Primordial magnetic field from chiral plasma instability with sourcing

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ABSTRACT

I. INTRODUCTION

There is a rich interplay between particle asymmetries and primordial magnetism in the early universe. Particle asymmetries are expected to arise in the early universe during the epoch of baryogenesis at which time the cosmological excess of matter over antimatter was established. Prior work has established that a chiral asymmetry, corresponding to an excess (or deficit) of right-chiral particles and antiparticles over left-chiral particles and antiparticles, opens an instability toward the growth of helical magnetic fields, which is known as the chiral plasma instability (CPI). Various studies have explored the development of chiral magnetic instability using both analytical methods and direct numerical simulation of the nonlinear equations of motion of chiral magnetohydrodynamics (χMHD).

The primordial magnetic field that is generated in this way may survive in the universe today as a relic of the early universe. This primordial magnetic field may have played an essential role in seeding the galactic dynamo, thus explaining the origin of observed micro-Gauss galactic magnetic fields. In the voids between galaxies, the primordial magnetic field would constitute an intergalactic magnetic field (IGMF). The presence of a nonzero IGMF in our Universe has not yet been firmly established, but astrophysical measurements involving observations of TeV blazars have provided evidence for a nonzero IGMF at the level of $10^{-14}\,$G on Mpc length scales. Since the origin of cosmological magnetism remains an open question, CPI makes it to ask whether such fields may have arisen in the early universe as a byproduct of baryogenesis's particle asymmetries.

While particle asymmetries associated with some charges, such as the baryon number minus the lepton number ($\mathsf{B}-\mathsf{L}$), are predicted to be exactly conserved, the particle asymmetry associated with chirality is not conserved in the Standard Model. Rather, scatterings of chiral fermions can change their chirality if the scattering is mediated by the Yukawa interaction with the Standard Model Higgs (or if the fermion's Dirac mass is involved). Once these scatterings come into thermal equilibrium in the primordial plasma, they deplete the chiral asymmetry to zero exponentially quickly (assuming that the asymmetry is not also being sourced). A study of these scattering processes reveals that the chirality should be depleted to zero when the primordial plasma cools to a temperature of approximately $T \approx 80\,\mathrm{TeV}$. At this time the age of the Universe was only $t \approx 4 \times 10^{-17}\,\mathrm{sec}$. This means that CPI, which requires the chiral asymmetry to be nonzero in order to open the instability, must take before the chiral asymmetry is erased.

However, the preceding discussion is modified if the chiral asymmetry is being sourced. For instance, baryogenesis generically entails an out-of-equilibrium scattering or decay that sources a chiral asymmetry and leads to the baryon asymmetry of the universe. In this work we study the development of the chiral plasma instability in the presence of a source for chirality in order to derive predictions for the relic primordial magnetic field. We argue that the source term allows the CPI to be active even at a temperature below the nominal chiral erasure temperature of 80 TeV.

II. A SOURCE FOR CHIRALITY

[AL: Andrew, add some references to this section by way of motivating that a chiral source comes from baryogenesis]

Although baryogenesis occurs in the early universe, chirality may be sourced by out-of-equilibrium scattering, decay, or oscillations. Even chirality-violation is not a necessary ingredient for baryogenesis, both charge- and parity-violation are necessary ingredients [1]. Given the close connection between chirality and parity, many mod-

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els of baryogenesis also entail some level of chirality violation. In this section we provide a simple particle physics toy model that leads to a nonzero chiral source $\mathbb{S}_5(t)$, and we use it to motivate the functional form of $\mathbb{S}_5(t)$ that we use for our numerical studies, which are presented in the next section.

[AL: Andrew, what's the Lagrangian for this model?]

