

MEAN FIELD DYNAMOS WITH ALGEBRAIC AND DYNAMIC α -QUENCHINGS

A. TWORKOWSKI

Math. Res. Centre, Queen Mary and Westfield College, London, UK¹

E. COVAS, R. TAVAKOL

Astronomy Unit, Queen Mary and Westfield College, London, UK²

A. BRANDENBURG

Dept. of Mathem., University of Newcastle upon Tyne, UK³

S u m m a r y : *Calculations for mean field dynamo models (in both full spheres and spherical shells), with both algebraic and dynamic α -quenches, show qualitative as well as quantitative differences and similarities in the dynamical behaviour of these models. We summarise and enhance recent results with extra examples.*

Overall, the effect of using a dynamic α appears to be complicated and is affected by the region of parameter space examined.

K e y w o r d s : Mean field dynamo, α quenching

1. INTRODUCTION

In most studies of axisymmetric mean field dynamo models (see for example, *Tavakol et al., 1995*), the nonlinearity is introduced through an algebraic form of α -quenching. Such models have produced a number of modes of behaviour in spherical and spherical shell dynamo models, including periodic, quasiperiodic and chaotic solutions. In addition it has recently been shown that spherical shell models with this type of quenching are capable of producing various forms of intermittent type behaviour (*Tworkowski et al., 1998*), which could be of relevance in understanding some of the intermediate time scale variability observed in the output of the Sun and stars.

Since such algebraic forms of α -quenching act instantaneously, it is possible that this may have a bearing on the occurrence of the more complicated modes of behaviour, such as chaos and intermittency, observed in these models. Here using recent results as well as new ones, we make a brief comparison between models with dynamic and algebraic α -quenching and examine whether features, such as chaotic and intermittent-type behaviour, survive as α -quenching is made dynamic.

2. THE MODEL

The standard mean field dynamo equation (cf. *Krause and Rädler, 1980*) is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B} - \eta_t \nabla \times \mathbf{B}), \quad (1)$$

¹ Address: Mile End Road, London E1 4NS (A.S.Tworkowski@qmw.ac.uk)

² Address: M.E. Rd., London E1 4NS (E.O.Covas@qmw.ac.uk; reza@maths.qmw.ac.uk)

³ Address: NE1 7RU, Newcastle upon Tyne (Axel.Brandenburg@ncl.ac.uk)

where as usual, the magnitudes of the α and ω effects are given by the dynamo parameters C_α and C_ω .

In the algebraic case we use a purely hydrodynamical α with $\alpha = \alpha_h$, for which we take the usual functional form of α -quenching

$$\alpha_h = \frac{\alpha_0 \cos \theta}{1 + \mathbf{B}^2}. \quad (2)$$

For the dynamical case, following to *Zeldovich et al. (1983)* and *Kleeorin and Ruzmaikin (1982)* (see also *Kleeorin et al., 1995*), α can be divided into a hydrodynamic and a magnetic part,

$$\alpha = \alpha_h + \alpha_m, \quad (3)$$

where α_h is given by Eq. (2) and the magnetic part satisfies an explicitly time dependent diffusion type equation with a nonlinear forcing in the form

$$\frac{\partial \alpha_m}{\partial t} = \frac{1}{\mu_0 \rho} \left(\mathbf{J} \cdot \mathbf{B} - \frac{\alpha \mathbf{B}^2}{\eta_t} \right) + \nu_\alpha \nabla^2 \alpha_m, \quad (4)$$

(see *Covas et al., 1997a,b; 1998a*). This equation differs from that used by *Kleeorin et al. (1995)* in that we take the damping term to be that of the form $\nu_\alpha \nabla^2 \alpha_m$ instead of $-\alpha_m/T$, where T is some damping time. Our approach is motivated by stability considerations.

We solved the above equations using spherical polar coordinates. We shall consider both axisymmetric spherical and spherical shell models, where the outer boundary in both cases is denoted by R and in the spherical shell models, the fractional radius of the inner boundary of the shell is denoted by r_0 . We discuss the behaviour of the dynamos by calculating the total magnetic energy, E , which is split into two parts, $E = E^{(A)} + E^{(S)}$, where $E^{(A)}$ and $E^{(S)}$ are respectively the energies of the field whose toroidal component is antisymmetric and symmetric about the equator. The overall parity P given by $P = (E^{(S)} - E^{(A)})/E$, with $P = -1, +1$ denoting the antisymmetric (dipole-like) and symmetric (quadrupole-like) pure parity solutions respectively.

