SPECTRUM OF THE GALACTIC MAGNETIC FIELDS

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(Received 20 July, 1981)

Abstract. The magnetic fields observed in the galactic disc are generated by the differential rotation and the helical turbulent motions of interstellar gas. On the scales $l = 2\pi k^{-1}$ which lie in the interval $l_0 < l < l_e$ ($l_0 \simeq 100$ pc is the energy-range scale of the galactic turbulence), the spectral density of the kinetic energy of turbulence ($\propto k^{-5/3}$) exceeds the magnetic energy spectral density ($\propto k^{-1}$). The equipartition between magnetic and kinetic energies is reached at $l = l_e \simeq 6$ pc in the intercloud medium and is maintained down to the scale $l = l_d \simeq 0.03$ pc. In dense interstellar clouds l_e is determined by the individual cloud size and $l_d \simeq 0.1$ pc. The internal turbulent velocities in H I clouds (cloud size is assumed to be 10 pc) lie in the range from 1.8 to 5.6 km s^{-1}, fitting well within the observed range of internal rms velocities. Dissipation of the interstellar MHD turbulence leads to creation of temperature fluctuations with amplitudes of 150 K and 65 K in dense clouds and intercloud medium, respectively. The small-scale fluctuations observed in the interstellar medium may arise from such perturbations due to the thermal instability, for instance. Dissipation of the MHD turbulence energy provides nearly half of the energy supply needed to maintain the thermal balance of the interstellar medium.

1. Introduction

Various observations provide strong evidence of a large-scale galactic magnetic field with a strength of about 2 μG (see Moffatt, 1978; Parker, 1979; Vainshtein *et al.*, 1980). In the galactic gaseous disc the mean large-scale magnetic field is predominantly azimuthal; its poloidal component is approximately ten times weaker than the azimuthal one. Along with the large-scale field, there are small-scale magnetic fields of comparable strength. The distinction between large and small scale is quite natural and is found at 100 pc, the dominant energy-range scale of the interstellar turbulence.

The mean galactic magnetic field is generated and maintained due to the turbulent dynamo-action. Only the laminar and spatially-averaged characteristics of the velocity field are essential for the theory of the generation of the large-scale magnetic field; that is differential rotation, turbulent diffusion and helicity of the turbulence. On the other hand, with regard to small-scale fields we must be aware of the detailed properties of the turbulence such as its spectrum, the width of the inertial range, the damping scale, etc. In the present paper we shall make an attempt to determine the statistical properties of the galactic turbulent magnetic fields and point out some observational tests designed to verify the theory.

Astrophysics and Space Science **82** (1982) 397–407. 0004-640X/82/0822-0397$01.65.

2. Generation of the Mean Magnetic Field

Consider first the energy balance of the large-scale galactic magnetic field. The hydromagnetic equation describing the evolution of the mean field $\mathbf{B}$ is

$$\partial \mathbf{B}/\partial t = \mathrm{rot}\{(\boldsymbol{\omega} \times \mathbf{r}) \times \mathbf{B} + \alpha \mathbf{B} - \beta \,\mathrm{rot}\, \mathbf{B}\}, \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the galactic gaseous disc, α is the helicity function, and β is the magnetic diffusivity. Below we shall use the cylindrical coordinates (r, φ, z) with the z-axis orthogonal to the plane of the disc. Denoting the scale, velocity and lifetime of the dominant turbulent eddies by l_0, v_0, and τ_0, respectively, we can write

$$\alpha = -\frac{\tau_0}{3}\langle \mathbf{v} \,\mathrm{rot}\, \mathbf{v}\rangle \simeq \frac{l_0^2 \omega}{h}\tilde{\alpha}(z), \tag{2}$$

where h is the half-thickness of the disc, $\tilde{\alpha}(z) = 0(1)$ is the dimensionless odd function of z, and the angular brackets symbolize an ensemble average (Moffatt, 1978; Parker, 1979; Vainshtein *et al.*, 1980). The magnetic diffusivity, ν, includes two dissipative processes: the turbulent diffusion, ν_t, and resistive diffusion, ν_m. In the galactic disc $\nu_t \gg \nu_m$, and we shall put $\nu \simeq \nu_t \simeq \frac{1}{3} l_0 v_0$ in (1).

