Schwinger effect in axion inflation on the lattice

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We present the first lattice simulations of non-linear evolution after axion inflation by self-consistently incorporating currents arising from Schwinger pair-production. The tachyonically amplified gauge fields trigger the growth of Schwinger currents, leading to a universal value for the conductivity and magnetic field at the onset of strong backreaction and subsequent quenching of gauge field production. We show that the Schwinger effect suppresses gauge field production, thereby excluding axion inflation magnetogenesis as a viable solution for blazar observations [**OI**: as a viable explanation for the absence of secondary GeV gamma rays in blazar spectra?], unless the Higgs field has a large displacement during inflation.

Introduction.

The inflationary paradigm provides a compelling framework for understanding the origin of large-scale structure and the observed homogeneity and isotropy of the universe [1–3]. Despite the abundance of inflationary models, axion (or natural) inflation [4, 5] has been receiving attention due to its theoretical robustness and its natural embedding within string theory and other ultraviolet (UV) complete frameworks. In such models, the inflaton is identified with a pseudo-scalar axion-like field, which naturally enjoys a shift symmetry—crucial for maintaining a flat potential across super-Planckian field excursions.

The shift symmetry of the axion forces it to couple only derivatively to gauge fields or fermions. Chern-Simons couplings $\phi F\tilde{F}$ lead to the exponential amplification of gauge field modes during and after inflation [6–8]. This amplification can generate distinctive non-Gaussian signatures, chiral gravitational waves and lead to almost instantaneous preheating. Furthermore, the induced electric fields from these amplified gauge modes can become large enough to trigger non-perturbative pair production of charged particles via the Schwinger effect [9, 10]. Due to its non-perturbative and non-linear nature and its importance for axion inflation, capturing the dynamics of the Schwinger effect has attracted significant attention and several methods have been proposed [**OI**: will add citations here].

This Letter contains results from the first lattice simulation of preheating after axion inflation, where the Schwinger effect is self-consistently taken into account. We demonstrate a suppression of the produced electric and magnetic fields, [**OI**: effectively ruling out primordial magnetogenesis from axion inflation]. We discover a universal value for both the electromagnetic fields as well as the conductivity of the Schwinger plasma at the onset of backreaction and present a surprisingly intuitive [**OI**: can we be more formal instead of use surprisingly intuitive?] derivation of these values. Furthermore, we explore the effect of the mass of the lightest Standard Model fermions and show that a large Higgs vacuum expectation value (VEV) during inflation can restore the viability of axion inflation magnetogenesis.

The rest of the Letter is organized as follows. We start by presenting the basics of the model and the different descriptions of the Schwinger-induced current [**OI**: ~~using electric and magnetic conductivities~~]. Following that, we present the results of our numerical simulations and analytic estimates. We conclude with the limitations of our method and outlook for future work.

Axion inflation and the Schwinger effect.

We consider a pseudoscalar inflaton (axion) ϕ coupled to the hypercharge sector of the Standard Model through a Chern-Simons interaction term in the presence of charged particles

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch}\right], \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu^{\rm ph} - \partial_\nu A_\mu^{\rm ph}$, α is the axion-gauge coupling, f is the axion decay constant, $V(\phi)$ is an axion potential and $\mathcal{L}_{ch} = \mathcal{L}_{ch}(A_\mu^{\rm ph}, \chi)$ describes all charged fields, χ, and their interaction with A_μ. With the superscript "ph" we denote *physical* fields. The physical electric four-current is then $J^\mu = -\partial\mathcal{L}_{\rm ch}/\partial A_\mu = \left(\rho_{\rm ch}, \boldsymbol{J}^{\rm ph}/a\right)$, where $a(t)$ is the scale factor and we assume charged particles initially absent (or exponentially diluted during inflation) and thus set the initial charge density to zero, $\rho_{\rm ch} = 0$. It is convenient to work with *comoving* fields that relate to physical as $\boldsymbol{E} = a^2\boldsymbol{E}^{\rm ph}$, $\boldsymbol{B} = a^2\boldsymbol{B}^{\rm ph}$, $\boldsymbol{J} = a^3\boldsymbol{J}^{\rm ph}$. Comoving electric and magnetic fields are defined as $\boldsymbol{E} = -\partial_\tau\boldsymbol{A} + \boldsymbol{\nabla}A_0$, $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, where we use derivatives with respect to conformal time $d\tau = dt/a(t)$. The dynamical equations that govern the evolution of the (comoving) gauge and axion fields are

