Supplemental Material to "Decay law of magnetic turbulence with helicity balanced by chiral fermions"

by Axel Brandenburg, Kohei Kamada, & Jennifer Schober

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Following Zhou+22 [6], the function $\mathcal{I}_{\rm H}(t,R)$ can be recast as a weighted integral

$$\mathcal{I}_{\rm H}(t,R)=\frac{1}{V}\int\frac{{\rm d}^3k}{(2\pi)^3}\;w_R(\boldsymbol{k})\;h^*(\boldsymbol{k})h(\boldsymbol{k}). \tag{1}$$

The weight function $w_R(\boldsymbol{k})$ depends on the shape of V_R. For a cubic region with length $2R$, we have

$$w_R(\boldsymbol{k})=w_{\rm cube}^{\rm BC}(\boldsymbol{k})\equiv 8R^3\prod_{i=1}^{3}j_0^2(k_iR); \tag{2}$$

see Fig. 1. Here $j_0(x)=\sin x/x$. For a spherical region with radius R,

$$w_R(\boldsymbol{k})=w_{\rm sph}^{\rm BC}(k)\equiv\frac{4\pi R^3}{3}\left[\frac{6j_1(kR)}{kR}\right]^2, \tag{3}$$

which is the version shown in the main paper.

Alternatively, by the correlation-integral (CI) method, we have

$$w_R(\boldsymbol{k})=w_{\rm cube}^{\rm CI}(\boldsymbol{k})\equiv 8R^3\prod_{i=1}^{3}j_0(k_iR), \tag{4}$$

for a cubic region V_R; see Fig. 2, and

$$w_R(\boldsymbol{k})=w_{\rm sph}^{\rm CI}(k)\equiv\frac{4\pi R^3}{3}\frac{6j_1(kR)}{kR}. \tag{5}$$

for a spherical region; Fig. 3.

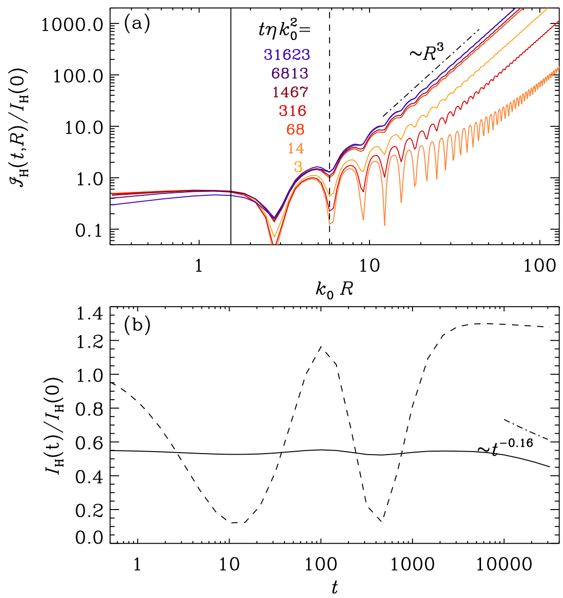

Figure 1: Box counting method with cubic volumes showing (a) $\mathcal{I}_H(R,t)$ versus R for different times (the normalized values of $t\eta k_0^2$ are indicated by different colors), and (b) $\mathcal{I}_H(R,t)$ versus t (normalized) for two choices of R, indicated by solid ($k_0R\approx 1.5$) and dashed ($k_0R\approx 6$) lines in both panels. The $t^{-0.16}$ scaling is indicated as the dashed-dotted curve for comparison.

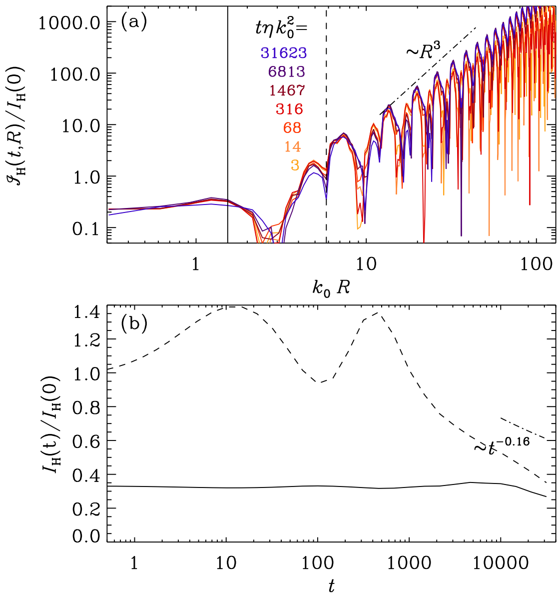

Figure 2: Similar to Fig. 1 using the correlation-integral method for cubic volumes.

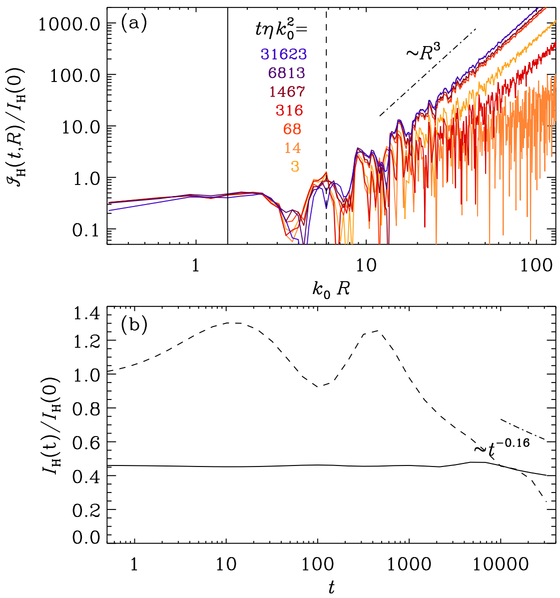

Figure 3: Similar to Fig. 1 using the correlation-integral method for spherical volumes.