

# Astronomical Notes

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## 1 Introduction

In a seminal paper, Pouquet et al. (1976) showed that in the nonlinear regime the alpha effect in mean field magnetohydrodynamics is no longer governed by the kinetic helicity (Steenbeck et al. 1966), but there is an additional contribution from the current helicity, so

$$\alpha = \frac{1}{3}\tau (-\overline{\boldsymbol{\omega} \cdot \mathbf{u}} + \rho_0^{-1} \overline{\mathbf{j} \cdot \mathbf{b}}), \quad (1)$$

where  $\tau$  is a correlation or relaxation time,  $\mathbf{u}$  is the small scale velocity,  $\boldsymbol{\omega} = \nabla \times \mathbf{u}$  is the small scale vorticity,  $\mathbf{b}$  is the small magnetic field, and  $\mathbf{j} = \nabla \times \mathbf{b} / \mu_0$  is the small scale current density. Overbars denote some suitable form of averaging. Equation (1) has been used to explain catastrophic (magnetic Reynolds number dependent) quenching of the alpha effect in the nonlinear regime (Gruzinov & Diamond 1994; Bhattacharjee & Yuan 1995; Field & Blackman 2002). Technically, the  $\overline{\mathbf{j} \cdot \mathbf{b}}$  term arises naturally when the  $\tau$  approximation is used (Kleeorin & Rogachevskii 1999; Rädler et al. 2003; Blackman & Field 2002, 2003; see review by Brandenburg & Subramanian 2005a).

In a recent paper, Brandenburg & Subramanian (2005b, hereafter BS05) presented results of numerical simulations that demonstrate the rise of the  $\overline{\mathbf{j} \cdot \mathbf{b}}$  term with magnetic Reynolds number in the presence of a finite imposed magnetic field,  $\overline{\mathbf{B}}_0$ . Recently, Rädler & Rheinhardt (2007) have pointed out that for finite values of  $\overline{\mathbf{B}}_0$  it may be important to consider instead the appropriate anisotropic expression, which can be written in the form

$$\alpha_{ip} = \tau \epsilon_{ijk} (-\overline{u_k u_{j,p}} + \rho_0^{-1} \overline{b_k b_{j,p}}). \quad (2)$$

The purpose of the present paper is to demonstrate that the values for both expressions, (1) and (2), are almost identical in the cases presented by BS05. We also show that the value of  $\tau$ , expressed in units of the turnover time, is in all cases close to unity, and in some cases better so than in BS05.

## 2 Formalism

As in BS05 we consider cases where the flow is driven either by a random body force in the momentum equation, or, alternatively, by random externally imposed currents in the induction equation. We calculated the isotropic expressions

$$\tilde{\alpha}_K = -\frac{1}{3} \overline{\boldsymbol{\omega} \cdot \mathbf{u}}, \quad \tilde{\alpha}_M = \frac{1}{3} \rho_0^{-1} \overline{\mathbf{j} \cdot \mathbf{b}}, \quad (3)$$

in the presence of an imposed mean field  $\overline{\mathbf{B}}_0$ , where the tilde indicates the absence of the  $\tau$  factor, so  $\alpha = \tau(\tilde{\alpha}_K + \tilde{\alpha}_M)$ . As in BS05, we use additional superscripts *k* and *m* to indicate cases with kinetic or magnetic driving. The resulting values of  $\tilde{\alpha}_K^{(k)}$ ,  $\tilde{\alpha}_K^{(m)}$ ,  $\tilde{\alpha}_M^{(k)}$ , and  $\tilde{\alpha}_M^{(m)}$ , presented below, are identical to those of BS05. In addition, we consider the appropriate component of the anisotropic expressions for the same simulation data. Since in our case the mean field points in the *y* direction, and because we use volume averages and periodic boundary conditions in all three directions, we can write the anisotropic expressions for  $\tilde{\alpha}_{yy}$  in the form

