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THE ONSET OF A SMALL-SCALE TURBULENT DYNAMO AT LOW MAGNETIC PRANDTL NUMBERS

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ABSTRACT

We study numerically the dependence of the critical magnetic Reynolds number Rm_c for the turbulent small-scale dynamo on the hydrodynamic Reynolds number Re. The turbulence is statistically homogeneous, isotropic, and mirror-symmetric. We are interested in the regime of low magnetic Prandtl number $\mathrm{Pm} = \mathrm{Rm}/\mathrm{Re} < 1$, which is relevant for stellar convective zones, protostellar disks, and laboratory liquid-metal experiments. The two asymptotic possibilities are $\mathrm{Rm}_c \to \mathrm{const}$ as $\mathrm{Re} \to \infty$ (a small-scale dynamo exists at low Pm) or $\mathrm{Rm}_c/\mathrm{Re} = \mathrm{Pm}_c \to \mathrm{const}$ as $\mathrm{Re} \to \infty$ (no small-scale dynamo exists at low Pm). Results obtained in two independent sets of simulations of MHD turbulence using grid and spectral codes are brought together and found to be in quantitative agreement. We find that at currently accessible resolutions, Rm_c grows with Re with no sign of approaching a constant limit. We reach the maximum values of $\mathrm{Rm}_c \sim 500$ for $\mathrm{Re} \sim 3000$. By comparing simulations with Laplacian viscosity, fourth-, sixth-, and eighth-order hyperviscosity, and Smagorinsky large-eddy viscosity, we find that Rm_c is not sensitive to the particular form of the viscous cutoff. This work represents a significant extension of the studies previously published by Schekochihin et al. (2004a) and Haugen et al. (2004a) and the first detailed scan of the numerically accessible part of the stability curve $\mathrm{Rm}_c(\mathrm{Re})$.

Subject headings: magnetic fields — methods: numerical — MHD — turbulence

Online material: color figures

The magnetic Prandtl number Pm, which is the ratio of the kinematic viscosity to the magnetic diffusivity, is a key parameter of MHD turbulence. In fully ionized plasmas, $\mathrm{Pm} \approx 2.6 \times 10^{-5} T^4/n$, where T is the temperature in kelvins and n the ion number density in units of cm^{-3}. In hot rarefied plasmas, such as the warm and hot phases of the interstellar medium or the intracluster medium, $\mathrm{Pm} \gg 1$. In contrast, in the Sun's convective zone, $\mathrm{Pm} \sim 10^{-7}$ to 10^{-4}, in planets, $\mathrm{Pm} \sim 10^{-5}$, and in protostellar disks, while estimates vary, it is also believed that $\mathrm{Pm} \ll 1$ (e.g., Brandenburg & Subramanian 2005). All these astrophysical bodies have disordered fluctuating small-scale magnetic fields and, in some cases, also large-scale "mean" fields. As they also have large Reynolds numbers and large-scale sources of energy, they are expected to be in a turbulent state. It is then natural to ask if their magnetic fields are a product of turbulent dynamo.

To be precise, there are two types of dynamo. The large-scale or *mean field dynamo* generates magnetic fields at scales larger than the energy-containing scale of the turbulence, as is, for example, the case in helical turbulence. *The small-scale dynamo* amplifies magnetic fluctuation energy below the energy-containing scale of the turbulence. The small-scale dynamo is due to random stretching of the magnetic field by turbulent motions and does not depend on the presence of helicity. Mean-field dynamos typically predict field growth on timescales associated with the energy-containing scale (or longer), while the small-scale dynamo amplifies magnetic energy at the turbulent rate of stretching. Thus, the small-scale dynamo is usually a much faster process than the mean-field dynamo, and the large-scale field produced by the latter can be treated as approximately constant on the timescale of the small-scale dynamo. The mean-field dynamo (or, more generally, a large-scale magnetic field of any origin) also gives rise to small-scale magnetic fluctuations because of the turbulent shredding of the mean field: this leads to algebraic-in-time growth of the small-scale magnetic energy—again, a slower generation process than the exponential-in-time small-scale dynamo.