Consider a metastable particle species ϕ that decays to chiral electron-positron pairs via two reactions:

$$\begin{aligned} \Gamma_R &= \Gamma(\phi \to e_R^- + e_R^+ + X) \\ \Gamma_L &= \Gamma(\phi \to e_L^- + e_L^+ + \bar{X}) \,. \end{aligned} \tag{1}$$

Here e_L^- denotes a left-chiral electron, e_R^- denotes a right-chiral electron, e_L^+ denotes a left-chiral positron, and e_R^+ denotes a left-chiral positron. There may be other particles in the final states, which we denote by X and $\bar{X}$, and these particles do not carry any chirality. The reaction with rate Γ_R increases the electron chirality by two units, while the reaction with rate Γ_L decreases it by two units. Without loss of generality we can write $\Gamma_R = \big(1+\epsilon/2\big)\,\beta\,\Gamma_\phi$ and $\Gamma_L = \big(1-\epsilon/2\big)\,\beta\,\Gamma_\phi$ where Γ_ϕ is the total decay rate of a ϕ particle, which may include additional channels beyond the two shown above, and where 2β is the branching ratio into either of the chirality-changing channels. Note that $0 < 2\beta \leq 1$ and $-2 < \epsilon < 2$.

We suppose that there's a population of ϕ particles having an approximately homogeneous number density $n_\phi(t_c)$ at time t_c.[1] We assume that these particles decay out of equilibrium, which means that the inverse processes, including $e_R^- + e_R^+ + X \to \phi$, effectively do not occur. If $\epsilon = 0$ such that $\Gamma_R = \Gamma_L$, then on average no chirality is generated when the population of particles ϕ decays. In contrast, if $\epsilon \neq 0$ then the decaying population of ϕ particles is a source of chirality. Let $n_5(\boldsymbol{x}, t_c)$ represent the density of chirality at position $\boldsymbol{x}$ and time t_c, i.e. the number of right-chiral particles minus the number of left-chiral particles per unit volume. For a system at temperature T with chemical potential μ_5 this density is given by $n_5 = (k_B^2/\hbar^3 c^3)(\mu_5 T^2/6)$. The rate of chirality generation per unit volume is given by

$$\tfrac{\partial}{\partial t_c} n_5(\boldsymbol{x}, t_c) = \mathbb{S}_5(t_c) + \cdots \,, \tag{2}$$

where there may be additional terms accounting for the washout or diffusion of chirality. In our toy model, the chirality source term is given by

$$\mathbb{S}_5(t_c) = \big(\Gamma_R - \Gamma_L\big)\, n_\phi(t_c) = \epsilon\beta\Gamma_\phi n_\phi(t_c) \,. \tag{3}$$

[1] In the flat Friedman-Lemaitre-Robertson-Walker (FLRW) cosmology the invariant interval is $(\mathrm{d}s)^2 = -(\mathrm{d}t_c)^2 + a(t_c)^2|\mathrm{d}\boldsymbol{x}|^2 = -a(t)^2(\mathrm{d}t)^2 + a(t)^2|\mathrm{d}\boldsymbol{x}|^2$ where t_c is called cosmic time and t is called conformal time, $ddt = \mathrm{d}t_c/a$ with $a(t_c)$ is the scale factor, normalized to be 1 at today moment.

The time dependence of $\mathbb{S}_5(t_c)$ follows the time dependence of $n_\phi(t_c)$, which we now proceed to calculate.

We can say that the ϕ particles decay at a time $t_c = \tau_\phi$ where $\tau_\phi = \Gamma_\phi^{-1}$ is their lifetime. We assume that the ϕ particles are non-relativistic when they decay, allowing us to approximate their energy density by only their rest energy $\rho_\phi \approx m_\phi c^2 n_\phi$ where m_ϕ is the mass of a ϕ particle. When the ϕ particles decay, we assume that the cosmological energy budget of the universe is dominated by radiation, having temperature T and energy density $\rho_{\text{rad}} = (k_B^4/\hbar^3 c^3)(\pi^2/30) g_E T^4$ where $g_E = 106.75$. The ϕ particles should compose a sub-dominant component of the energy budget, and therefore it is useful to define $\Omega_\phi = \rho_\phi/\rho_{\text{rad}}$, which gives their fraction of the energy budget at the time when they decay. We can use this relation to exchange the variable $n_\phi(\tau_\phi)$ for the variable $\Omega_\phi(\tau_\phi)$.