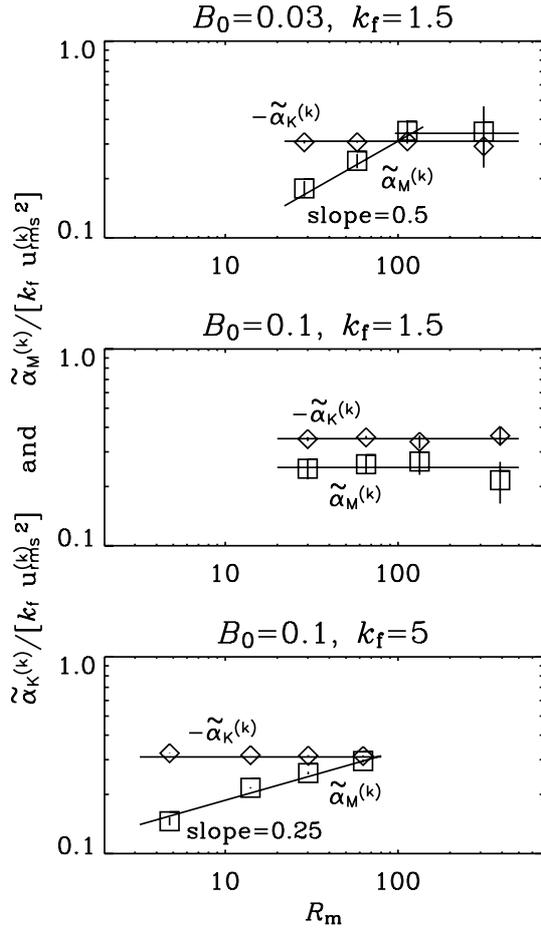
$$\tilde{\alpha}_K^{(a)} = -2 \overline{u_x u_{z,y}}, \quad \tilde{\alpha}_M^{(a)} = 2 \rho_0^{-1} \overline{b_x b_{z,y}}, \quad (4)$$

where the superscript (a) indicates anisotropy. Again, we consider cases with kinetic and magnetic driving and thus obtain the 4 values,  $\tilde{\alpha}_K^{(ak)}$ ,  $\tilde{\alpha}_K^{(am)}$ ,  $\tilde{\alpha}_M^{(ak)}$ , and  $\tilde{\alpha}_M^{(am)}$ . The resulting values are normalized with respect to the corresponding rms turbulent velocities,

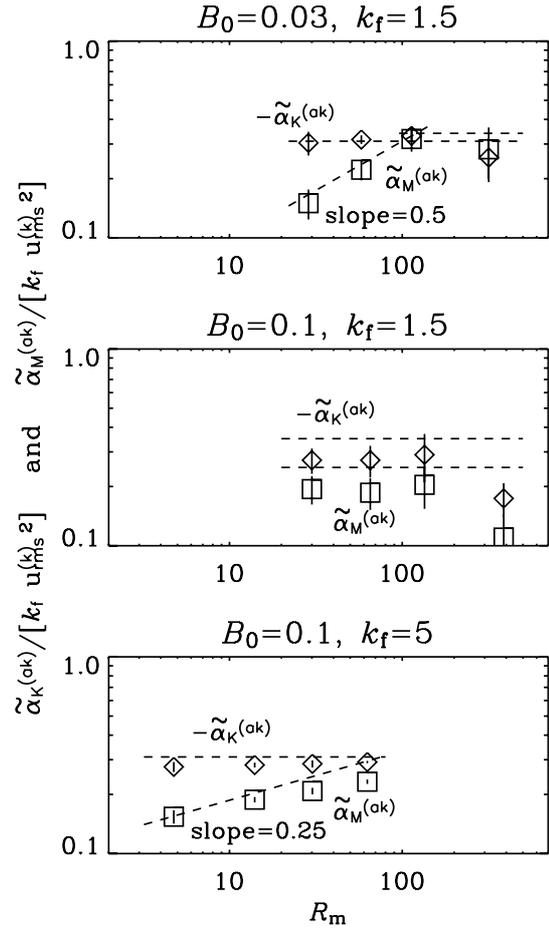
$$\tilde{a}_{K,M}^{(k,m)} = \tilde{\alpha}_{K,M}^{(k,m)} / \left[ k_f u_{\text{rms}} u_{\text{rms}}^{(k,m)} \right], \quad (5)$$

where  $u_{\text{rms}} = [u_{\text{rms}}^{(k)} u_{\text{rms}}^{(m)}]^{1/2}$  is the geometrical mean of the rms velocities for kinetically and magnetically driven runs. This particular normalization emerges naturally when deriving the time scale  $\tau$  in Eq. (2). In the following we only consider the case of a statistically steady state, so  $\overline{b_x b_{z,y}}$  and  $\overline{u_x u_{z,y}}$ , and hence also  $\tilde{\alpha}_M^{(a)}$  and  $\tilde{\alpha}_K^{(a)}$ , have converged to a stationary value.