In the systems with $\mathrm{Pm} \gg 1$, the existence of the small-scale dynamo is well established numerically and has a solid theoretical basis (see Schekochihin et al. 2004b for an account of the relevant theoretical and numerical results and for a long list of references). The situation is much less well understood for the case of small Pm. It has been largely assumed that a small-scale dynamo should be operative in this regime as well. For example, the presence of large amounts of small-scale magnetic flux in the solar photosphere (e.g., Title 2000) has been attributed to small-scale dynamo action. This appeared to be confirmed by numerical simulations of the MHD turbulence in the convective zone (Cattaneo 1999; Cattaneo et al. 2003; Nordlund 2003). However, such simulations are usually done at $\mathrm{Pm} \geq 1$ ($\mathrm{Pm} = 5$ in Cattaneo's simulations). Previous attempts to simulate MHD turbulence in various contexts with $\mathrm{Pm} < 1$ found achieving dynamo in this regime much more difficult than for $\mathrm{Pm} \geq 1$ (Nordlund et al. 1992; Brandenburg et al. 1996; Nore et al. 1997; Christensen et al. 1999; Maron et al. 2004). A systematic numerical investigation of the effect of Pm on the efficiency of the small-scale dynamo was carried out by Schekochihin et al. (2004a), who found that the critical magnetic Reynolds number Rm_c required for the small-scale dynamo to work increases sharply at $\mathrm{Pm} < 1$. An independent numerical study by Haugen et al. (2004a) confirmed this result.

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What are the basic physical considerations that should guide us in interpreting this result? First of all, let us stress that all working numerical small-scale dynamos are of the large-Pm kind (the case of Pm = 1 is nonasymptotic, but its properties that emerge in numerical simulations suggest that it belongs to the same class). Two essential features of the large-Pm dynamos are (1) the scale of the velocity field is larger than the scale of the magnetic field, and (2) the velocity field that drives the dynamo is spatially smooth and locally looks like a random linear shear, so the dynamo is due to exponential-in-time separation of Lagrangian trajectories and the consequent exponential stretching of the magnetic field. The basic physical picture of such dynamos (Zeldovich et al. 1984; see discussion in Schekochihin et al. 2004b; see also a review of an alternative but complementary approach by Ott 1998) explicitly requires these two conditions to hold. The map dynamos and the dynamos in deterministic chaotic flows that were extensively studied in the 1980s–1990s (see review by Childress & Gilbert 1995) are all of this kind. The numerical dynamos with $\mathrm{Pm} \geq 1$ (the first due to Meneguzzi et al. 1981) are of this kind as well because they are driven by the spatially smooth viscous-scale turbulent eddies, which have the largest turnover rate.

When $\mathrm{Pm} \ll 1$ with both $\mathrm{Rm} \gg 1$ and $\mathrm{Re} \gg 1$, the characteristic scale l_B of the magnetic field lies in the inertial range. For Kolmogorov turbulence, a simple estimate gives $l_B \sim \mathrm{Rm}^{-3/4} l_0$, where l_0 is the energy-containing scale. As the viscous scale is $l_\nu \sim \mathrm{Re}^{-3/4} l_0$, we have $l_0 \ll l_B \ll l_\nu$. In a rough way, one can think of the turbulent eddies at scales $l > l_B$ as stretching the field at the rate u_l/l and of the eddies at scales $l < l_B$ as diffusing the field with the turbulent diffusivity $u_l l$. In Kolmogorov turbulence, $u_l \sim l^{1/3}$, so both the dominant stretching and the dominant diffusion are due to the eddies at scale $l \sim l_B$. The resulting rates of stretching and of turbulent diffusion are of the same order, so the outcome of their competition cannot be determined on this qualitative level (Kraichnan & Nagarajan 1967). An important conclusion, however, can be drawn. If the bulk of the magnetic energy is at the scale l_B, the existence of the dynamo is entirely decided by the action of the velocities at the scale l_B. Then it cannot matter where in the inertial range l_B lies. But $l_B/l_\nu \sim \mathrm{Pm}^{-3/4}$, so the value of Pm does not matter as long as it is asymptotically small. Therefore, there are two possibilities: either there is a dynamo at low Pm and $\mathrm{Rm}_c \to \mathrm{const}$ as $\mathrm{Re} \to \infty$ or there is not and there exists a finite $\mathrm{Pm}_c = \mathrm{Rm}_c/\mathrm{Re} \to \mathrm{const}$ as $\mathrm{Re} \to \infty$. Strictly speaking, the third possibility is that $\mathrm{Rm}_c \propto \mathrm{Re}^\alpha$, where α is some fractional power, but this can only happen if the intermittency of the velocity field (non–self-similarity of the inertial range) is important for the existence of the dynamo.[9]

