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1.1 Force-free model

$$\boldsymbol{B} = \boldsymbol{p} B_\phi + \boldsymbol{\nabla} \times \boldsymbol{p} A_\phi. \tag{1}$$

For the quadrupole, we have

$$\boldsymbol{B} = \begin{pmatrix} +0.53\, r^{-4}\, P_2(\cos\theta) \\ -0.53\, r^{-4}\, P_2^1(\cos\theta) \\ -0.43\, r^{-2}\, P_1^1(\cos\theta) \end{pmatrix} = \begin{pmatrix} +0.53\, r^{-4}\, \frac{1}{2}(3\cos^2\theta - 1) \\ +0.53\, r^{-4}\, 3\cos\theta\sin\theta \\ +0.43\, r^{-2}\, \sin\theta \end{pmatrix} \tag{2}$$

so

$$\mathrm{d}_\theta \tfrac{1}{2}(3\cos^2\theta - 1) r^{-1} 3\cos\theta\sin\theta \tag{3}$$

Note that

$$\mathrm{d}_\theta P_2(\cos\theta) = r^{-1}\, P_2^1(\cos\theta). \tag{4}$$

$$J_\phi = \mathrm{D}_r B_\theta - \mathrm{d}_\theta B_r = r^{-5}(-9\cos\theta\sin\theta) \tag{5}$$

$$J_\phi = -(\mathrm{D}_r^2 + \mathcal{D}_\theta^2) A_\phi. \tag{6}$$

For $A_\phi = r^{-2}\sin\theta$, we have

$$J_\phi = -(\mathrm{D}_r^2 + \mathcal{D}_\theta^2) A_\phi. \tag{7}$$

and then

$$\boldsymbol{J} = \begin{pmatrix} +0.53\, r^{-4}\, P_2(\cos\theta) \\ -0.53\, r^{-4}\, P_2^1(\cos\theta) \\ -0.43\, r^{-2}\, P_1^1(\cos\theta) \end{pmatrix} \tag{8}$$