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**Fig. 1** Dependence of  $\tilde{\alpha}_K^{(k)}$  and  $\tilde{\alpha}_M^{(k)}$  on  $R_m$  in the kinetically forced case. Vertical bars give error estimates. Adapted from BS05.



**Fig. 2** Same as Fig. 1, but for the relevant component of the anisotropic expressions,  $\tilde{\alpha}_K^{(ak)}$  and  $\tilde{\alpha}_M^{(ak)}$ . The dashed lines represent the fit to the data of Fig. 1, not the present data!

### 3 Results

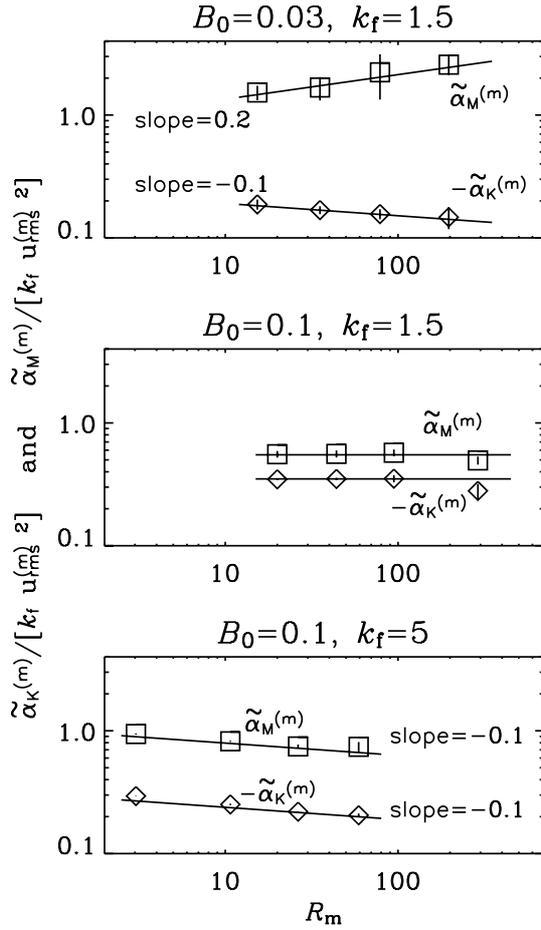
We consider the values of  $\tilde{\alpha}_K$  and  $\tilde{\alpha}_M$  and compare with the results of the appropriate component of the anisotropic expressions; see Figs. 1 and 2 for the kinetically driven case and Figs. 3 and 4 for the magnetically driven case. The straight lines in Figs. 1 and 3 denote fits to the data points, while in Figs. 2 and 4 the same lines are just repeated as dashed lines and still represent only the fits to the isotropic data. This helps demonstrating that the results change very little when the anisotropic expressions are used.

It is remarkable that the differences between the isotropic and anisotropic expressions are rather systematic. Generally speaking, the anisotropic expressions give either the same or slightly smaller values than the isotropic expressions if the flow is driven hydrodynamically. The differences are larger for stronger fields ( $B_0 = 0.1$ ) and especially when the forcing is at larger scales ( $k_f = 1.5$ ). In that case the differences are around 15% and 25% for the kinetic and magnetic  $\alpha$  effects, respectively. In the magnetically driven case the kinetic alpha effect tends to be smaller for the anisotropic expressions, but the magnetic alpha ef-