As a result of the cosmological expansion, the number density of ϕ particles will decrease $\propto a(t_c)^{-3}$; In a radiation-dominated universe $a(t_c) = a(\tau_\phi)\,(t_c/\tau_\phi)^{1/2}$. As a result of the decays, the density will decrease further $\propto \exp(-\Gamma_\phi t)$. Putting together these various factors, we can write the chirality source term as follows,

$$\mathbb{S}_5(t) = \epsilon\beta\Gamma_\phi \frac{\rho_{\text{rad}}\Omega_\phi}{m_\phi c^2} \big(t_c/\tau_\phi\big)^{-3/2} \exp[-\Gamma_\phi(t_c - \tau_\phi)] \,. \tag{4}$$

Notice that $\mathbb{S}_5(t)$ diverges as $t_c \to 0$, due to the $a(t_c)^{-3}$ factor. However, this divergence is removed if we work instead with comoving densities and conformal time. The comoving density obeys an equation like Eq. (2) but where the term on the right side is instead $S_5(t_c) = a(t_c)^4 \mathbb{S}_5(t_c)$. We also exchange cosmic time t_c for conformal time t using $t_c = a(\tau_\phi)^2 t^2/4\tau_\phi$, since $a \sim t_c^{1/2} \sim t^1$ in the radiation-dominated era. After these changes, the new source term is

$$S_5(t) = \bar{S}_5 \, \frac{t}{t_\phi} \, \mathrm{e}^{-t^2/t_\phi^2} \,, \tag{5a}$$

where the parameters $\bar{S}_5$ and t_ϕ are given by

$$\bar{S}_5 = \epsilon\beta\mathrm{e}\frac{\rho_{\text{rad}}\Omega_\phi}{m_\phi c^2}\frac{a(\tau_\phi)^4}{\tau_\phi} \quad \text{and} \quad t_\phi = \frac{2\tau_\phi}{a(\tau_\phi)} \,. \tag{5b}$$

Here we have used $\Gamma_\phi = \tau_\phi^{-1}$ and $\mathrm{e} \approx 2.718$ is the base of the natural logarithm. Note that t_ϕ is the value of the conformal time t when the cosmic time is $t_c = \tau_\phi$ in the radiation era. We present a graph of $S_5(t)$ in Fig. 1. Note that $S_5(t)$ increases linearly in the early stages, reaches a maximum of $\bar{S}_5/\sqrt{2\mathrm{e}}$ in $t = t_\phi/\sqrt{2}$, and falls exponentially in the later stages. We use this parametrization for our numerical studies.

III. EQUATIONS OF χMHD

The dynamical variables in χMHD are the magnetic vector potential $\boldsymbol{A}(\boldsymbol{x}, t)$, the magnetic field $\boldsymbol{B}(\boldsymbol{x}, t)$, the

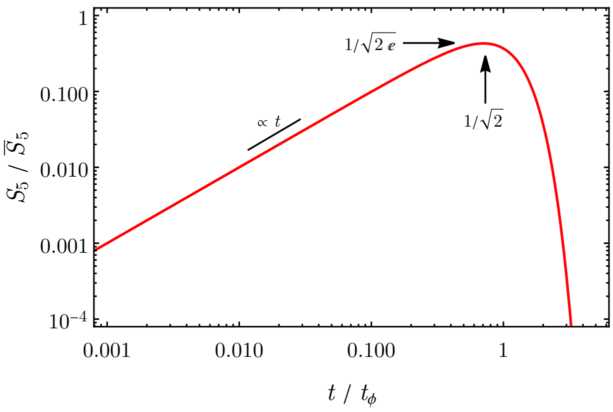

FIG. 1: Time dependence of the chiral source term.