The two possibilities identified above are illustrated in Figure 1. Several authors (Vainshtein 1982; Rogachevskii & Kleeorin 1997; Boldyrev & Cattaneo 2004) showed that, given certain reasonable assumptions, the first possibility ($\mathrm{Rm}_c \to \mathrm{const}$) is favored by the Kazantsev (1968) model: the small-scale dynamo in a Gaussian white-in-time velocity field. In particular, Boldyrev & Cattaneo (2004) found that the Kazantsev model gives Rm_c that is roughly 10 times larger in the $\mathrm{Pm} \ll 1$ regime than in the $\mathrm{Pm} \gg 1$ regime (Rogachevskii & Kleeorin 1997 predict $\mathrm{Rm}_c \sim 400$, which is consistent with that). This

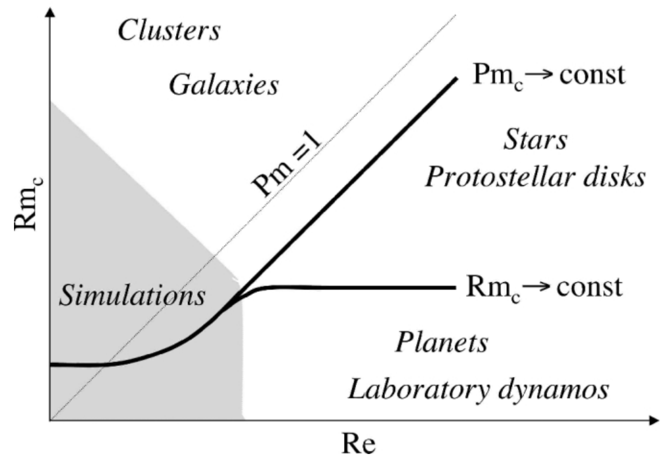

FIG. 1.—Sketch of the two possible shapes of the stability curve Rm_c vs. Re for the small-scale dynamo. [*See the electronic edition of the Journal for a color version of this figure.*]

prompted them to declare the issue settled on the grounds that the failure of the dynamo in numerical experiments at current limited resolutions is compatible with such an increase in Rm_c. However, the $\mathrm{Pm} \ll 1$ dynamo in the Kazantsev model is a quantitative mathematical consequence of the model, and it is not known if and how it is affected by such drastic and certainly unrealistic assumptions as the Gaussian white-noise statistics for the velocity field.[10] The existence of a dynamo in real turbulence is also a quantitative question (see discussion above), so it cannot be decided by a model that is not a quantitative approximation of turbulence.

Thus, the issue cannot be considered settled until definitive numerical evidence is produced. This is an especially hard task because we do not know just how high a magnetic Reynolds number we must achieve in order to clearly see the distinction between $\mathrm{Rm}_c \to \mathrm{const}$ and $\mathrm{Rm}_c/\mathrm{Re} \to \mathrm{const}$. In this Letter, we have collected numerical results from two independent computational efforts: simulations using an incompressible spectral MHD code (see code description in Maron & Goldreich 2001 and Maron et al. 2004) and weakly compressible simulations using a grid-based high-order MHD code (the Pencil Code[11]).