To obtain the equation for the magnetic field energy $B^2/8\pi$ multiply Equation (1) by $\mathbf{B}$ and integrate over the disc volume. The result can be written as

$$\frac{\partial}{\partial t}\int \frac{\mathbf{B}^2}{2}\,\mathrm{d}^3\mathbf{r} = \int \mathbf{B}\,\mathrm{rot}\{(\boldsymbol{\omega}\times\mathbf{r})\times\mathbf{B}\}\,\mathrm{d}^3\mathbf{r} + \int \alpha\mathbf{B}\,\mathrm{rot}\,\mathbf{B}\,\mathrm{d}^3\mathbf{r} - \int \nu_t(\mathrm{rot}\,\mathbf{B})^2\,\mathrm{d}^3\mathbf{r}. \tag{3}$$

Due to the influence of the differential rotation the radial lines of force (component B_r) are sheared in the azimuthal direction, producing the φ-component of the large-scale field. This effect is represented by the first term on the right-hand side of (1): i.e.,

$$|\mathbf{B}\,\mathrm{rot}\{(\boldsymbol{\omega}\times\mathbf{r})\times\mathbf{B}\}| \simeq B_r B_\varphi r\frac{\mathrm{d}\omega}{\mathrm{d}r}. \tag{4}$$

The second term on the right-hand side describes the inverse process: production of the radial component by the helical turbulence in the presence of the azimuthal field. If the rotation is strongly non-uniform the role of the second term in the energy balance is negligible. This becomes evident after writing

$$|\alpha\mathbf{B}\,\mathrm{rot}\,\mathbf{B}| \simeq \frac{l_0^2\omega}{h}\frac{B_r B_\varphi}{h} = \left(\frac{l_0}{h}\right)^2 \omega B_r B_\varphi.$$

From this we estimate the ratio of the second term to the first one to be $(l_0/h)^2\,\mathrm{d}\ln r/d\ln\omega \simeq 0.05$ in the solar neighbourhood (Ruzmaikin and Shukurov, 1981). The last term on the right-hand side of Equation (1) represents the loss of the mean magnetic field.

Using (4) we obtain the following estimate of the first integrand on the right-hand side of Equation (1) with $h(r) = \text{const.}$:

$$\mathscr{E}_G \simeq 4\pi h R \bar{B}_r \bar{B}_\varphi (V_R - 2\bar{V}), \tag{5}$$

where $\bar{B}_r$ and $\bar{B}_\varphi$ are the field components (slowly varying with r) averaged over the disc volume, $V_R = \omega(R)R$ is the rotational velocity at the disc edge $r = R$ and $\bar{V} = (1/R)\int_0^R V(r)\,\mathrm{d}r$ is the mean velocity of rotation. The dissipation integrand is estimated as

$$\mathscr{E}_D \simeq -4\pi\nu_t \frac{R^2}{h} \bar{B}_\varphi^2 .$$

Therefore, in the stationary situation, $\partial/\partial t = 0$, we arrive at

$$\left|\frac{\bar{B}_r}{\bar{B}_\varphi}\right| \simeq \frac{\nu_t R}{h^2(V_R - 2\bar{V})} . \tag{6}$$

Substituting such typical values for the galaxy as $\nu_t \simeq 10^{26}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$, $R \simeq 15\,\mathrm{kpc}$, $\bar{V} \simeq V_R \simeq 250\,\mathrm{km\,s}^{-1}$, $h \simeq 400\,\mathrm{pc}$, we come to an estimate of $\bar{B}_r/\bar{B}_\varphi \simeq 1/10$. In the innermost region of the galaxy the α^2-dynamo action is possible (Ruzmaikin and Shukurov, 1981). If this is the case, the second term in (1) is not negligible and $\bar{B}_r/\bar{B}_\varphi \simeq 1$. Note that, in the solar neighbourhood, the ratio B_r/B_φ is close to 0.1 (Ruzmaikin *et al.*, 1977).

Altogether, the rate of magnetic energy production (5) is proportional to the rotational velocity of the disc, but not to the rotational energy. The omitted term $\int \alpha \mathbf{B}\,\mathrm{rot}\,\mathbf{B}\,\mathrm{d}^3\mathbf{r} \simeq 4\pi h R^2 \bar{B}_r \bar{B}_\varphi \bar{\alpha}/h$ does not include the energy of the helical turbulence. Hence, the differential rotation and the helicity of turbulence are just the 'driving belts' which transfer energy from the interstellar turbulence to the mean large-scale magnetic field. The interstellar turbulence, in turn, is fed by supernovae explosions and the strong outflow of gas from the luminous young stars. The efficiency of the magnetic field generation depends, of course, on the capacity of driving mechanism. The sink of the mean magnetic field energy is provided by turbulent motions, the most destructive of which is on scale l_0.