For \mathbf{B} we assume vacuum boundary conditions and for α_m we use $\alpha_m = 0$ on the inner and outer boundary.

3. RESULTS

For the the algebraic α -quenching model, we solved equations (1) and (2), whilst for the dynamical case we solved equations (1) and (4). Clearly our conclusions are subject to the finiteness of the resolution of parameter space that we chose. We also point out that the parameters C_α and C_ω do not play identical roles in these models and therefore the comparison of the behaviours in the two cases is not straightforward.

3.1 Spherical dynamo models

Taking $C_\omega = -10^4$, which is a typical value that has been used in other dynamo studies, for example *Tavakol et al. (1995)* (and references therein), we found that as C_α became large, the algebraic model tended to an antisymmetric (dipolar) parity whilst the dynamic case tended towards a symmetric (quadrupolar) one. Overall, no chaos was observed in either case although the dynamic case did show evidence of more complicated behaviour in its transitions from one parity form to another. We also wished to study the allowed forms of behaviour in the supercritical regimes, so we chose C_ω values which were effectively the highest values numerically allowed by our code, and these turned out to be -10^4 and -10^5 in the dynamic and algebraic cases respectively. For the algebraic model, we found chaotic behaviour at large C_α value when $C_\omega = -10^5$ was used.

3.2 Spherical shell dynamo models

Using $C_\omega = -10^4$, we found that for the dynamical α -quenching case, the behaviour observed for large C_α values depended on the shell thickness. For thick shells ($r_0 = 0.2$) we observed symmetric parity solutions whilst for medium ($r_0 = 0.5$) and thin ($r_0 = 0.7$) shells we found antisymmetric asymptotic parity solutions. In particular, for the medium shell we observed multiple attractors at $C_\alpha \approx 15$, that is, we observed the occurrence of different dynamical solutions possessing different mean energies at the same value of C_α but having different initial parities. Also, chaotic behaviour was observed in the thin shell at C_α values around 35. However, the algebraic α -quenching model did not exhibit any chaotic behaviour nor the presence of multiple attractors at this value of C_ω . The asymptotic behaviour obtained for the algebraic case was a symmetric parity solution, an oscillatory mixed parity solution which varied between symmetric and antisymmetric values but was periodic and a non-oscillatory mixed parity solution for the thick, medium and thin shells respectively. However, when C_ω was set to -10^5 , thus in the supercritical regime for this case, the asymptotic behaviour was chaotic for all shell thicknesses.

Our results appear to indicate that dynamic α -quenching smoothes out chaotic behaviour possibly because of the back reaction of α_m via diffusion.

4. INTERMITTENCY

Spherical shell dynamo models with algebraic α -quenching have been shown to be capable of producing various forms of intermittent-type of behaviour (*Tworkowski et al., 1998*), in which the dynamical modes of behaviour for which the statistics taken over different time intervals are different. One example of such behaviour is given in Figure 1 (see *Tworkowski et al., 1998* for other examples). As can be seen, for times around 20 to 60 units, the parity is almost antisymmetric but that there are also time intervals which are interrupted by excursions of the parity to values well away from -1 . We have also searched for occurrences of intermittency in the dynamic α -quenching model using the algebraic α -quenching given by equation