The equations we solved numerically (in a triply periodic cube) are

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{\nabla p}{\rho} + \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi\rho} + \boldsymbol{F}_{\mathrm{visc}} + \boldsymbol{f}, \quad (1)$$

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}, \quad (2)$$

where $\boldsymbol{u}$ is the velocity and $\boldsymbol{B}$ is the magnetic field (the Pencil Code, in fact, solves the evolution equation for the vector potential $\boldsymbol{A}$ and then computes $\boldsymbol{B} = \nabla \times \boldsymbol{A}$). All runs reported below are in the kinematic regime, $|\boldsymbol{B}| \ll |\boldsymbol{u}|$, so the Lorentz force in equation (1) plays no role. Turbulence is driven by a random white-in-time nonhelical body force $\boldsymbol{f}$ concentrated at

[9] The role of coherent structures can be prominent in quasi–two-dimensional dynamos (three-component velocity field depending on two spatial variables), where the inverse cascade characteristic of the two-dimensional turbulence gives rise to persistent large-scale vortices, which drive the dynamo (Smith & Tobias 2004).

[10] Vainshtein & Kichatinov (1986) argue that the equations that arise from the Kazantsev model are valid for nonwhite velocity fields if n-point joint probability density functions of Lagrangian displacements satisfy Fokker-Planck equations with some diffusion tensor. They further assume (on dimensional grounds) that this diffusion tensor scales as the scale-dependent turbulent diffusion $\sim u_l l$. This is, in fact, a closure scheme that we believe to be equivalent to Kazantsev's zero-correlation-time theory.

[11] See http://www.nordita.dk/software/pencil-code.

$k = k_0$, where k_0 is the wavenumber associated with the box size. The (hyper)viscous force is

$$\boldsymbol{F}_{\rm visc} = \frac{1}{\rho}\boldsymbol{\nabla}\cdot\left[2\langle\rho\rangle\,\nu_n(-\nabla^2)^{n-1}\hat{\boldsymbol{S}}\right], \quad (3)$$

where ν_n is the fluid viscosity and

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{u}. \quad (4)$$

In the spectral simulations, the density $\rho = 1$, and the incompressibility constraint $\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$ is enforced exactly via the determination of the pressure p. The grid simulations are isothermal: $p = c_s^2\rho$ with sound speed $c_s = 1$, and the density satisfies

$$\partial_t\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) = 0. \quad (5)$$

We stay in the weakly compressible regime of low Mach numbers $M = \langle u^2\rangle^{1/2}/c_s \sim 10^{-1}$ and $\rho \simeq \langle\rho\rangle = 1$ (angular brackets denote volume averages). Some numerical results on the onset of dynamo action at larger Mach numbers are given in Haugen et al. (2004b).

The dissipation in the induction equation (2) is always Laplacian with magnetic diffusivity η (we choose not to tamper with magnetic dissipation because we are interested in the sensitive question of field growth or decay). With regard to the viscous dissipation, we perform three kinds of simulations:

1. Simulations with Laplacian viscosity: $n = 1$ in equation (3).
2. Simulations with fourth-, sixth-, and eighth-order hyperviscosities: $n = 2$, 3, and 4, respectively, in equation (3).
3. Large-eddy simulations (LES) with the Smagorinsky effective viscosity (e.g., Pope 2000): in equation (3), $n = 1$, and ν_1 is replaced by $\nu_{\rm S} = \left(C_{\rm S}\Delta\right)^2\left(2\hat{\boldsymbol{S}}:\hat{\boldsymbol{S}}\right)^{1/2}$, where Δ is the mesh size and $C_{\rm S} = 0.2$ is an empirical coefficient.

The magnetic Reynolds number is defined by $\mathrm{Rm} = \langle u^2\rangle^{1/2}/k_0\eta$, where k_0 is the box wavenumber (the smallest wavenumber in the problem). For the runs with Laplacian viscosity ($n = 1$), the hydrodynamic Reynolds number is $\mathrm{Re} = \langle u^2\rangle^{1/2}/k_0\nu_1$. For hyperviscous runs and for LES, we define Re by replacing ν_1 with the effective viscosity:

$$\nu_{\rm eff} = \epsilon/\langle 2\hat{\boldsymbol{S}}:\hat{\boldsymbol{S}}\rangle = \epsilon/\langle|\boldsymbol{\nabla}\boldsymbol{u}|^2\rangle \quad (6)$$

(the second expression is for the spectral simulations, where $\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$ exactly). Here $\epsilon = \langle\boldsymbol{f}\cdot\boldsymbol{u}\rangle$ is the total injected power and is equal to the total energy dissipation. As the forcing $\boldsymbol{f}$ is white in time, $\epsilon = $ const; indeed, given $\langle f^i(t,\boldsymbol{x})f^j(t',\boldsymbol{x}')\rangle = \delta(t-t')\epsilon^{ij}(\boldsymbol{x}-\boldsymbol{x}')$, it is easy to show that $\epsilon = \frac{1}{2}\epsilon^{ii}(0)$.

