1 From axion inflation through the inverse magnetic cascade to today

Axion inflation provides a possible means of amplifying magnetic fields through Chern-Simons coupling of a pseudoscalar axion-like field to gauge fields [1–5]. Towards the end of inflation, Schwinger pair production leads to an electrically conducting plasma [6–9]. It produces therefore the conditions necessary for a magnetohydrodynamic description

The resulting electric currents lead to a Lorentz force that drives bulk motions in the plasma. Those motions are turbulent and affect the magnetic field evolution. They produce not only an accelerated decay of the magnetic field, but they also transfer magnetic energy to progressively larger length scales [10]. This process has previously been studied in isolation without the underlying magnetogenesis that produces the magnetic field [11, 12].

Here we study the magnetohydrodynamic (MHD) evolution of such a plasma in combination with the magnetogenesis process.

2 The model

The equations of motion for the MHD evolution at the end of inflation we already discussed in Ref. []. Owing to the presence of an axion-like field field $\phi(\boldsymbol{x},t)$, the Maxwell–Ampère equation takes the form

$$\partial_\tau^2\phi + 2\mathcal{H}\partial_\tau\phi - \boldsymbol{\nabla}^2\phi + a^2\frac{dV}{d\phi} = \frac{\alpha}{a^2 f}\boldsymbol{E}\cdot\boldsymbol{B}, \quad (2.1)$$

$$\partial_\tau\boldsymbol{E} - \mathrm{rot}\,\boldsymbol{B} + \frac{\alpha}{f}\left(\partial_\tau\phi\boldsymbol{B} + \boldsymbol{\nabla}\phi\times\boldsymbol{E}\right) + \boldsymbol{J} = 0, \quad (2.2)$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -\frac{\alpha}{f}\boldsymbol{\nabla}\phi\cdot\boldsymbol{B}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0, \quad (2.3)$$

$$\partial_\tau\boldsymbol{B} + \mathrm{rot}\,\boldsymbol{E} = 0, \quad (2.4)$$

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}a^2\left(\rho_\phi + \rho_E + \rho_B + \rho_\chi\right), \quad (2.5)$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the comoving electric and magnetic fields, $\boldsymbol{J}$ is the comoving current density, α is the coupling constant, f is the axion decay constant, and $\mathcal{H} = \partial_\tau a/a$ is the conformal Hubble parameter.

The physical energy densities are $\rho_\phi = \left\langle(\partial_\tau\phi)^2/2a^2 + (\boldsymbol{\nabla}\phi)^2/2a^2 + V\right\rangle$ for the axion, $\rho_E = \left\langle\boldsymbol{E}^2\right\rangle/2a^4$ for the electric field, $\rho_B = \left\langle\boldsymbol{B}^2\right\rangle/2a^4$ for the magnetic field, and ρ_χ for the plasma. Here, $\langle ... \rangle$ denotes volume averaging over the simulation domain. We use the temporal gauge.

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