3. The Spectrum of Turbulent Magnetic Fields

The mean large-scale magnetic field maintained by the turbulent dynamo action is inevitably accompanied by the small-scale magnetic fields of turbulent cells. Moreover, the generation of the large-scale field is impossible without the small-scale fields, because the source term in Equation (1) is the correlator $\langle \mathbf{v} \times \mathbf{b} \rangle \simeq \alpha \mathbf{B}$, where $\mathbf{b}$ is the small-scale random component of the magnetic field. In the galactic disc the magnetic Reynolds number of the dominant eddies is rather high, $R_m \simeq 10^5$ (see below). It suggests that the magnitude of the galactic small-scale field is not less than the magnitude of the mean field, $|\mathbf{b}| \gtrsim |\mathbf{B}|$. In this section we discuss the spectrum of the small-scale fields arising due to the mean-field dynamo action.

To obtain the equation governing the evolution of **b**, subtract (1) from the hydromagnetic equation for the total magnetic field, $\mathbf{B}+\mathbf{b}$; the result is

$$\partial \mathbf{b}/\partial t = \operatorname{rot}\{(\boldsymbol{\omega}\times\mathbf{r})\times\mathbf{b}\} + \operatorname{rot}(\mathbf{v}\times\mathbf{B}) + \operatorname{rot}\mathbf{G} + \nu_m \nabla^2 \mathbf{b}\,, \tag{7}$$

where $\mathbf{G} = \mathbf{v}\times\mathbf{b} - \langle \mathbf{v}\times\mathbf{b}\rangle$ (cf. Moffatt, 1978; Vainshtein *et al.*, 1980). If we view the fluid from the local frame of reference rotating with the angular velocity $\boldsymbol{\omega}$, the first term on the right-hand side of (7) is eliminated. Now, note that the growth rate of **b** is obviously the same as the growth rate of the mean field, $b \propto B \propto \exp(\gamma t)$. The rate at which the magnetic field grows, $\gamma \simeq 1/(5\times 10^8\ \text{yr})$ (Ruzmaikin and Shukurov, 1981), is much less than both the rate at which the turbulent energy cascades toward the smaller scales and the inverse lifetime of the turbulent eddies. Thus, the small-scale fields may be considered as the quasistationary, $\partial\mathbf{b}/\partial t \simeq 0$, on the span $t < \gamma^{-1}$. Expressing the hydromagnetic equation (7) in terms of the spatial Fourier transforms, b_k and v_k, of **b** and **v**, respectively, we have the following order of magnitude estimates

$$\langle|\operatorname{rot}(\mathbf{v}\times\mathbf{B})|\rangle_k \simeq k v_k B\,; \qquad \langle \mathbf{b}\operatorname{rot}\mathbf{G}\rangle_k \simeq -b_k^2/\tau_k\,; \qquad \langle \mathbf{b}\,\nu_m\nabla^2\mathbf{b}\rangle_k \simeq -b_k^2/\tau_d\,, \tag{8}$$

where k is the wave-number, τ_k is the time-scale of the spectral energy cascade at the wave-number k, $\tau_d = 1/(\nu_m k^2)$ is the magnetic dissipation time on the scale k^{-1}, and the subscript 'k' denotes the spatial Fourier transforms. The estimate of $\langle \mathbf{b}\operatorname{rot}\mathbf{G}\rangle_k$ follows from the arguments that are similar to Orszag (1970) τ-approximation arguments. Now we can readily calculate the magnetic field spectrum, b_k, and the magnetic energy spectral density, $M_k \propto k^{-1}b_k^2$. Equations (7) and (8) yield

$$b_k \simeq k v_k \tau_k B \quad \text{for} \quad \tau_d \gg \tau_k\,.$$

We should bear in mind that Equations (1) and (7) are the kinematic dynamo equations, i.e. they do not incorporate the dynamical back-action of the magnetic force on the velocity field. It follows that for these equations to be valid, the magnetic energy density must be much less than the turbulent kinetic energy density. Thus, the turbulence presumably should exhibit a Kolmogorov spectrum and