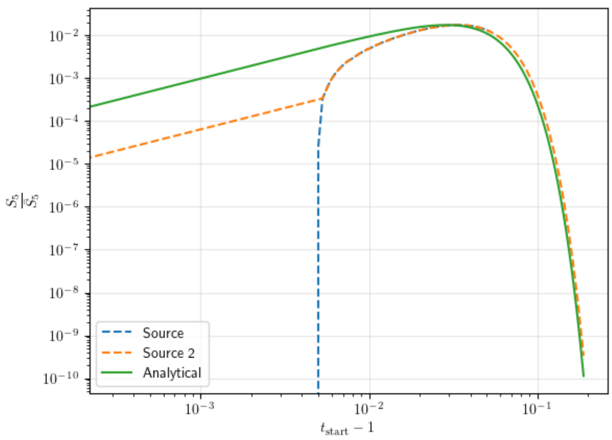

FIG. 2: Time dependence of the chiral source term. Numerical case

electric scalar potential $V(\boldsymbol{x},t)$, the electric field $\boldsymbol{E}(\boldsymbol{x},t)$, the electric charge density $Q(\boldsymbol{x},t)$, the electric current density $\boldsymbol{J}(\boldsymbol{x},t)$, the Lorentz-invariant fluid energy density $\rho(\boldsymbol{x},t)$, the Lorentz-invariant fluid pressure $p(\boldsymbol{x},t)$, the fluid velocity $\boldsymbol{u}(\boldsymbol{x},t)$, and the chiral chemical potential $\mu_5(\boldsymbol{x},t)$. The fields are related by $\boldsymbol{E} = -\boldsymbol{\nabla} V - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{A}$ and $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, and we work in the Weyl gauge $V(\boldsymbol{x},t) = 0$. We assume a relativistic equation of state, which restricts $p = \rho/3$. We assume a charge-neutral plasma, which restricts $Q = 0$. We assume a constitutive relation for the current, which restricts $\boldsymbol{J} = \boldsymbol{J}_{\rm Ohm} + \boldsymbol{J}_{\rm CME}$ where $\boldsymbol{J}_{\rm Ohm} = \sigma\boldsymbol{E}$ is the Ohmic current, σ is the electric conductivity, $\boldsymbol{J}_{\rm CME} = c\tilde{\mu}_5\boldsymbol{B}$ is the chiral magnetic effect current, and $\tilde{\mu}_5(\boldsymbol{x},t) = 2\alpha\mu_5(\boldsymbol{x},t)/\pi\hbar c$ is the rescaled chiral chemical potential.

The equations of motion for these dynamical variables are a system of coupled system partial differential equations [2–4]

$$\frac{\partial}{\partial t}\boldsymbol{A} = \boldsymbol{u}\times\boldsymbol{B} + \eta\big(\tilde{\mu}_5\boldsymbol{B} - \tfrac{1}{c}\boldsymbol{J}\big)\,, \tag{6a}$$

$$\begin{aligned}\big(\tfrac{\partial}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\big)\tilde{\mu}_5 &= -\tilde{\mu}_5\boldsymbol{\nabla}\cdot\boldsymbol{u} + D_5\nabla^2\tilde{\mu}_5 \\ &\quad - \lambda\eta\big(\tilde{\mu}_5\boldsymbol{B} - \tfrac{1}{c}\boldsymbol{J}\big)\cdot\boldsymbol{B} - \Gamma_5\tilde{\mu}_5 + \tilde{S}_5\,,\end{aligned} \tag{6b}$$

$$\begin{aligned}\rho\big(\tfrac{\partial}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\big)\boldsymbol{u} &= 2\boldsymbol{\nabla}\cdot(\rho\nu\mathsf{S}) - \tfrac{c^2}{4}\boldsymbol{\nabla}\rho \\ &\quad + \tfrac{1}{3}\boldsymbol{u}\boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) + \tfrac{3c}{4}\boldsymbol{J}\times\boldsymbol{B} \\ &\quad - \boldsymbol{u}\big[\tfrac{1}{c}\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \tfrac{1}{c^2}\eta|\boldsymbol{J}|^2\big]\,,\end{aligned} \tag{6c}$$

$$\begin{aligned}\tfrac{\partial}{\partial t}\rho &= -\tfrac{4}{3}\rho\boldsymbol{\nabla}\cdot\boldsymbol{u} - \tfrac{4}{3}\boldsymbol{u}\cdot\boldsymbol{\nabla}\rho \\ &\quad + \tfrac{1}{c}\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \tfrac{1}{c^2}\eta|\boldsymbol{J}|^2\,,\end{aligned} \tag{6d}$$