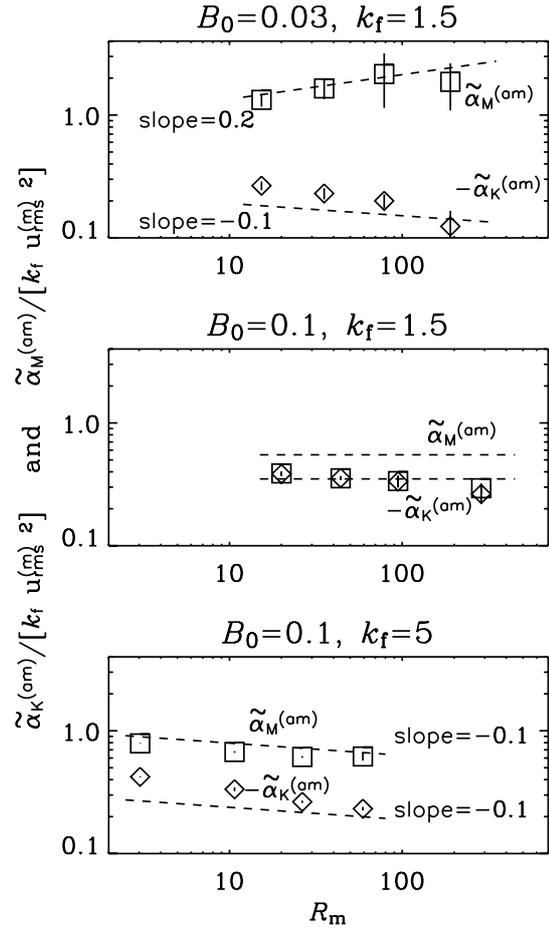
fect is either the same or larger for the anisotropic expressions.

Following BS05, we also compare the results for all runs in tabular form; see Table 1. As in BS05, we non-dimensionalize the measurements for kinetically and magnetically driven cases independently, because the root mean square velocities,  $u_{rms}^{(k)}$  and  $u_{rms}^{(m)}$ , are different in the two cases; see Eq. (5).

There are two important aspects of the  $R_m$  dependence of kinetic and magnetic  $\alpha$  effects. One is the fact that, at least for moderate values of  $R_m$ , the two approach each other for finite field strength and increasing strength of the mean field. Furthermore, in the case of isotropic expressions,  $|\tilde{\alpha}_M|$  could even slightly exceed the value of  $|\tilde{\alpha}_K|$ . But when the anisotropic expressions are used, this is no longer the case – or at least less drastically so, e.g. in the middle panel of Fig. 2. The other aspect is the tendency for  $\tilde{\alpha}_K$  to stay asymptotically independent of  $R_m$ , even though the actual  $\alpha$  effect decreases like  $1/R_m^n$ , with  $n = 0.5 \dots 1$ , as was shown in Fig. 2 of BS05 for the same data. This property is critical to understanding the catastrophic quenching of the  $\alpha$  effect for closed or periodic domains where magnetic he-



**Fig. 3** Dependence of  $\tilde{\alpha}_K^{(m)}$  and  $\tilde{\alpha}_M^{(m)}$  on  $R_m$  in the magnetically forced case. Vertical bars give error estimates. Adapted from BS05.



**Fig. 4** Same as Fig. 3, but for the relevant component of the anisotropic expressions,  $\tilde{\alpha}_K^{(am)}$  and  $\tilde{\alpha}_M^{(am)}$ . The dashed lines represent the fit to the data of Fig. 3, not the present data!

licity is a conserved quantity in the high conductivity limit. (We recall that, in contrast to the expressions for  $\tilde{\alpha}_K^{(a)}$  and  $\tilde{\alpha}_M^{(a)}$ ,  $\alpha$  itself was always calculated as  $\alpha = \langle \vec{\mathcal{E}} \cdot \mathbf{B}_0 \rangle_t / B_0^2$ , which does already account for the anisotropy for  $\alpha$ . So the results for  $\alpha$  remain unchanged from those obtained in BS05.) Let us also note in this connection that, within error bars, the off-diagonal components of the  $\alpha$  tensor are found to be zero, i.e.  $|\langle \vec{\mathcal{E}} \times \mathbf{B}_0 \rangle_t| = 0$ .