$$\partial_\tau^2\phi + 2\mathcal{H}\partial_\tau\phi - \boldsymbol{\nabla}^2\phi + a^2\frac{dV}{d\phi} = \frac{\alpha}{a^2 f}\boldsymbol{E}\cdot\boldsymbol{B}, \tag{2}$$

$$\partial_\tau\boldsymbol{E} - \mathrm{rot}\,\boldsymbol{B} + \frac{\alpha}{f}\left(\partial_\tau\phi\boldsymbol{B} + \boldsymbol{\nabla}\phi\times\boldsymbol{E}\right) + \boldsymbol{J} = 0, \tag{3}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -\frac{\alpha}{f}\boldsymbol{\nabla}\phi\cdot\boldsymbol{B}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0, \tag{4}$$

$$\partial_\tau\boldsymbol{B} + \mathrm{rot}\,\boldsymbol{E} = 0, \tag{5}$$

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\rm pl}^2}a^2\left(\rho_\phi + \rho_E + \rho_B + \rho_\chi\right), \tag{6}$$

where $\mathcal{H} = \partial_\tau a/a$ is the conformal Hubble parameter. The energy densities are defined as $\rho_\phi = \left\langle(\partial_\tau\phi)^2/2a^2 + (\boldsymbol{\nabla}\phi)^2/2a^2 + V\right\rangle$ for the axion, $\rho_E = \left\langle\boldsymbol{E}^2\right\rangle/2a^4$ for the electric field, $\rho_B = \left\langle\boldsymbol{B}^2\right\rangle/2a^4$ for the magnetic field, and ρ_χ for the the plasma. In the simulation $\langle...\rangle$ denotes volume averaging over the whole simulation domain (box).

Strong backreaction from Schwinger currents. The induced Schwinger current generated by the created particles for the case of constant and spatially uniform (anti-)collinear electric and magnetic fields in de Sitter space takes the form [11–13]

$$J = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}E|B|\coth\left(\frac{\pi|B|}{E}\right)e^{-\frac{\pi m^2 a^2}{e|Q|E}}, \tag{7}$$

where $E = |\boldsymbol{E}|$ is the magnitude of the electric field and J, B are the electric current and magnetic field, projected onto the direction of the electric field, e is the gauge coupling constant, Q is the particle's charge and m is the particle's mass. We focus our attention on the strong-field limit, defined as [14] $|eQE| \gg \mathcal{H}^2$, meaning that we choose couplings that would generate an E-field that satisfies the above inequality in the absence of a Schwinger plasma. We also neglect the fermion masses by assuming $m\pi a^2 \ll e|Q|E$, unless otherwise stated. When electric and magnetic fields are (anti-)collinear, the induced current is proportional to both $\boldsymbol{E}$ and $\boldsymbol{B}$. [es - Is this the case without a Plasma? If so we should write it.] This results in an ambiguity in writing a vector form for the Ohm's law for the Schwinger current and allows for different formulations, dubbed the "electric", "magnetic" and "mixed" picture

$$\boldsymbol{J} = \sigma_E\boldsymbol{E}, \quad \sigma_E = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}|B|\coth\left(\frac{\pi|B|}{E}\right), \tag{8}$$

$$\boldsymbol{J} = \sigma_B\boldsymbol{B}, \quad \sigma_B = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}\mathrm{sign}(B)E\coth\left(\frac{\pi|B|}{E}\right), \tag{9}$$

$$\boldsymbol{J} = \sigma_E\boldsymbol{E} + \sigma_B\boldsymbol{B}, \tag{10}$$

where for the mixed picture the conductivities σ_E, σ_B are chosen to satisfy equation (7). We refer to this description as *collinear*, to emphasize the underlying assumption of (anti-)collinearity of the fields.