$$E_k \propto k^{-1}v_k^2 \propto k^{-5/3}; \qquad v_k \propto k^{-1/3}; \qquad \tau_k = (k v_k)^{-1}\,,$$

where E_k is the kinetic energy spectrum. Hence, for $\tau_d \gg \tau_k$, i.e. at wave-numbers $k \ll k_m = k_0 R_m^{3/4}$ ($2\pi k_0^{-1} = l_0$, $R_m = v_0 l_0/\nu_m$ is the magnetic Reynolds number) the magnetic spectrum has the form

$$M_k \propto k^{-1}; \qquad b_k \propto k^0\,. \tag{9}$$

If $\tau_d \ll \tau_k$, i.e. in the range $k_\eta \gg k \gg k_m$ where the magnetic field dissipation is essential, we have

$$M_k \propto k^{-11/13}; \qquad b_k \simeq k v_k \tau_d B \propto k^{-4/3}$$

(the inequality $k_\eta \gg k_m$ certainly holds true in the galactic disc). The magnetic and kinetic energy spectra in the linear regime ($E_k \gg M_k$) are sketched in Figure 1a.

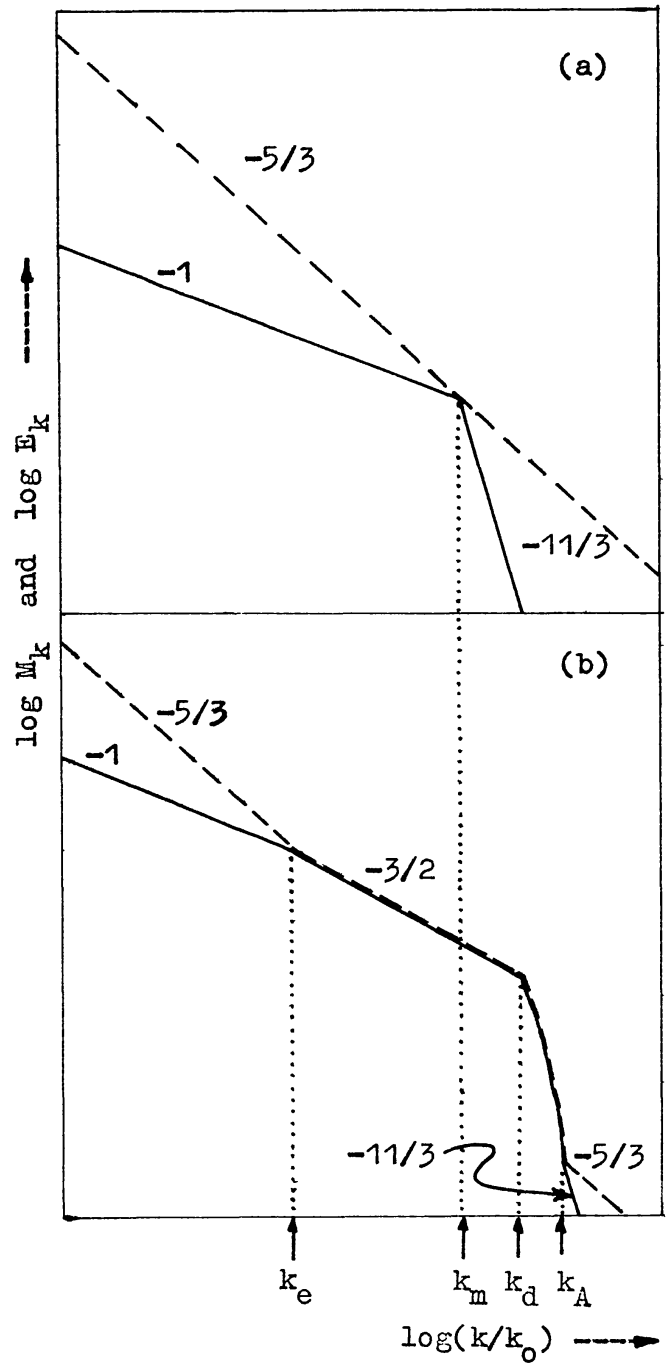

Fig. 1. The spectra of magnetic (solid curves) and kinetic (dashed curves) energy of MHD turbulence (both in arbitrary units) when the large-scale turbulent dynamo is in action. The spectral indices are written alongside the corresponding part of the curve. (a) The linear regime when $E_k \gg M_k$. (b) The equipartition range $E_k = M_k$ is well developed.