The results of all our simulations are presented in Figure 2, where Rm_c is plotted versus Re. Each value of Rm_c was computed by interpolating between least-squares–fitted growth/decay rates of a growing and a decaying run. Error bars are based on Rm and Re for these pairs of runs. The only exception is the point enclosed in a circle, which corresponds to (Rm, Re) for a run that appeared to be marginal (in this case we could not afford the resolution necessary to achieve a decaying case). The run times in all cases were long enough for the least-squares–fitted growth rates to stop changing appreciably (typically this required about 20 box-crossing times, but cases close to marginal needed longer running times).

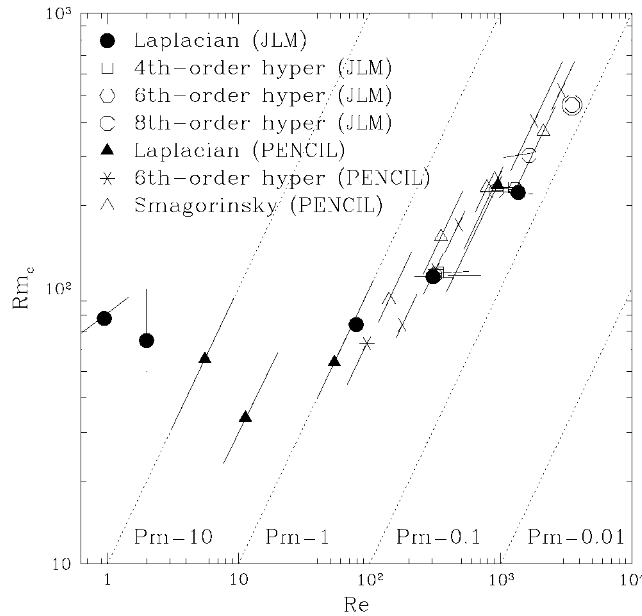

FIG. 2.—Dependence of Rm_c on Re. "JLM" refers to simulations done with the incompressible spectral code written by J. L. Maron: runs with Laplacian viscosity and fourth-, sixth-, and eighth-order hyperviscosity (resolutions 64^3–256^3). In this set of simulations, hyperviscous runs were done at the same values of η as the Laplacian runs, so the difference between the results for these runs is nearly imperceptible. "PENCIL" refers to weakly compressible simulations done with the Pencil Code: runs with Laplacian viscosity, sixth-order hyperviscosity, and Smagorinsky large-eddy viscosity (resolutions 64^3–512^3). [*See the electronic edition of the Journal for a color version of this figure.*]

We see that there is good agreement between the results for runs with different forms of viscous dissipation; this confirms the natural assumption that the field-generation properties of the turbulence at low Pm are not sensitive to the way the velocity spectrum is cut off. It is also encouraging that results from two very different codes are in quantitative agreement.

Our previous studies (Schekochihin et al. 2004a; Haugen et al. 2004a) had the maximum value of $\mathrm{Rm}_c \sim 200$. The results reported here raise it to ∼500, with the corresponding values of Pm_c around 0.15. While a roughly 10-fold increase with respect to Rm_c for the $\mathrm{Pm} = 1$ dynamo has now been achieved, there is thus far no sign of Rm_c reaching an asymptotically constant value. This said, the current resolutions are clearly still insufficient to make a definitive judgement, although we are now very close to values of Rm_c predicted by the theories based on the Kazantsev model (Rogachevskii & Kleeorin 1997; Boldyrev & Cattaneo 2004)—whether or not the model yields quantitatively correct predictions should become clear in the near future.