Finally we address the question of the relaxation time  $\tau$ . In BS05 we calculated  $\tau$  based on the values of  $\alpha$ ,  $\tilde{\alpha}_K^{(k,m)}$ , and  $\tilde{\alpha}_M^{(k,m)}$ . In the following we repeat the same analysis using the anisotropic expressions,  $\tilde{\alpha}_K^{(ak,am)}$  and  $\tilde{\alpha}_M^{(ak,am)}$ . We recall that we allowed for different and unknown prefactors  $g_K$  and  $g_M$  in front of  $\tilde{\alpha}_K$  and  $\tilde{\alpha}_M$ . We therefore wrote our unknowns in the form  $\tau g_K$  and  $\tau g_M$ , and expressed them in normalized form as

$$\text{St } g_{K,M} = u_{\text{rms}} k_f \tau g_{K,M}. \quad (6)$$

These unknowns can be obtained by solving a matrix equation which, in the present case, reads

$$\begin{pmatrix} a^{(ak)} \\ a^{(am)} \end{pmatrix} = \begin{pmatrix} \tilde{\alpha}_K^{(ak)} & \tilde{\alpha}_M^{(ak)} \\ \tilde{\alpha}_K^{(am)} & \tilde{\alpha}_M^{(am)} \end{pmatrix} \begin{pmatrix} \text{St } g_K \\ \text{St } g_M \end{pmatrix}. \quad (7)$$

The result is shown in Fig. 5 for the old case using isotropic expressions of  $\tilde{\alpha}$ , and in Fig. 6 for the present case using the anisotropic expressions.

One of the most remarkable results from Fig. 6 is that the values of the magnetic and kinetic Strouhal numbers are in all three cases close to unity, whereas in the middle panel of Fig. 5 the Strouhal numbers were only about 0.3. In all other aspects the new results are rather similar to the old ones. For example, the values of magnetic and kinetic Strouhal numbers are rather close to each other except in the case  $B_0 = 0.1$  with  $k_f = 1.5$ , where the magnetic Strouhal numbers are somewhat larger than the kinetic ones. This is also the parameter regime for which the largest differences were found between Figs. 1 and 2. Furthermore, like in BS05, we still find a drop in the Strouhal numbers in the case where  $R_m$  is around 300. As argued in BS05, this may be connected with these simulations not having run for long enough.

**Table 1** Comparison of the results using the isotropic and anisotropic expressions for the various values of the normalized  $\alpha$  for kinetically and magnetically forced runs. For  $k_f = 1.5$  the resolution varies between  $64^3$  and  $512^3$  mesh points for  $\eta = 2 \times 10^{-3}$  and  $2 \times 10^{-4}$ , corresponding to magnetic Reynolds numbers of 20 and 300, respectively, while for  $k_f = 5$  the resolution varies between  $32^3$  and  $256^3$  mesh points for  $\eta = 5 \times 10^{-3}$  and  $5 \times 10^{-4}$ , corresponding to magnetic Reynolds numbers of 4 and 60, respectively. The magnetic Prandtl number is always equal to unity, i.e. the viscosity  $\nu$  is always equal to the magnetic diffusivity,  $\eta$ .