The large-scale field B grows at the rate γ and this causes the turbulent magnetic energy to grow at the rate 2γ until the equipartition between magnetic and kinetic energies is reached at the wave-number k_m. From this moment, the inertial range of magnetohydrodynamic (MHD) turbulence with $M_k = E_k$ appears and spreads down over the spectrum. The foregoing arguments become inapplicable when the equipartition is reached. Spectra of the magnetic and kinetic energies in the inertial range are determined by the conservation of the energy flux flowing along the spectrum from the dominant wave-number up to the cut-off wave-number,

$$\frac{kM_k}{\tau_{ki}} = \frac{kE_k}{\tau_{ki}} = \epsilon = \text{const.}\,,$$

and by the physical nature of the energy cascade. In the inertial range MHD turbulence can be regarded as an ensemble of random hydromagnetic waves moving at nearly the Alfvén speed, $V_A^2 = \langle b^2 \rangle / 4\pi\rho$, where ρ is a gas density. Now the time-scale, τ_{ki}, at which the energy cascades across a wave-number magnitude k, is not equal to the eddy lifetime $(kv_k)^{-1}$, as was the case in the linear regime. τ_{ki} is essentially the time-scale for the nonlinear interaction of random waves of the scale k^{-1}. For weak MHD turbulence the three-wave interaction prevails (Sagdeev and Galeev, 1969; Kaburaki and Uchida, 1971). If the turbulence is helical, the interaction of Fourier components b_k and v_k at the wave-number k_0 excites the harmonics of smaller wave-numbers, $k < k_0$ (Pouquet *et al.*, 1976; Moffatt, 1978); that is, the large-scale magnetic field is intensified. The process of generation of higher harmonics provides the flow of energy towards higher wave-numbers, i.e. the turbulent energy cascade. The three-wave interaction equations are quadric with respect to the energy density, so we can write $\tau_{ki}^{-1} = \omega_k(kM_k/U)$, where $\omega_k = kV_A$ is the Alfvén frequency and U is a homogeneous function of dimension cm^2 s^{-2} (cf. Sagdeev and Galeev, 1969). Upon writing $U = V_A^2$, we have $\tau_{ki}^{-1} = k^2 M_k/V_A$; hence, $kM_k/\tau_{ki} = k^3 M_k^2/V_A = \epsilon$. This brings us to the spectrum of MHD turbulence in the inertial range,

$$M_k \propto k^{-3/2}; \qquad b_k \propto k^{-1/4}\,,$$

as Kraichnan (1965) determined with his use of dimensional arguments. For those wave-numbers where $M_k \ll E_k$ the earlier developed spectra are still valid, of course. In Figure 1b we have sketched the spectra of magnetic and kinetic energies after the inertial range has evolved.

The lower wave-number boundary of the inertial range, k_e, is determined from the equation $M_{k_e} = E_{k_e}$, where $E_k \propto k^{-5/3}$. Thus,

$$k_e = k_0(v_0/V_A)^3\,. \tag{10}$$

The wave-number at which the inertial range cuts-off, k_d, is determined by the equality $\tau_{ki} = \tau_d$; hence,

$$k_d = (\epsilon / V_A \nu_m^2)^{1/3} = k_0 R_m^{2/3} (v_0 / V_A)^{1/3} . \tag{11}$$

When the equipartition range has spread over the whole spectrum up to the dominant scale, then $v_0 = V_A$ and $k_d = k_0 R_m^{2/3}$.

Note that if a small-scale turbulent dynamo of some type acts, then the spectrum in the range $k_0 < k < k_e$ ($k_0 < k < k_m$ in the linear regime) has the form $M_k \propto k^4$ (Kraichnan and Nagarajan, 1967; Ruzmaikin and Sokolov, 1981) and sharply differs from the spectrum (9). Therefore, even a crude experimental estimate of the magnetic spectrum index in the range $k_0 < k < k_e$ would allow us to determine whether the small-scale turbulent dynamo action is possible.