where $\mathsf{S}_{ij} = (\partial_j u_i + \partial_i u_j)/2 - \delta_{ij}\partial_k u_k/3$ is the rate-of-strain tensor. Note that all of the dynamical variables are comoving quantities (i.e., they have been scaled by a number of $a(t)$ factors equal to their mass dimension), $\boldsymbol{x}$ is the comoving spatial coordinate, and t is the conformal time coordinate. The model parameters are the electromagnetic fine-structure constant $\alpha = e^2/4\pi\hbar c \approx 1/137$, the magnetic diffusivity $\eta = c^2/\sigma$, the kinematic viscosity ν, the chiral diffusion coefficient D_5, the chiral depletion parameter $\lambda = (\hbar c/k_B^2)(12\alpha^2/\pi^2 T^2)$, the chiral erasure rate Γ_5, and the rescaled chiral source $\tilde{S}_5 = (\hbar^2 c^2/k_B^2)(\alpha S_5/3\pi T^2)$. We use Eq. (5) to model the source $S_5(t)$, and we can write $\tilde{S}_5(t) = \bar{\tilde{S}}_5 \times (t/t_\phi)\exp(-t^2/t_\phi^2)$. Ampere's Law with Maxwell's correction implies $\boldsymbol{J} = c\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{\partial}{\partial t}\boldsymbol{E}$, and we neglect $\frac{\partial}{\partial t}\boldsymbol{E}$.

IV. CHIRAL PLASMA INSTABILITY

The equations of χMHD admit an instability, known as the chiral plasma instability, toward the growth of a helical magnetic field To identify the instability, it is sufficient to set $\boldsymbol{u} = 0$ in Eq. (6a), which becomes

$$\frac{\partial}{\partial t}\boldsymbol{A} - \eta\nabla^2\boldsymbol{A} + \eta\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A}) - \eta\tilde{\mu}_5\boldsymbol{\nabla}\times\boldsymbol{A} = 0\,. \tag{7}$$

The vector potential may be decomposed into Fourier modes $\boldsymbol{A}(\boldsymbol{x},t) = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi\hbar)^3}\boldsymbol{A}_{\boldsymbol{k}}(t)\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$, and further decomposed onto a basis of right- and left-handed circular polarization modes $\boldsymbol{A}_{\boldsymbol{k}}(t) = A_{\boldsymbol{k},+}(t)\hat{\boldsymbol{\epsilon}}_{\boldsymbol{k},+} + A_{\boldsymbol{k},-}(t)\hat{\boldsymbol{\epsilon}}_{\boldsymbol{k},-}$. In terms of these variables, the equation of motion becomes

$$\frac{\partial}{\partial t}A_{\boldsymbol{k},\pm} + \big(\eta|\boldsymbol{k}|^2 \mp \eta\tilde{\mu}_5|\boldsymbol{k}|\big)A_{\boldsymbol{k},\pm} = 0\,. \tag{8}$$

If $\tilde{\mu}_5 = 0$ then $A_{\boldsymbol{k},\pm} \propto \exp[-\eta|\boldsymbol{k}|^2 t]$ decays at a rate controlled by the magnetic diffusivity η. If $\tilde{\mu}_5 > 0$ then the positive-helicity modes $A_{\boldsymbol{k},+}$ with $|\boldsymbol{k}| < \tilde{\mu}_5$ are unstable. If $\tilde{\mu}_5 < 0$ then the negativity-helicity modes $A_{\boldsymbol{k},-}$ with $|\boldsymbol{k}| < -\tilde{\mu}_5$ are unstable. This is a tachyonic instability that only impacts long-wavelength modes. It is useful to identify $k_{\rm CPI} = |\tilde{\mu}_5|$ and note that the instability develops most quickly for modes with wavenumer $|\boldsymbol{k}| = k_{\rm CPI}/2$ and wavelength $l_{\rm CPI} = 4\pi/k_{\rm CPI}$, which grow exponentially (while $\tilde{\mu}_5$ is constant) on a time scale $t_{\rm CPI} = 4/\eta k_{\rm CPI}^2$. The growth saturates after a few e-foldings, when an order-one fraction of the chiral asymmetry is depleted.