However, in axion inflation, electric and magnetic fields may not remain collinear or anti-collinear at all times. Relaxing the assumption of collinearity was addressed by performing a Lorentz boost from the comoving coordinate frame to a frame in which the electric and magnetic fields are collinear, and then transforming back. This was first explored perturbatively by considering small deviations around constant, anti-collinear background fields in Ref. [15], and later extended to a non-perturbative treatment in Ref. [14]. Since no assumption is made about the collinearity of the fields, and they can take arbitrary configurations, we refer to this case as *non-collinear*. This procedure leads to the induced current in the mixed picture (10), where it is described through both an electric and magnetic conductivities as [14]

$$\sigma_E = \frac{|\boldsymbol{J}'|E'}{\gamma}\frac{1}{\sqrt{(\boldsymbol{E}^2 - \boldsymbol{B}^2)^2 + 4(\boldsymbol{E}\cdot\boldsymbol{B})^2}}, \tag{11}$$

$$\sigma_B = \frac{|\boldsymbol{J}'|}{E'\gamma}\frac{(\boldsymbol{E}\cdot\boldsymbol{B})}{\sqrt{(\boldsymbol{E}^2 - \boldsymbol{B}^2)^2 + 4(\boldsymbol{E}\cdot\boldsymbol{B})^2}}, \tag{12}$$

where prime quantities are fields in the collinear frame defined through an arbitrary configuration of comoving $\boldsymbol{E}$ and $\boldsymbol{B}$ fields as

$$J' = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}E'|B'|\coth\left(\frac{\pi|B'|}{E'}\right), \tag{13}$$

$$E' = \frac{1}{\sqrt{2}}\left[\boldsymbol{E}^2 - \boldsymbol{B}^2 + I^2\right]^{1/2}, \tag{14}$$

$$B' = \frac{\mathrm{sign}(\boldsymbol{E}\cdot\boldsymbol{B})}{\sqrt{2}}\left[\boldsymbol{B}^2 - \boldsymbol{E}^2 + I^2\right]^{1/2}, \tag{15}$$

$$\gamma = \frac{1}{\sqrt{2}}\left[1 + \frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{I^2}\right]^{1/2}, \tag{16}$$

where $I^2 \equiv \sqrt{(\boldsymbol{E}^2-\boldsymbol{B}^2)^2+4(\boldsymbol{E}\cdot\boldsymbol{B})^2}$. To obtain a closed system of equations, one needs to account for the evolution of the fermion energy density, ρ_χ. Incorporating energy conservation in an expanding universe, the equation for ρ_χ can be written phenomenologically as [14, 16]

$$\partial_\tau \rho_\chi + 4\mathcal{H}\rho_\chi = \frac{1}{a^3}\left(\langle\sigma_E\rangle\,\langle\boldsymbol{E}^2\rangle + \langle\sigma_B\rangle\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle\right), \quad (17)$$

where it is assumed that the plasma is comprised of relativistic particles possessing a statistically isotropic momentum distribution, $p_\chi = \rho_\chi/3$. It is worth noting that in Eq. (17) one could use $\langle\sigma_E \boldsymbol{E}^2\rangle$, $\langle\sigma_B \boldsymbol{E}\cdot\boldsymbol{B}\rangle$. Choosing a different prescription does not significantly alter our results [**OI**: this we did not check, we checked average/non-average for conductivities. I would omit "Choosing a different prescription".. and only say "we defer"]; we defer a detailed comparison of different prescriptions for a subsequent publication.

Numerical simulations.
To determine the evolution of the system, we solve equations (2)–(6) together with equation (17). This is done numerically on a lattice using the Pencil Code [17]. The simulation includes a grid of 512^3 points and starts around 2 e-folds before the end of inflation. For simplicity we choose a quadratic potential for the axion $V(\phi) = \frac{1}{2}m^2\phi^2$ with $m = 1.04\times10^{-6}\,m_{\rm pl}$. Even though this is observationally ruled out during inflation, it is a valid approximation during preheating and as such has been widely used in the preheating literature [8]. We do not expect qualitative differences for more complicated potentials, like axion monodromy [8].

As outlined above, there are several possible descriptions of the Schwinger current. This is due to the non-perturbative nature of the effect and the fact that the solution is only known in the constant field limit. Thus extrapolating from this to more realistic scenarios leads to different prescriptions. We begin our analysis with the simplest parametrization: the collinear current description in the electric picture, given by Eq. (8). We follow the definitions of Ref. [14] for the charge and the gauge coupling constant. In the expression for the conductivities we set $Q^3 = 41/12$, which equals half the sum of the cubes of the hypercharges of all Standard Model Weyl fermions (while (7) is written for a single Dirac fermion). [es - repetitive, why do we need "the sum is taken..."?] [**OI**: I changed]. The gauge coupling constant is $e = g' = \sqrt{4\pi/137} \simeq 0.303$, however, a realistic description of the Schwinger effect requires taking into account its running. Hence in our simulations we use the gauge coupling constant $e = g'(\tilde\mu)$ defined as