5. The Small-Scale Fields in the Galactic Disc

To obtain numerical estimates of typical parameters of the galactic MHD turbulence, we should begin with the magnetic diffusivity, ν_m. Collisions between hydrogen ions, H^+, and neutral hydrogen atoms, H, produce the most important damping mechanism which destroys the small-scale magnetic fields in the partially ionized interstellar gas. This diffusion coefficient can be written as

$$\nu_m \simeq 2 \times 10^{20}\ \mathrm{cm}^2\ \mathrm{s}^{-1} \left(\frac{0.03\ \mathrm{cm}^{-3}}{n_e}\right)\left(\frac{1\ \mathrm{cm}^{-3}}{n_a}\right)\left(\frac{10^4\ \mathrm{K}}{T}\right)\left(\frac{B}{1\ \mu\mathrm{G}}\right)^2\left(\frac{10^{-14}\ \mathrm{cm}^2}{\sigma_{ia}}\right), \tag{12}$$

where T is the gas temperature, n_e and n_a are the number densities of free electrons and neutral atoms respectively, and σ_{ia} is the ion-neutral collisions cross-section ($\sigma_{H^+H} \simeq 10^{-14}\ \mathrm{cm}^2$ for $T = 10^4$ K; Dalgarno, 1960). The mean electron number density in the galactic disc is $n_e \simeq 0.03\ \mathrm{cm}^{-3}$ (Kaplan and Pikel'ner, 1979). Then, adopting the values $B = 2\ \mu$G, $n_a = 1\ \mathrm{cm}^{-3}$ and $T = 10^4$ K, we have $\nu_m \simeq 10^{21}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ and $R_m \simeq 3 \times 10^5$.

Following the interpretation of the Faraday rotation measures of pulsars (Ruzmaikin and Sokolov, 1977), the magnetic field fluctuations on the dominant scale $l_0 \simeq 100$ pc are of the magnitude $\delta B \simeq 1.2B$ – i.e., $b_0 \simeq 2.5\ \mu$G. The spectral energy density of turbulent magnetic fields is a decreasing function of wave-number, so that the main contribution to the total magnetic energy comes from the dominant scale, $\langle b \rangle^2 \simeq b_0^2$. Thus, the Alfvén speed is $V_A \simeq b_0/\sqrt{4\pi\rho} \simeq 5.5\ \mathrm{km\ s}^{-1}$ (for $\rho = 1.7 \times 10^{-24}\ \mathrm{g\ cm}^{-3}$). For $M_k \propto k^{-1}$ the lower boundary of the equipartition region lies at the wave-number k_e defined by (10). In the galaxy, where $v_0 \simeq 10\ \mathrm{km\ s}^{-1}$, we have $k_e \simeq 6k_0$. The associated scale is $l_e \simeq l_0/6 \simeq 20$ pc. The inertial range cuts-off at $k = k_d \simeq 5.5 \times 10^3\ k_0$ (see (11)). In the region $k > k_d$ both magnetic and kinetic energy spectra cut-off exponentially, but the equipartition is maintained until the Alfvén frequency exceeds the dissipation rate (Kraichnan and Nagarajan, 1967) – i.e., if $k < k_A = V_A/\nu_m \simeq 3 \times 10^4\ k_0$. For $k > k_A$

the linear-regime spectra $E_k \propto k^{-5/3}$ and $M_k \propto k^{-11/3}$ are re-established. Due to a kinematic viscosity ν (and/or a thermal conductivity), the ultimate damping of the turbulence occurs at the wave-number $k_\nu = k_A \,\mathrm{Re}_A^{3/4}$ ($\mathrm{Re}_A = v_{k_A}/\nu k_A$ is the Reynolds number on the scale k_A^{-1}).

Thus, the inequality $k_e \ll k_d$ holds true in the galactic disc, so that the inertial range $k_e < k < k_d$ is well developed and the magnetic and kinetic energy spectra have the form, represented in Figure 1b.

Recently Vainshtein (1980) proposed a small-scale dynamo model for the turbulent velocity field with zero mean helicity. He claims that the turbulent magnetic fields grow if the Reynolds number of the dominant scale Re is far less than the magnetic Reynolds number $R_m \gg \mathrm{Re}$. The same conclusion was reached by Batchelor (1950) in his pioneering paper. However, with regard to the galactic disc, the sense of the inequality is the reverse. Indeed, in H I regions the kinematic viscosity is $\nu \simeq 3 \times 10^9\, n^{-1}\, T^{1/2}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$, thus $\mathrm{Re} \simeq 2 \times 10^7$ while $R_m \simeq 2 \times 10^4$. In the intercloud medium $\nu \simeq 10^8\, n^{-1}\, T^{5/2}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$; therefore, $\mathrm{Re} \simeq 6 \times 10^7 \gg R_m \simeq 3 \times 10^5$ (for the values of R_m see below).