$B_0$	$\eta$	$k_f$	$u_{\text{rms}}^{(k)}$	$a^{(k)}$	$\tilde{a}_K^{(k)}$	$\tilde{a}_K^{(ak)}$	$\tilde{a}_M^{(k)}$	$\tilde{a}_M^{(ak)}$	$u_{\text{rms}}^{(m)}$	$a^{(m)}$	$\tilde{a}_K^{(m)}$	$\tilde{a}_K^{(am)}$	$\tilde{a}_M^{(m)}$	$\tilde{a}_M^{(am)}$
0.01	$2 \times 10^{-3}$	1.5	0.10	-0.261	-0.46	-0.44	0.04	0.04	0.05	4.79	-0.11	-0.21	1.44	2.53
0.03	$2 \times 10^{-4}$	1.5	0.09	-0.048	-0.38	-0.33	0.46	0.36	0.06	0.29	-0.12	-0.10	2.23	1.44
0.03	$5 \times 10^{-4}$	1.5	0.09	-0.062	-0.37	-0.40	0.42	0.38	0.06	0.88	-0.13	-0.17	1.85	1.80
0.03	$1 \times 10^{-3}$	1.5	0.09	-0.099	-0.39	-0.40	0.32	0.28	0.05	0.88	-0.13	-0.18	1.31	1.29
0.03	$2 \times 10^{-3}$	1.5	0.09	-0.143	-0.42	-0.42	0.24	0.21	0.05	0.74	-0.14	-0.19	1.12	0.97
0.06	$1 \times 10^{-3}$	1.5	0.09	-0.030	-0.40	-0.39	0.36	0.28	0.06	0.23	-0.24	-0.28	0.61	0.46
0.06	$2 \times 10^{-3}$	1.5	0.08	-0.054	-0.40	-0.40	0.35	0.28	0.05	0.22	-0.24	-0.30	0.58	0.44
0.10	$2 \times 10^{-4}$	1.5	0.12	-0.003	-0.42	-0.20	0.24	0.13	0.09	0.07	-0.25	-0.23	0.41	0.25
0.10	$5 \times 10^{-4}$	1.5	0.10	-0.008	-0.41	-0.35	0.32	0.24	0.07	0.08	-0.29	-0.28	0.48	0.28
0.10	$1 \times 10^{-3}$	1.5	0.10	-0.010	-0.43	-0.33	0.32	0.23	0.07	0.08	-0.29	-0.29	0.46	0.29
0.10	$2 \times 10^{-3}$	1.5	0.09	-0.019	-0.43	-0.33	0.30	0.24	0.06	0.07	-0.28	-0.31	0.45	0.32
0.14	$2 \times 10^{-3}$	1.5	0.10	-0.009	-0.43	-0.25	0.26	0.20	0.06	0.04	-0.28	-0.28	0.45	0.26
0.20	$2 \times 10^{-3}$	1.5	0.11	-0.004	-0.43	-0.18	0.21	0.16	0.06	0.02	-0.27	-0.24	0.43	0.22
0.30	$2 \times 10^{-3}$	1.5	0.12	-0.002	-0.42	-0.14	0.18	0.13	0.06	0.01	-0.24	-0.19	0.41	0.19
0.06	$5 \times 10^{-4}$	5	0.16	-0.080	-0.31	-0.30	0.25	0.22	0.15	0.08	-0.25	-0.20	1.10	0.45
0.06	$1 \times 10^{-3}$	5	0.16	-0.121	-0.32	-0.30	0.20	0.18	0.14	0.01	-0.12	-0.09	2.03	0.17
0.06	$2 \times 10^{-3}$	5	0.15	-0.172	-0.49	-0.46	0.22	0.20	0.06	0.34	-0.16	-0.22	0.52	0.44
0.06	$5 \times 10^{-3}$	5	0.13	-0.215	-0.41	-0.37	0.10	0.11	0.08	0.54	-0.18	-0.23	0.81	0.72
0.10	$5 \times 10^{-4}$	5	0.16	-0.035	-0.32	-0.30	0.30	0.24	0.15	0.36	-0.20	-0.23	0.72	0.60
0.10	$1 \times 10^{-3}$	5	0.15	-0.058	-0.34	-0.31	0.27	0.22	0.13	0.35	-0.21	-0.25	0.70	0.57
0.10	$2 \times 10^{-3}$	5	0.14	-0.091	-0.36	-0.32	0.25	0.22	0.11	0.34	-0.22	-0.29	0.72	0.59
0.10	$5 \times 10^{-3}$	5	0.12	-0.131	-0.41	-0.35	0.18	0.19	0.08	0.31	-0.24	-0.34	0.75	0.63

