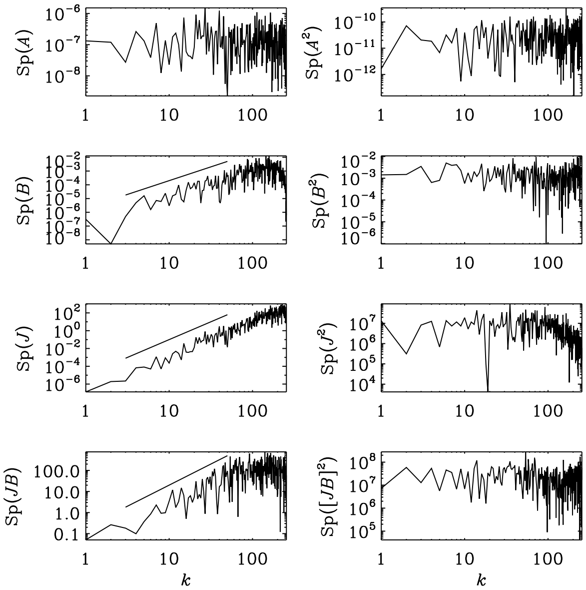

Figure 3: 1-D spectra of scalar functions A, $B = A'$, $J = B'$, and JB (left column) and their squares (right column), where A has a white noise spectrum.

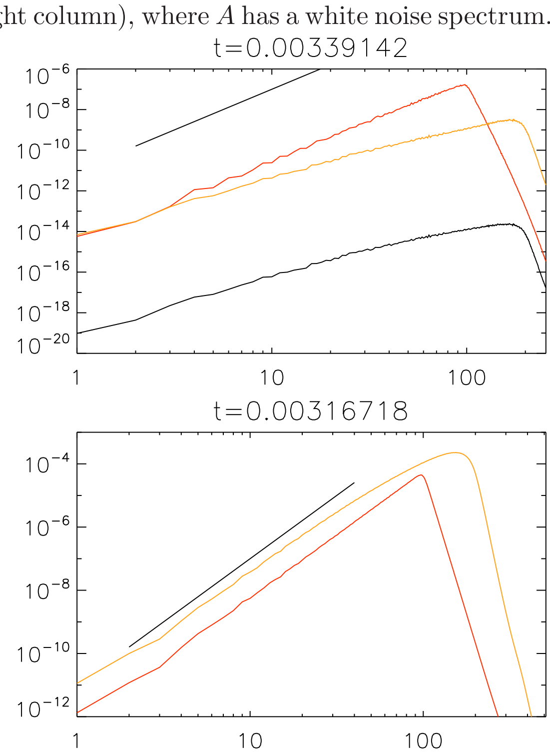

Figure 4: 2D and 3D.

5.3 Spectra of other products

We now verify that the spectra of other products, e.g., UB or $\boldsymbol{U} \times \boldsymbol{B}$, remain white. This is demonstrated in Figure 5. In the last panel of Figure 5, we also demonstrate that, if $\boldsymbol{U} \rightarrow \boldsymbol{J} \times \boldsymbol{B}$, the spectrum is also white.

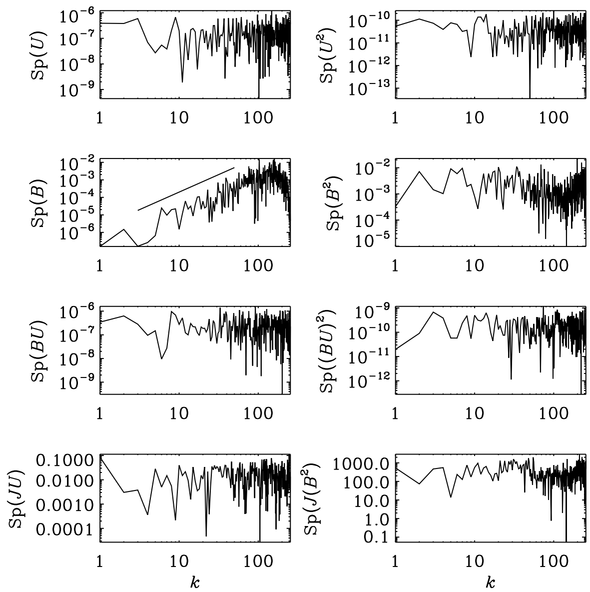

Figure 5: 1-D spectra of scalar functions U, $B = A'$ (same as in Figure 3), UB, and UJ (left column) and their squares (right column), where U independent of A, but also has a white noise spectrum.

6 Dependence of the low wavenumber velocity on the spectral magnetic cutoff

We now compare spectra with an extended UV tail, i.e., we compute the spectrum of JB or $\boldsymbol{J} \times \boldsymbol{B}$ in cases where Sp(A) follows a broken power law, which peaks at a wavenumber $k_{\rm p}$ and is followed by a tail k^{-m} for $k > k_{\rm p}$, where $m > 0$ takes different values. The result is shown in Figure 6.

Note that the IR level of all spectra (perhaps with the exception of that for A) decreases with increasing values of m. This explains why for the Lorentz force spectra and the resulting velocity spectra the subinertial ranges contain progressively less power as m increases. In our simulations, a loss of spectral power at high wavenumbers is caused by magnetic