If a small-scale turbulent dynamo of some other type acts in the galactic disc, then the spectrum in the range of wave-numbers $k_0 < k < k_e$ is $M_k \propto k^4$; therefore, $k_e = k_0(v_0/V_A)^{6/11} \simeq 1.4 k_0$ and $l_e \simeq 70$ pc. Thus, even a crude observational estimate of the small-scale magnetic field spectral index for the scales $70\,\mathrm{pc} \gtrsim l \gtrsim 20\,\mathrm{pc}$ would be sufficient to verify the theory of the small-scale turbulent dynamo. If b_k in this interval is a decreasing function of l, then the small-scale dynamo is in action. On the other hand, if it turns out that $b_k = \mathrm{const.}$, one can be sure that the turbulent magnetic fields are entirely due to mixing and stretching of the lines of force of the large-scale field. It is clear that if the turbulent magnetic fields are generated by a small-scale dynamo of any type, their spectrum reaches the maximum on a scale, which is a little smaller than the dominant one. We should note, that the analysis of the Faraday rotation measures of the pulsars (Ruzmaikin and Sokolov, 1977) has not yet revealed any indication of such a maximum.

The foregoing numerical estimates of the characteristics of the galactic MHD turbulence are based on the mean values of the ionization degree, temperature, matter density and, hence, magnetic diffusivity of the interstellar gas. However, the interstellar medium is highly inhomogeneous. Recent discussions (see, for instance, Cox and Smith, 1974; McKee and Ostriker, 1977) suggest that there are three main phases of the interstellar medium: cool, dense clouds; 'standard' hot intercloud medium; and very hot tunnels formed by merging of supernova remnants. Of course, the magnetic diffusivity and, therefore, width of the inertial range of MHD turbulence are not the same in all the phases. As it was shown by the detailed calculations (McIvor, 1977), ion-neutral collisions provide the main damping mechanism in clouds and the intercloud medium. The gas in the tunnels is highly ionized and, in addition, a large fraction of turbulent energy is stored in the longitudinal magnetosonic waves – but not in the transverse Alfvén waves.

For these reasons, the main contribution to the damping rate in the tunnels comes from the thermal conductivity. This contribution is so considerable, that the turbulence is strongly suppressed and presumably there is not any inertial range (McIvor, 1977). Hence, all of both the magnetic and kinetic energies of MHD turbulence dissipates out of the tunnels, and it is thought that the tunnels may occupy half of the galactic disc volume (Jenkins and Meloy, 1974). We shall calculate the wavenumbers at which the inertial range of turbulence starts and terminates in clouds and the intercloud medium. We shall also briefly discuss the role of MHD turbulence in the energy balance of the interstellar gas.

We shall adopt the following values for the H I cloud parameters: $T \simeq 100$ K, $n \simeq 20\ \mathrm{cm}^{-3}$, $n_e \simeq 0.05\ \mathrm{cm}^{-3}$ (Kaplan and Pikel'ner, 1979). The cross-section of the H^+–H collisions in the cloud is $\sigma_{\mathrm{H^+H}} \simeq 1.5 \times 10^{-14}\ \mathrm{cm}^2$ (Osterbrock, 1961). The matter density in the clouds is higher than the mean value, so the magnetic fields there are higher too, $B \propto n^{2/3}$. Thus, for the clouds we use the values $B \simeq 15\ \mu$G ($B \simeq 2\ \mu$G for $n = 1\ \mathrm{cm}^{-3}$) and $b_0 \simeq 1.2B \simeq 18\ \mu$G. The corresponding Alfvén speed is $V_A \simeq 10\ \mathrm{km\ s}^{-1}$. Hence, the beginning of the inertial range is found immediately on the dominant scale, $l_e = l_0$. The typical cloud size, 10 pc, is considerably smaller than l_0; therefore, the maximum turbulent scale, velocity and the consequent Alvén speed are determined by the actual size of an individual cloud. Substituting to (12) the adopted values for the relevant parameters, we have the magnetic diffusivity $\nu_m \simeq 1.5 \times 10^{22}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ and magnetic Reynolds number $R_m \simeq 2 \times 10^4$. It follows, that $k_d = k_0 R_m^{2/3} \simeq 8 \times 10^2\ k_0$ and $l_d \simeq 0.12$ pc. The turbulent velocities on the scales 0.12 pc and 10 pc are $1.8\ \mathrm{km\ s}^{-1}$ and $5.6\ \mathrm{km\ s}^{-1}$, respectively. It is interesting to note that the small-scale r.m.s. velocities in H I clouds lie in the range from $1.2\ \mathrm{km\ s}^{-1}$ to $4.5\ \mathrm{km\ s}^{-1}$ (Weaver, 1970; Hobbs, 1974) which is inferred from the observed line broadening. This range is sufficiently close to the turbulent velocity range calculated above.