The numerical results reported above concerned the dependence $\mathrm{Rm}_c(\mathrm{Re})$ for the turbulent small-scale dynamo, i.e., the ability of turbulent velocity fluctuations to amplify magnetic energy at scales smaller than the energy-containing scale of the turbulence. The $\mathrm{Rm}_c(\mathrm{Re})$ dependence is also an interesting issue for other kinds of dynamo.

If the velocity field is non–mirror-symmetric, it can often drive the mean-field dynamo (MFD), which means the growth of the magnetic field at scales larger than the energy-containing scale of the turbulence (Krause & Rädler 1980). This large-scale field generated by the MFD, just like a mean field imposed externally, can induce small-scale magnetic fluctuations as it is shredded by the turbulence, so the total field has both a mean (large-scale) and a fluctuating component. In many cases, the breaking of the mirror symmetry leads to a nonzero value of the net helicity $\langle \boldsymbol{u}\cdot(\boldsymbol{\nabla}\times\boldsymbol{u})\rangle \neq 0$ (the average is over all scales that are smaller than the energy-containing scale of the turbulence, *not* over the entire volume of the system). The mean-field generation is then referred to as the α-effect. The stability curve $\mathrm{Rm}_c(\mathrm{Re})$ for the α-effect is different than for the small-scale dynamo: it is essentially a condition for at least one unstable large-scale mode to fit into the system. In a numerical study done with the same code as the grid simulations reported above but with fully helical random forcing, Brandenburg (2001) found much lower values of Rm_c than for the small-scale dynamo and very little dependence of Rm_c on Pm for $\mathrm{Pm} \geq 0.1$.

A nonzero net helicity is not a necessary condition for the MFD (e.g., Gilbert et al. 1988). In fact, it has been suggested recently by Rogachevskii & Kleeorin (2003) that the MFD can be driven simply by the presence of a constant mean velocity shear (shear-current or δ-effect)—a very generic possibility of obvious relevance to systems with mean flows. Mean flows are present in many astrophysical cases and in all current laboratory dynamo experiments (Gailitis et al. 2004; Müller et al. 2004; Bourgoin et al. 2002; Lathrop et al. 2001; Forest et al. 2002). A mean flow can be a dynamo in its own right: an MFD (field growth at scales above the flow scale) and, if the flow has chaotic trajectories in three dimensions, also a small-scale dynamo (field growth at scales $\sim\mathrm{Rm}^{-1/2}$ times the scale of the flow; see Childress & Gilbert 1995—as noted above, small-scale dynamos in deterministic chaotic flows are equivalent to the large-Pm case). When Re is large, the energy of the turbulent velocity fluctuations is comparable to the energy of the mean flow. The critical Rm required for field growth will have some dependence on Re, which reflects the effect of the turbulence on the structure of the mean flow and/or on the effective value of the magnetic diffusivity (the β-effect; see Krause & Rädler 1980). This dependence was the subject of two recent numerical studies: of the dynamo in a turbulence with a constant Taylor-Green forcing by Ponty et al. (2005) and of the Madison dynamo experiment (propeller driving in a spherical domain) by Bayliss & Forest (2004). The Re dependence of Rm_c that emerges from such simulations is distinct from that for a pure small-scale dynamo. Indeed, Y. Ponty et al. (2005, private communication) have shown that, in the limit of large Re, the value of Rm_c in their simulations tends to a constant that coincides with Rm_c calculated for the mean flow alone, i.e., for the velocity field with fluctuations removed by time averaging. In contrast, the subject of the present Letter has been the possibility of a small-scale dynamo driven solely by turbulent fluctuations, in the absence of a mean flow. The importance of this possibility or lack thereof is that such a dynamo, if it exists, occurs at the turbulent stretching rate associated with the resistive scale. This is much faster (by a factor of $\sim\mathrm{Rm}^{1/2}$; see discussion above) than the growth rate of any MFD or of a small-scale dynamo associated with the mean flow, i.e., than the stretching rate at the energy-containing scale or at the scale of the mean flow.