$$g'(\tilde\mu) = \left([g'(m_Z)]^{-2} + \frac{41}{48\pi^2}\ln\frac{m_Z}{\tilde\mu}\right)^{-1/2}, \quad (18)$$

where $g'(m_Z) \simeq 0.35$, $m_Z \simeq 91.2\,\mathrm{GeV}$, with the characteristic energy scale $\tilde\mu$

$$\tilde\mu = (\rho_E+\rho_B)^{1/4} = \frac{1}{a}\left(\frac{1}{2}\langle\boldsymbol{E}^2\rangle + \frac{1}{2}\langle\boldsymbol{B}^2\rangle\right)^{1/4}. \quad (19)$$

The conductivity in Eq. (8) depends itself on the electric and magnetic fields. In our numerical simulations, we consider fields in (8) *locally*, at each point in the grid, fully taking their time and space dependence. It is worth noting that averaging the fields in the definition of conductivity over a domain yields a very similar result (see [18]).

We perform a full non-linear computation of the system for the collinear case (Eq. (8)) and the non-collinear case (Eq. (10), (11), (12)), treating the fields locally. [es - local treatment is repeated in the paragraph above][**OI**: that was for conductivity, I want to highlight again we use local treatment in all equations.] The result is shown in Figure [**OI**: to add Figure (the evolution of conductivities and $B_{\rm rms}$, $E_{\rm rms}$)]. Where $B_{\rm rms} = \sqrt{\int d\log k\,\cdot P_B(k)}$.

We also show the result in the linear regime, where $\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle = 0$ and the inflaton is homogeneous (i.e., $\boldsymbol{\nabla}\phi = 0$), dominating the dynamics of the universe such that $\mathcal{H}^2 = \frac{8\pi}{3m_{\rm pl}^2}a^2\rho_\phi$. The evolution in the strong backreaction regime without fermions has been analyzed in detail in [19, 20].

We see that all cases which operate in the large-coupling regime exhibit very similar suppression, regardless of the current description or whether the regime is treated as linear or non-linear.

Universality of the Schwinger backreaction.
It is known that the largest amplification of gauge fields during axion inflation occurs close to the end of inflation [8]. This can be simply understood, as the tachyonic amplification depends on the axion velocity, which is maximal close to the end of inflation. The growth of E and B fields can be described by a simple exponential growth rate (see [8] for a WKB analysis). Furthermore, the E and B fields are almost equal during this growth. By examining the equation of motion for the electric field (equivalently for the gauge field modes $A_\pm$), we see two competing terms: $(\alpha/f)(\partial_\tau\phi)\mathbf{B}$ supports the tachyonic amplification, whereas the current $\mathbf{J} = \sigma_E\mathbf{E}$ opposes it. Initially the tachyonic amplification term dominates and thus the fields undergo the usual exponential enhancement (one of the two polarizations). Since $|E| \approx |B|$, we can compare the two terms by comparing the $(\alpha/f)\partial_\tau\phi$ to the conductivity σ_E. For simplicity we take $a = 1$, as the amplification takes place mostly within one e-fold around the end of inflation, thereby derivatives with respect to conformal time coincide with their cosmic time counterparts.

Using $\partial_\tau\phi = m_{\rm Pl}\mathcal{H}\sqrt{\epsilon/4\pi}$ we get $(\alpha/f)\partial_\tau\phi \sim \mathcal{O}(100)\mathcal{H}$, where we took $\epsilon \sim 1$ close to the end of inflation and $\alpha M_{\rm Pl}/f \sim 60-100$. For $\mathcal{H} \sim 10^{-5}m_{\rm Pl}$ we see that the back-reaction from Schwinger pair-production occurs at $\sigma_E \sim 10^{-3}m_{\rm Pl}$. We can now also estimate the typical value of the electric and magnetic field. First we observe that $\coth(\pi E/B) \simeq 1$ for $E/B = \mathcal{O}(1)$. Furthermore $eQ \simeq 1$, leading to $B \sim 6\pi^2(\alpha m_{\rm Pl}/f)\mathcal{H}^2 \sim 10^{-6}m_{\rm Pl}^2$.