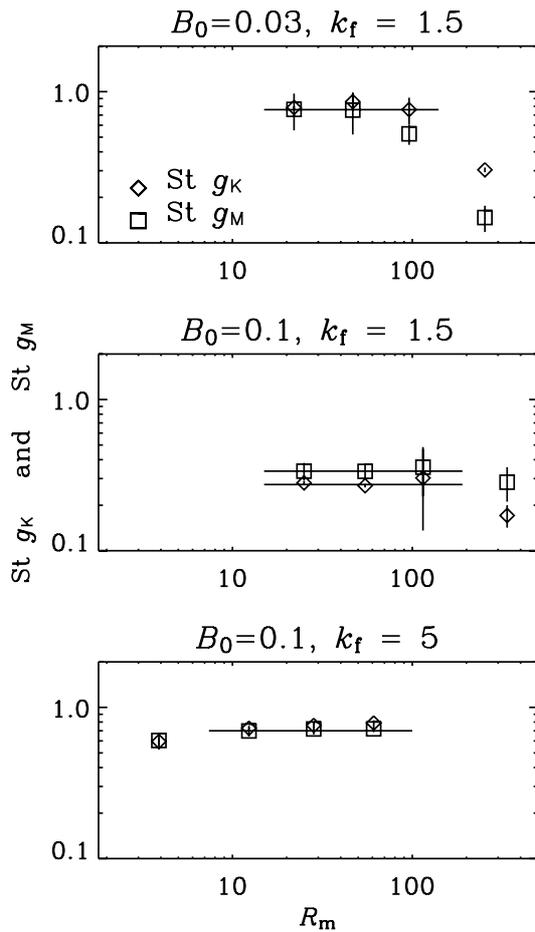
## 4 Discussion

The work of BS05 was mainly an extension of earlier work on passive scale diffusion (Brandenburg et al. 2004), where certain aspects of MTA were tested. In particular, it was shown that the relaxation time  $\tau$  in the  $\tau$  approximation is of the order of the turnover time ( $St = \tau u_{\text{rms}} k_f \approx 3$ ). In the case with a magnetic field, the  $\alpha$  effect was assumed to be expressible as  $\alpha = \tau(\tilde{\alpha}_K + \tilde{\alpha}_M)$ . The main result of BS05 was that  $St$  is independent of  $R_m$ . This is important because neither  $\tilde{\alpha}_K$  nor  $\tilde{\alpha}_M$  decline with increasing values of  $R_m$ . Instead,  $-\tilde{\alpha}_M$  approaches  $\tilde{\alpha}_K$ , resulting in near cancellation. Together with the finding that  $\tau$  is approximately independent of  $R_m$ , this supports the validity of the assumed formula for  $\alpha$ . It should be noted, however, that for  $R_m \approx 300$  the result is not convincing and our present data suggest a drop in the Strouhal number.

However, as RR07 have pointed out, several other issues remained open or unsatisfactory. In particular the comparative use of kinetically and magnetically forced models may be questionable. This was done to change the relative importance of kinetic and magnetic  $\alpha$  effects. The problem is that the nature of the turbulence can change considerably in the two cases. On the other hand, there is no reason why the expressions for  $\alpha$  should not apply equally well in both regimes

Another problem is the use of isotropic expressions for  $\tilde{\alpha}_K$  and  $\tilde{\alpha}_M$ . Surprisingly enough, as we have shown here, the isotropic expressions are indeed good proxies for the relevant component of the full anisotropic expressions. One advantage of using the anisotropic expressions is that the need for adopting (slightly) different coefficients in front of  $\tilde{\alpha}_K$  and  $\tilde{\alpha}_M$  is now less severe, if at all present.

Finally, there is the puzzle that, on the one hand, when using the first order smoothing approximation (FOSA),  $\alpha$  is given by an expression involving just the actual velocity field while, on the other hand, according to the  $\tau$  approximation, it is the sum of magnetic and kinetic  $\alpha$  effects. Obviously, a rigorous comparison between FOSA and  $\tau$  approximation is only permissible when the magnetic Reynolds number is below unity. In the present paper this is not the case, so the neglect of the higher order (triple) correlation terms under FOSA cannot be justified, given that the Strouhal numbers are always around unity. So this comparison may not have been permissible. However, the puzzle seems to exist even in the low magnetic Reynolds number limit, when the triple correlations can be neglected altogether. This case has been analyzed recently by Sur et al. (2007), who showed that the formulations in terms of FOSA and  $\tau$  approximation are in fact equivalent (as they have to be, because the starting equations are the same!), but that