Similar calculations for the intercloud medium, for which we adopt the values $T \simeq 10^4$ K, $n \simeq 0.2\ \mathrm{cm}^{-3}$ and $n_e \simeq 0.02\ \mathrm{cm}^{-3}$ (Kaplan and Pikel'ner, 1979), lead to the following estimates: $b_0 \simeq 0.8\ \mu$G, $V_A \simeq 4.0\ \mathrm{km\ s}^{-1}$, $R_m \simeq 3 \times 10^5$, $k_e \simeq 16 k_0$ ($l_e \simeq$ 6 pc) and $k_d \simeq 3 \times 10^3\ k_0$ ($l_d \simeq 0.03$ pc).

Now we are ready to estimate the contribution of MHD turbulence to the heating of the interstellar medium. In the H I clouds the rate at which the turbulent energy dissipates is $\tilde{\epsilon} = 2\epsilon\rho \simeq 2 \times 10^{-25}\ \mathrm{erg\ cm}^{-3}\ \mathrm{s}^{-1}$ (the factor 2 comes from the equal contributions of magnetic field and turbulent velocity field). This rate is comparable to the energy supply from the other sources (Kaplan and Pikel'ner, 1979). However, cosmic rays and soft X-rays, for example, heat the medium more or less uniformly, whereas the damping of the MHD turbulence causes temperature fluctuations of the scale l_d. The magnitude of the fluctuations is $\delta T = \epsilon\tau_t/(nk)$, where k is the Boltzmann constant and τ_t is the lifetime of the smallest eddies. Taking into account that $\tau_t = (v_0 k_0)^{-1} (k_0/k_d)^{3/4} \simeq 7 \times 10^4$ yr, we get $\delta T \simeq 150$ K ($\delta T/T \simeq 1.5$) for H I clouds. Similar calculations for the intercloud medium yield: $\tilde{\epsilon} \simeq 2 \times 10^{-27}$ erg cm^{-3} s^{-1}, $\tau_t = (v_e k_e)^{-1}$ $(k_e/k_d)^{3/4} \simeq 3 \times 10^4$ yr, and therefore $\delta T \simeq 65$ K (i.e., $\delta T/T \simeq 6.5 \times 10^{-3}$). The typical scale of these fluctuations is $l_d \simeq 0.03$ pc. The fluctuations may grow due to thermal instability, for example, this may produce the small-scale temperature and density fluctuations of a characteristic scale of the order of 0.03 pc, which are observed in H II regions (Kaplan and Pikel'ner, 1979). For example, the small-scale fluctuations in the Orion nebula have the scale of the order of 0.02–0.05 pc (Pikel'ner and Shajn, 1954; Mezger, 1970).

Finally, note that for the observed thermal and ionization state of the interstellar medium to be maintained, energy must be supplied at the rate of about 2×10^{-26} erg cm^{-3} s^{-1} (Kaplan and Pikel'ner, 1979). Compare this figure with the mean rate of dissipation of the energy of MHD turbulence, of the order of 10^{-26} erg cm^{-3} s^{-1} (based on the mean value $n = 1$ cm^{-3}). It is clear that the MHD turbulence plays an important role in maintaining the present thermal and ionization state of the interstellar medium.

Acknowledgments

The authors wish to thank Drs N. G. Bochkarev and D. D. Sokolov for valuable discussions. Special thanks to Miss Masha Yovanovich for her helpful comments on English grammar and stylistics which made this paper understandable.

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