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REFERENCES

Bayliss, R. A., & Forest, C. B. 2004, Phys. Rev. Lett., submitted
Boldyrev, S., & Cattaneo, F. 2004, Phys. Rev. Lett., 92, 144501
Bourgoin, M., et al. 2002, Phys. Fluids, 14, 3046
Brandenburg, A. 2001, ApJ, 550, 824
Brandenburg, A., Jennings, R. L., Nordlund, Å., Rieutord, M., Stein, R. F., & Tuominen, I. 1996, J. Fluid Mech., 306, 325
Brandenburg, A., & Subramanian, K. 2005, Phys. Rep., in press (astro-ph/0405052)
Cattaneo, F. 1999, ApJ, 515, L39
Cattaneo, F., Emonet, T., & Weiss, N. 2003, ApJ, 588, 1183
Childress, S., & Gilbert, A. D. 1995, Stretch, Twist, Fold: The Fast Dynamo (Berlin: Springer)
Christensen, U., Olson, P., & Glatzmaier, G. A. 1999, Geophys. J. Int., 138, 393
Forest, C. B., Bayliss, R. A., Kendrick, R. D., Nornberg, M. D., O’Connell, R., & Spence, E. J. 2002, Magnetohydrodynamics, 38, 107
Gailitis, A., Lielausis, O., Platacis, E., Gerbeth, G., & Stefani, F. 2004, Phys. Plasmas, 11, 2838
Gilbert, A. D., Frisch, U., & Pouquet, A. 1988, Geophys. Astrophys. Fluid Dyn., 42, 151
Haugen, N. E. L., Brandenburg, A., & Dobler, W. 2004a, Phys. Rev. E, 70, 016308
Haugen, N. E. L., Brandenburg, A., & Mee, A. J. 2004b, MNRAS, 353, 947
Kazantsev, A. P. 1968, Soviet Phys.–JETP, 26, 1031
Kraichnan, R. H., & Nagarajan, S. 1967, Phys. Fluids, 10, 859
Krause, F., & Rädler, K.-H. 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory (Oxford: Pergamon)
Lathrop, D. P., Shew, W. L., & Sisan, D. R. 2001, Plasma Phys. Controlled Fusion, 43(12A), A151
Maron J., & Goldreich, P. 2001, ApJ, 554, 1175
Maron, J. L., Cowley, S. C., & McWilliams, J. C. 2004, ApJ, 603, 569
Meneguzzi, M., Frisch, U., & Pouquet, A. 1981, Phys. Rev. Lett., 47, 1060
Müller, U., Stieglitz, R., & Horanyi, S. 2004, J. Fluid Mech., 498, 31
Nordlund, Å. 2003, in Dynamic Sun, ed. B. N. Dwivedi (Cambridge: Cambridge Univ. Press), 148
Nordlund, Å., Brandenburg, A., Jennings, R. L., Rieutord, M., Ruokolainen, J., Stein, R. F., & Tuominen, I. 1992, ApJ, 392, 647
Nore, C., Brachet, M. E., Politano, H., & Pouquet, A. 1997, Phys. Plasmas, 4, 1
Ott, E. 1998, Phys. Plasmas, 5, 1636
Ponty, Y., Mininni, P. D., Montgomery, D. C., Pinton, J.-F., Politano, H., & Pouquet, A. 2005, Phys. Rev. Lett., in press (physics/0410046)
Pope, S. B. 2000, Turbulent Flows (Cambridge: Cambridge Univ. Press)
Rogachevskii, I., & Kleeorin, N. 1997, Phys. Rev. E, 56, 417
———. 2003, Phys. Rev. E, 68, 036301
Schekochihin, A. A., Cowley, S. C., Maron, J. L., & McWilliams, J. C. 2004a, Phys. Rev. Lett., 92, 054502
Schekochihin, A. A., Cowley, S. C., Taylor, S. F., Maron, J. L., & McWilliams, J. C. 2004b, ApJ, 612, 276
Smith, S. G. L., & Tobias, S. M. 2004, J. Fluid Mech., 498, 1
Title, A. 2000, Philos. Trans. R. Soc. London A, 358, 657
Vainshtein, S. I. 1982, Magnetohydrodynamics, 28, 123
Vainshtein, S. I., & Kichatinov, L. L. 1986, J. Fluid Mech., 168, 73
Zeldovich, Ya. B., Ruzmaikin, A. A., Molchanov, S. A., & Sokoloff, D. D. 1984, J. Fluid Mech., 144, 1