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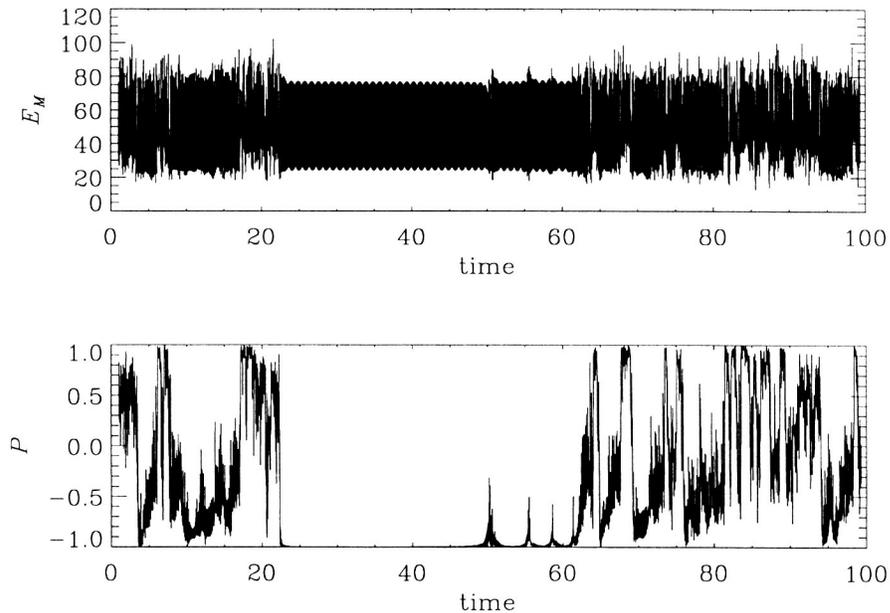


Fig. 1. Total energy and parity for the α -profile given by (2). $C_\alpha = 1.94$, $r_o = 0.4$, $F = 0$

(2) with no success, at least to the resolution of our parameter regime. However, we did find examples of intermittency using another form of algebraic quenching, namely, that due to *Kitchatinov (1987)*. This form was derived in the context of Λ -quenching by rapid rotation and is essentially an interpolation formula having the correct asymptotic behaviour. An example of the intermittent behaviour found using this form is given in Figure 2. Although these two examples appear similar in that they both have intervals where the parity is nearly antisymmetric, this does not mean that these two are examples of the same type of intermittency. To resolve this issue, one needs to examine the appropriate theoretical framework for the detailed properties of the corresponding model such as scaling laws (see, for example, *Ashwin et al., 1998; Covas et al., 1998b*).

Overall, then, it appears that dynamic α -quenching appears to suppress the existence of intermittency, at least to the resolution of our parameter search.

5. CONCLUSION

Our studies indicate that in the full sphere models, the main similarities are the presence of similar modes of parity behaviour and the absence of intermittency, whilst the differences are in the details of the transitions between the different modes of behaviour and the presence of chaotic behaviour in the algebraic α -quenching case.