Intriguingly, the above estimates for the conductivity and the value of the electromagnetic fields at the onset of Schwinger backreaction are consistently supported by a wide range of simulations. In the large coupling regime, where the universe preheats almost instantaneously to gauge fields in the absence of the Schwinger effect, this universal plasma conductivity is reached during the early gauge field growth at the end of inflation. That is true for example for $\alpha m_{\rm Pl}/f = 60, 90$. For $\alpha M_{\rm Pl}/f = 35$, the same point is reached, albeit slightly less violently (see also Refs. [19, 20] [**OI**: correct ref (fig. 14)][es - I think we need 45 here, not 35. The IC's can play a role here]. Our analysis points to the existence of a universal behavior for axion inflation magnetogenesis, where the Schwinger effect is significant when gauge fields reach a value of $E, B \sim \mathcal{O}(10^{-6})m_{Pl}$ close to the end of inflation. The Schwinger suppression will be less pronounced (largely irrelevant) for couplings that lead to smaller field values.

Consequences for magnetogenesis.
The non-detection of secondary GeV photons from blazars provides indirect evidence for the presence of extragalactic magnetic fields in the intergalactic medium.[es - possibly helical [21]] This observation motivates investigations into their origin in the early universe. The prospect of magnetogenesis in axion inflation has been explored for couplings up to $\alpha m_{\rm pl}/f \le 60$ in Ref. [20] and, more recently, for $\alpha m_{\rm pl}/f = 75, 90$ in Ref. [19]. These studies conclude that, for $\alpha m_{\rm pl}/f \ge 60$, the axion–U(1) inflation model is already marginally compatible with generating magnetic fields strong enough to account for the non-observation of GeV photons in blazar spectra. Moreover, as we have seen, the Schwinger effect significantly reduces the final amplitude. To quantify the suppression, let us consider the present-day magnetic field strength and its coherence length (after accounting for the nonlinear evolution of the fields after conductivity becomes much larger than the Hubble parameter) are given by [es - mention inverse cascade here?] [19]

$$B^{\rm ph}_{\rm rms}|_0 = 9.2\times10^{-15}\,{\rm G}\sqrt{\frac{\int d\log k\,\cdot P_B}{\rho_{\rm tot}}}\left(\frac{10^{-6}m_{\rm pl}}{H}\right)r_A^{1/3}, \tag{20}$$

$$L_c|_0 = 0.8\,{\rm pc}\,(\mathcal{H}L_c)\left(\frac{10^{-6}m_{\rm pl}}{H}\right)r_A^{-2/3}, \tag{21}$$

where H is the Hubble parameter in comic time, ρ_{tot} is the total energy density at the end of simulation, $L_c = \frac{\int d\log k\cdot k^{-1}\cdot P_B}{\int d\log k\cdot P_B}$ is the coherence length, and the parameter r_A is defined as $r_A = \max(1, \mathcal{H}L_c/V_A)$, where $V_A = \sqrt{B^2/(2(\rho_{\rm tot}+p))}$ is the Alfvén velocity. The lower bound on the present-day magnetic field strength depends on the coherent length as $B_{\rm bd} = 1.8\times10^{-17}\,{\rm G}\,(L_c|_0/0.2{\rm Mpc})^{1/2}$ [22]. The results of [19] show that even without the Schwinger suppression for $\alpha m_{\rm pl}/f \ge 60$ and a coherence length $L_c|_0 \simeq 10^{-1}-10^{-2}\,{\rm pc}$ the primordial magnetic field has an amplitude $B^{\rm ph}_{\rm rms}|_0 \simeq 10^{-14}-10^{-15}{\rm G}$, which is already barely consistent with the observational lower bound of $B_{\rm bd} \simeq 10^{-14}\,{\rm G}$ for this coherence length. When the Schwinger effect is taken into account, it suppresses $B_{\rm rms}$ by at least two orders of magnitude for large couplings, and reduces the electromagnetic energy density by approximately four orders of magnitude. As a result, the ratio $\sqrt{B^2_{\rm rms}/\rho_{\rm tot}}$ in (20) remains well below unity (see [**OI**: Fig for energy densities]), effectively ruling out magnetogenesis from axion inflation.