On time scales that are longer than Γ_5^{-1}, chirality-violating reactions are in equilibrium, and the washout term in Eq. (6b) drives $\tilde{\mu}_5(\boldsymbol{x},t)$ to zero. However, the presence of a nonzero source $\tilde{S}_5(\boldsymbol{x},t)$ shifts the equilibrium point away from zero, and prevents the complete erasure of the chiral asymmetry. The new equilibrium point can be estimated as

$$\tilde{\mu}_5(\boldsymbol{x},t) \approx \tilde{S}_5(\boldsymbol{x},t)/\Gamma_5 \ . \tag{9}$$

Using the parametrization of the source in Eq. (5), we infer the CPI length and time scales:

$$l_{\text{CPI}} = 4\sqrt{2e}\pi\Gamma_5/\tilde{\bar{S}}_5 \quad \text{and} \quad t_{\text{CPI}} = 8e\Gamma_5^2/\eta\tilde{\bar{S}}_5^2 \ . \tag{10}$$

It is useful to compare t_{CPI} and t_ϕ. Since the source drops exponentially to zero after a time $t \gtrsim t_\phi$, in order for the CPI to develop it is necessary that $t_{\text{CPI}} < t_\phi$.

V. PARAMETER ESTIMATES

The parameters of sourced χMHD (6) are as follows:

$$\eta \ , \quad D_5 \ , \quad \lambda \ , \quad \nu \ , \quad \Gamma_5 \ , \quad \tilde{\bar{S}}_5 \ , \quad \text{and} \quad t_\phi \ . \tag{11}$$

In addition we hold fixed $\alpha = 1/137$, c, $\hbar$, and k_B. [AL: andrew, include numerical estimates of the parameters in our desired units]

In order to solve Eq. (6), we must also specify initial conditions at time t_i, which are as follows [AL: Murman/Alberto, write Pencil Code initial conditions – use same units as above]

$$t_i = \text{what is initial time?} \tag{12a}$$
$$\boldsymbol{A}(\boldsymbol{x},t_i) = \text{what's the initial spectrum?} \tag{12b}$$
$$\tilde{\mu}_5(\boldsymbol{x},t_i) = \text{is it zero?} \tag{12c}$$
$$\boldsymbol{u}(\boldsymbol{x},t_i) = \text{what's the initial spectrum?} \tag{12d}$$
$$\rho(\boldsymbol{x},t_i) = \text{what's the initial spectrum?} \ . \tag{12e}$$

Note that we initialize the chiral chemical potential $\tilde{\mu}_5 = 0$, since the washout term sends it to zero before the start of the simulation.

To provide a numerical estimate of $\bar{S}_5$ and t_ϕ, suppose that the ϕ particles are decaying at the electroweak epoch in the early universe. At this time we take the fiducial plasma temperature to be $T = 100\,\text{GeV}$, which corresponds to cosmic time $t \approx 2.3 \times 10^{-11}\,\text{sec}$ and scale factor $a \approx 7.8 \times 10^{-16} a_0$. For these estimates we have assumed radiation domination with $g_E = g_S = 106.75$ effective relativistic degrees of freedom, we assume comoving entropy conservation between the electroweak epoch and today, and a_0 denotes the scale factor today. It follows that $t_\phi \approx 6.0 \times 10^4\,\text{sec}/a_0$ and $\bar{S}_5 \approx (1.3 \times 10^{-16}\,\text{eV/cm}^3)(a_0^4 \beta\epsilon\Omega_\phi)(m_\phi c^2/100\,\text{GeV})^{-1}$

[AL: Murman, include a table showing your benchmark parameter sets]

VI. NUMERICAL χMHD SIMULATIONS

We solve Eqs (6a)–(6d) using the Pencil Code [5], which is a massively parallel MHD code using sixth-order finite differences and a third-order time stepping scheme.