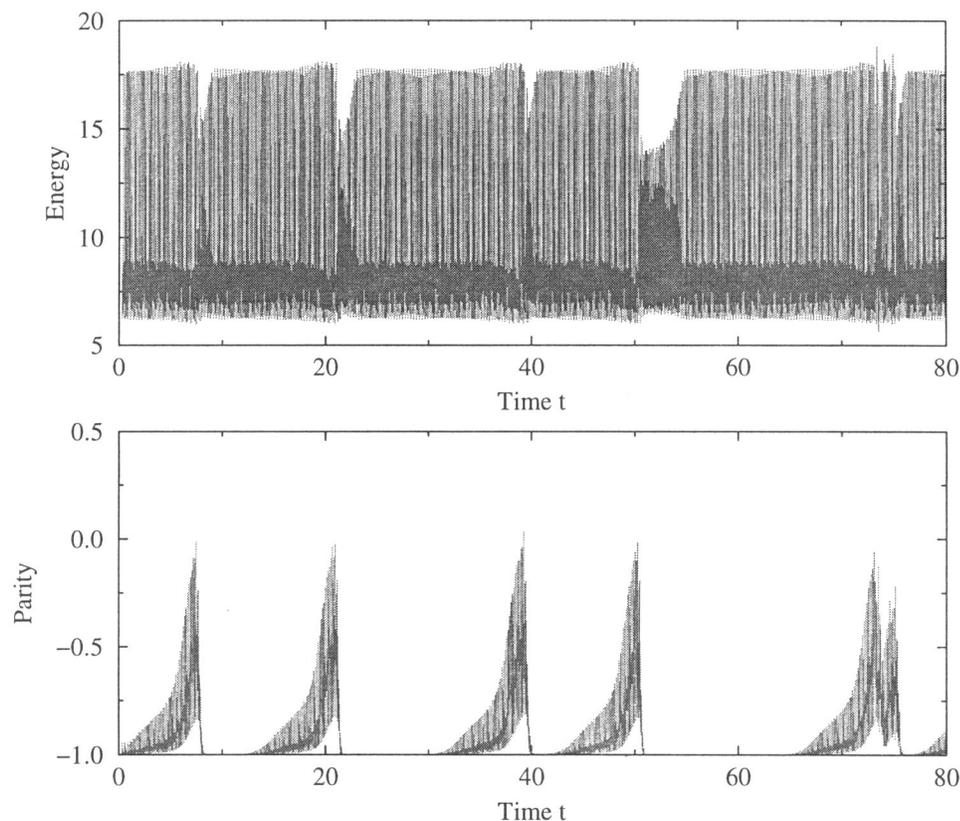


Fig. 2. Result for $r_0 = 0.5$ with the dynamic α model and a particular form of α_n (see text). The parameters are $C_\alpha = 9.34$, $C_w = -10^4$ and $F = 0$.

For the spherical shell models, we observe differences which depend on the region of the parameter space considered. The dynamic α models appear to produce more varied modes of behaviour for $C_w = -10^4$. However, taking the numerical upper bound of C_w in each case appears to indicate that the introduction of dynamic α -quenching drastically reduces the likelihood of the occurrence of chaotic behaviour and intermittency, which was observed in models with algebraic α -quenching. Our dynamic α -quenching models also show multi-attractor regimes with the possibility of the final state sensitivity (fragility) with respect to small changes in the initial parity.

With regards to the extra complexity introduced by a time dependent α -quenching, our present results show that the behaviour seen in the solutions is complicated and with the outcome depending on the region of parameter space considered, rather than a simplistic decrease or increase in complexity.

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References

- Ashwin, P., Covas, E. and Tavakol, R., 1998: Transverse instability for non-normal parameters. *Nonlinearity*, in press
- Covas, E., Tworkowski, A., Brandenburg, A. and Tavakol, R., 1997a: Dynamos with different formulations of a dynamic α -effect. *Astron. and Astrophys.* **317**, 610
- Covas, E., Tworkowski, A., Tavakol, R. and Brandenburg, A., 1997b: Robustness of truncated $\alpha\omega$ dynamos with a dynamic α . *Solar Physics* **172**, 3
- Covas, E., Tavakol, R., Tworkowski, A. and Brandenburg, A., 1998a: Axisymmetric mean field dynamos with dynamic and algebraic α -quenchedings. *Astron. and Astrophys.* **329**, 350
- Covas, E., Tavakol, R., Ashwin, P., Tworkowski, A. and Brooke, J. M., 1998b: In-out intermittency in PDE and ODE models of axisymmetric mean-field dynamos. Submitted to *Phys. Rev. Lett.*
- Kitchatinov, L.L., 1987: A mechanism for differential rotation based on angular momentum transport by compressible convection. *Geophys. Astrophys. Fluid Dyn.* **38**, 273
- Kleeorin, N. I. and Ruzmaikin, A.A., 1982: Dynamics of the average turbulent helicity in a magnetic field. *Magnetohydrodynamica* **N2**, 17
- Kleeorin, N. I., Rogachevskii, I. and Ruzmaikin, A., 1995: Magnitude of the dynamo-generated magnetic field in solar-type convective zones. *Astron. and Astrophys.* **297**, 159
- Krause, F. and Rädler, K.-H., 1980: *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Pergamon Press, Oxford
- Tavakol, R.K., Tworkowski, A. S., Brandenburg, A., Moss, D. and Tuominen, I., 1995: Structural stability of axisymmetric dynamo models. *Astron. and Astrophys.* **296**, 269
- Tworkowski, A., Tavakol, R., Brandenburg, A., Brooke, J. M., Moss, D. and Tuominen, I., 1998: Intermittent behaviour in axisymmetric mean-field dynamo models in spherical shells. *Mon. Not. Roy. Astr. Soc.*, in press
- Zeldovich, Ya.B., Ruzmaikin, A.A. and Sokoloff, D.D., 1983: *Magnetic Fields in Astrophysics*, Gordon and Breach, New York