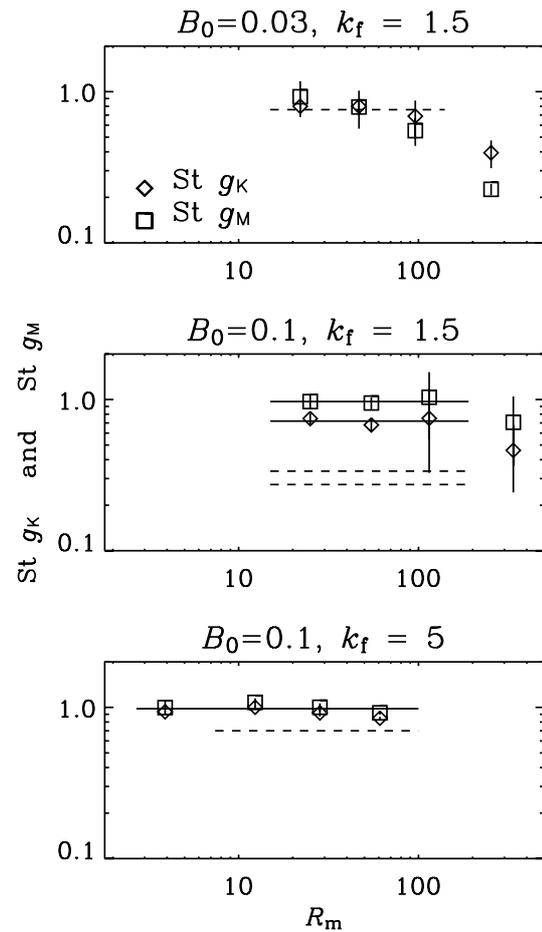


**Fig. 5** Magnetic and kinetic Strouhal numbers as a function of  $R_m$  for different values of  $B_0$  and  $k_f$ . Here, kinetically and magnetically forced runs have been used to calculate separately  $g_K \neq g_M$ . The horizontal lines are drawn to indicate the range over which the Strouhal numbers are approximately constant. Adapted from BS05.

the individual components contributing to the total  $\alpha$ -effect in the two formulations are different. In fact, it turns out that in the  $\tau$  approximation there is, in addition to the kinetic and magnetic alpha effects, in general also one more term resulting from the correlation between the small scale magnetic field and the forcing function. Only in the special case of  $\delta$ -correlated forcing, that is adopted in many numerical investigations, does this extra term vanish. Nevertheless, even then the kinetic part of the alpha effect in the  $\tau$  approximation is not simply related to the alpha effect obtained from the first order smoothing approximation, even if the actual velocity field is used in both cases. Therefore there is actually no puzzle in the limit of small magnetic Reynolds numbers either.

## 5 Conclusions

We have shown that the basic conclusions obtained in BS05 carry over to the case where the anisotropic expressions for



**Fig. 6** Same as Fig. 5, but for Strouhal numbers calculated from the expressions for the anisotropic alpha coefficients. The dashed lines represent the fits used in Fig. 5, and the solid lines represent new fits.

$\tilde{\alpha}_K$  and  $\tilde{\alpha}_M$  are used. The present work provides an extra piece of evidence that the  $\tau$  approximation may provide a useable formalism for describing simulation data and for predicting the behavior in situations that are not yet accessible to direct simulations. There are currently no other approaches capable of this. The basic hypothesis that the triple correlations are expressible as a damping term may not be well justified, although some important properties of this approach seem to be borne out by simulations. A number of further practical tests of the  $\tau$  approximations could be envisaged. One such example might be the so-called  $\overline{\mathbf{W}} \times \overline{\mathbf{J}}$  effect of Rogachevskii & Kleeorin (2003, 2004), which was derived using the  $\tau$  approximation. Direct simulations of hydromagnetic turbulence with shear give qualitative support to this idea (Brandenburg 2005a), although it is not clear under which conditions the anticipated effect has the appropriate sign for dynamo action (Brandenburg 2005b; Rüdiger & Kitchatinov 2006; Rädler & Stepanov 2006). Further work in this direction would be worthwhile for establishing the real usefulness of the  $\tau$  approximation.

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