Figure 3 shows the time evolution of the source $\tilde{S}_5(t)$ and the induced chiral chemical potential $\tilde{\mu}_5(t)$. [AL: Murman, include a time-evolution figure ... we want to discuss how $\tilde{\mu}_5$ is initially equal to zero b/c of the washout term, how $\tilde{\mu}_5$ rises to track the growing source, how the growth of $\tilde{\mu}_5$ leads to a growth in the B-field energy, and how $\tilde{\mu}_5$ returns to zero after the source drops to zero]

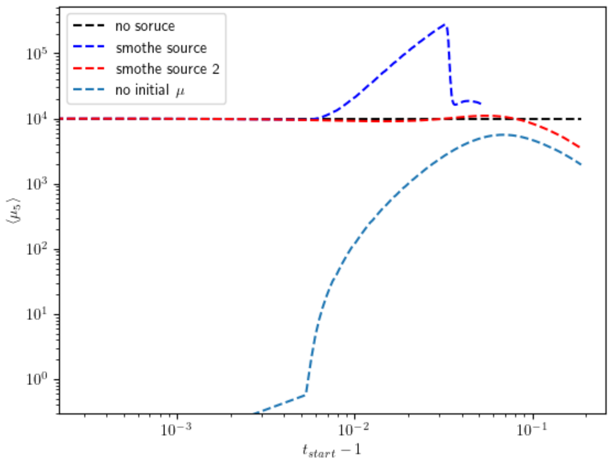

FIG. 3: [AL: Murman, add a evolution figure.]

Figure 4 shows the spectrum of magnetic energy $E_M(k)$ and fluid kinetic energy $E_K(k)$ at several different times while the CPI is developing. [AL: Murman, include a spectrum figure ... we want to discuss how the maximally unstable mode is the one with $k \approx k_{\text{CPI}}/2$ where $k_{\text{CPI}} = \max[\tilde{\mu}_5(t)]$ ... how this mode grows on a time scale $t \approx 4\sigma_Y/k_{\text{CPI}}^2$ ... how the magnetic energy and fluid kinetic energy grows on this scale ... how the growth saturates (after an order 1 fraction of the chemical potential is used up) and then the inverse cascade occurs]

Figure 5 shows the helicity fraction of the magnetic field. [AL: Murman, include a figure that shows how the magnetic field is helical ... and how the sign of the helicity is linked to the sign of the chemical potential, which is linked to the sign of the source]

Figure **??** shows a two-dimensional parameter space with contours indicating the resultant magnetic field strength and coherence length. [AL: Murman, include a figure that shows the total magnetic field generated as a function of some model parameters ... maybe $\bar{S}_5$ and t_ϕ]

fig_spectrum.pdf

FIG. 4: [AL: Murman, add a spectrum figure.]

fig_helicity.pdf

FIG. 5: [AL: Murman, add a helicity figure.]

TABLE I: Simulation Parameters for Different Configurations

Parameter	**no_source**	**Smooth**	**Smooth2**
source5	0	1.000×10^9	1.000×10^7
source5_expt2	0.0	600	600
gammaf5	0	1.000×10^1	1.000×10^1
t1_source5	0.0	1.005	1.005

VII. SUMMARY AND CONCLUSION

Since chirality-violating interactions come into equilibrium when the temperature of the primordial plasma drops below $T \approx 80$ TeV, any preexisting chiral asymmetry will be erased. However, a source for chirality counteracts its washout, allowing for a nonzero chiral asymmetry even below 80 TeV. The presence of a chiral asymmetry opens the possibility for helical magnetic field generation via the chiral plasma instability. In this work we have studied the chiral plasma instability in the presence of such a source.

We model the system as a fluid, whose constituents carry both electric and chiral charge, which is therefore coupled to electromagnetism. We study this system's dynamics with the equations of χMHD extended to include a source term. We assume a particular functional form for the source's time dependence, which we motivate by considering the out-of-equilibrium decay of a metastable scalar field. When selecting parameters, it is important to ensure that the time scale for the CPI is short compared to the lifetime of this scalar field, since otherwise the source will deactivate before the instability has time to develop. We solve the equations of χMHD using numerical methods, implemented by the PENCIL CODE.

Our numerical calculation reveals that the source is sufficient to induce the CPI even at plasma temperature below 80 TeV where the chirality-washout reactions are in equilibrium. [AL: Murman, update based on your figures]

When a source for chirality is present, the chiral charge erasure is avoided. This allows the chiral plasma instability to develop even after the chirality-erasing reactions are in thermal equilibrium at plasma temperatures $T \lesssim 80\,\mathrm{TeV}$.

Data availability—The source code used for the simulations of this study, the PENCIL CODE, is freely available from Ref. [6]. The simulation setups and the corresponding data are freely available from Ref. [7].

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