Heavy fermion effects.
So far, we have neglected the effects arising from finite fermion masses. From (7) it follows that a significant suppression of the Schwinger effect requires $\frac{\pi m^2 a^2}{e|Q|E} > 1$ or $m^2/E \gtrsim \mathcal{O}(1)$, where we took $a \approx 1$, as we are focusing to the era close to the end of inflation. [**OI**: in our simulation $a = 1$ is at the start of inflation.] At the end of inflation $E^2 \simeq B^2 \sim 3H^2m^2_{\rm Pl}/8\pi$, or $E \sim 0.1Hm_{\rm Pl}$. The masses of Standard Model (SM) fermions are given by $m = yh$, where y is the Yukawa coupling and h is the Higgs VEV, which is expected to be nonzero during inflation, if the Higgs is a light field subject to de-Sitter fluctuations. Among the electrically charged fermions, the electron has the smallest Yukawa coupling, $y_e \simeq 3\times10^{-6}$, making it the lightest. Thus, if electrons are too heavy to be efficiently produced via the Schwinger effect, all other charged fermions will be even more suppressed. To suppress the Schwinger effect, the fermion mass must satisfy

$$m^2 \gtrsim 0.1Hm_{\rm Pl}\,. \tag{22}$$

Figure [**OI**: Fig and its description goes here] shows how the presence of a large electron mass suppresses the Schwinger current and can restore magnetogenesis. However, the constraint (22) implies a lower bound on the Higgs VEV $h \gtrsim (0.3/y_e)\sqrt{H m_{\rm Pl}} \simeq 10^5 m_{\rm Pl}\sqrt{H/M_{\rm Pl}}$. To avoid super-Planckian values for the Higgs field, one requires a low inflationary scale: $H \lesssim 10^{-10} m_{\rm Pl} \sim 10^9$ GeV. This provides a key result: a suppression of the Schwinger effect through fermion masses requires both low-scale inflation and large (possibly Planckian) field excursions. If one relies solely on de Sitter fluctuations to generate a Higgs VEV, one expects $h \sim H\lambda^{-1/4}$, which leads to $H/m_{\rm Pl} \gtrsim 10^{12}\sqrt{\lambda}$. This requirement is difficult to satisfy given observational upper bounds on H, unless we consider an almost vanishing Higgs self-coupling. However, alternative mechanisms such as a direct coupling between the Higgs and the inflaton, or a non-minimal coupling to gravity can dynamically induce a large Higgs VEV during inflation.

These considerations point to an intriguing model-building challenge: any realistic suppression of the Schwinger effect involving SM fermions may lead to observable consequences at collider experiments through a modification of the Higgs sector. We must note before concluding that all simulations presented here refer to high-scale inflation, $H \sim 10^{-6} m_{\rm Pl}$, and thus need to be re-done for different Hubble scales. However, given our qualitative understanding for the efficient preheating and the universal onset of the Schwinger plasma backreaction, simple estimates for different Hubble scales are easy to make.

Summary and outlook.

We presented the first lattice simulation of non-linear evolution after axion inflation that self-consistently incorporate Schwinger pair production. Our results demonstrate that the induced Schwinger current provides a robust backreaction that quenches gauge field amplification once a *universal conductivity threshold* $\sigma_E \sim 10^{-3} m_{\rm Pl}$ and *magnetic field strength* $B_{rms} \sim 10^{-6} m_{\rm Pl}^2$ are reached. This threshold is independent of the specific formulation of the current and marks the onset of plasma domination.

The resulting suppression of EM field production significantly reduces the amplitude of primordial magnetic fields, effectively ruling out axion inflation as a source of intergalactic magnetogenesis. We further showed that avoiding this suppression requires fermion masses large enough to inhibit Schwinger production—necessitating a large Higgs VEV during inflation and favoring low-scale inflation scenarios. These findings motivate future studies on low-scale inflationary models and on connections between the axion and Higgs sectors, with potential implications for collider signatures. [es - should we hint on our long paper here and mention the Schwinger time-integration issue?]

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Appendix: Full integration of the current - spurious interpretation for time evolving current.
[**OI**: I wrote here the current from [12] in our convention and a reference in case you would like to go for this section. Here it is still not in a vector form, I kept "c" for comoving. BTW K. Mukaida is coming for a conference, we might want to talk to him :)] [es - Yes, it's always useful AND fun to talk to people like Kyohei] The differential equation for the current is [12] (equation (4.13)-(4.14))

$$\partial_\tau(J^{com}) = \frac{(e|Q|)^3}{2\pi^2} E_c B_c \coth\left(\frac{\pi B_c}{E_c}\right) \tag{23}$$

After integration it gives (assumption is the constant H here, and static physical (!) fields)

$$J^{com} = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} |B_c| E_c \coth\left(\frac{\pi |B_c|}{E_c}\right) \tag{24}$$

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[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. B **108**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).
[5] P. Svrcek and E. Witten, JHEP **06**, 051 (2006), hep-th/0605206.
[6] M. M. Anber and L. Sorbo, Phys. Rev. D **81**, 043534 (2010), 0908.4089.
[7] N. Barnaby, R. Namba, and M. Peloso, JCAP **04**, 009 (2011), 1102.4333.
[8] P. Adshead, J. T. Giblin, T. R. Scully, and E. I. Sfakianakis, JCAP **12**, 034 (2015), arXiv:1502.06506 [astro-ph.CO].
[9] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).

[10] Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Phys. Rev. D **45**, 4659 (1992).
[11] E. Bavarsad, S. P. Kim, C. Stahl, and S.-S. Xue, Phys. Rev. D **97**, 025017 (2018), arXiv:1707.03975 [hep-th].
[12] V. Domcke and K. Mukaida, JCAP **11**, 020 (2018), arXiv:1806.08769 [hep-ph].
[13] V. Domcke, Y. Ema, and K. Mukaida, JHEP **02**, 055 (2020), arXiv:1910.01205 [hep-ph].
[14] R. von Eckardstein, K. Schmitz, and O. Sobol, Journal of High Energy Physics **2025**, 96 (2025), arXiv:2408.16538 [hep-ph].
[15] T. Fujita, J. Kume, K. Mukaida, and Y. Tada, JCAP **09**, 023 (2022), arXiv:2204.01180 [hep-ph].
[16] O. O. Sobol, E. V. Gorbar, and S. I. Vilchinskii, Phys. Rev. D **100**, 063523 (2019), arXiv:1907.10443 [astro-ph.CO].
[17] A. Brandenburg *et al.* (Pencil Code), J. Open Source Softw. **6**, 2807 (2021), arXiv:2009.08231 [astro-ph.IM].
[18] A. Brandenburg, O. Iarygina, and E. I. Sfakianakis, Manuscript in preparation.
[19] R. Sharma, A. Brandenburg, K. Subramanian, and A. Vikman, JCAP **05**, 079 (2025), arXiv:2411.04854 [astro-ph.CO].
[20] P. Adshead, J. T. Giblin, T. R. Scully, and E. I. Sfakianakis, JCAP **10**, 039 (2016), arXiv:1606.08474 [astro-ph.CO].
[21] T. Vachaspati, Phys. Rev. D **95**, 063505 (2017), arXiv:1606.06186 [astro-ph.CO].
[22] V. A. Acciari *et al.* (MAGIC), Astron. Astrophys. **670**, A145 (2023), arXiv:2210.03321 [astro-ph.HE].

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Supplemental Material

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1. Full integration of the current

It is worth pointing out that the formulation for the Schwinger current derived through [es - mention here the QFT calculation] is given in terms of the its derivative [12]

$$\partial_\tau (J^{com}) = \frac{(e|Q|)^3}{2\pi^2} E_c B_c \coth\left(\frac{\pi B_c}{E_c}\right) \tag{S1}$$

where the assumption for static electric and magnetic field is made and the fermion mass is again dropped for simplicity. One can integrate the above, by adding the assumption of a constant Hubble scale to the already static EM fields, and arrive at

$$J^{com} = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} |B_c| E_c \coth\left(\frac{\pi |B_c|}{E_c}\right) \tag{S2}$$

which is the form of the current that we used in our simulations and has been widely used in the literature [es - some citations].

Since we are putting the full system on the lattice though, there is no difficulty in numerically integrating Eq. (S1) instead of using Eq. (S2). Figure [es - insert figure] shows the results of a simulation run, where the axion and electromagnetic fields are computed at each point on the lattice, while the current is computed by integrating Eq. (S2), similarly at each point on the lattice. We first of all observe that until the universal threshold of $(\sigma_E, B) \sim (10^{-3}, 10^{-6} m_{\rm Pl}^2)$. However, after the threshold value has been reached, the evolution is qualitatively different.