Axisymmetric mean field dynamos with dynamic and algebraic α-quenchings

Eurico Covas[*1], **Reza Tavakol**[**1], **Andrew Tworkowski**[***1]**, and Axel Brandenburg**[†2]

[1] Astronomy Unit, School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, UK
[2] Department of Mathematics, University of Newcastle upon Tyne NE1 7RU, UK

Abstract. We study axisymmetric mean field spherical and spherical shell dynamo models, with both dynamic and algebraic α-quenchings. Our results show that there are qualitative as well as quantitative differences and similarities between these models. Regarding similarities, both groups of models exhibit symmetric, antisymmetric and mixed modes of behaviour. As regards differences, the important feature in the full sphere models is the occurrence of chaotic behaviour in the algebraic α-quenching models. For the spherical shell models with dynamic α the main features include the possibility of multi-attractor regimes with final state sensitivity with respect to small changes in the magnitude of α and the initial parity.

We find the effect of introducing a dynamic α is likely to be complicated and depend on the region of the parameter space considered, rather than a uniform change towards simplicity or complexity.

Key words: MHD – Sun: magnetic fields – stars: magnetic fields

1. Introduction

Axisymmetric mean field dynamo models have been extensively studied as a possible way of understanding some aspects of solar and stellar dynamos (see for example, Brandenburg et al. 1989a,b; Moss et al. 1991; Tavakol et al. 1995). In most such studies the nonlinearity is introduced through an algebraic (as opposed to a dynamic) form of α-quenching. These studies have produced a number of novel modes of behaviour in spherical and spherical shell dynamo models, which include periodic, quasiperiodic and chaotic solutions, with the latter mode of behaviour previously observed only in spherical shell

Send offprint requests to: Eurico Covas
[*]   (E.O.Covas@qmw.ac.uk)
[**]   (reza@maths.qmw.ac.uk)
[***] (A.S.Tworkowski@qmw.ac.uk)
[†]   (Axel.Brandenburg@ncl.ac.uk)

(Tavakol et al. 1995), torus (Brooke & Moss 1994), and accretion disc dynamos (Torkelsson & Brandenburg 1994, 1995). In addition it has recently been shown that spherical shell models are capable of producing various forms of intermittent type behaviour (Tworkowski et al. 1997). Similar results have also been obtained in torus models by Brooke & Moss (1995), Brooke (1997), Brooke et al. (1997). Such intermittent behaviour could be of relevance in understanding some of the intermediate time scale variability observed in the output of the Sun (Eddy 1975; Ribes & Nesme-Ribes 1993) and stars (Baliunas & Vaughan 1985).

An important point regarding the α-quenching terms, usually employed in such models, is that they are approximate in nature. This is a direct consequence of the fact that small scale turbulent effects cannot be prescribed precisely and parametrisations are always required in order to estimate second order correlations. As a result the question arises as to whether the modes of behaviour observed in such studies are in some sense a consequence of the particular forms of α-quenching employed. The question of robustness of such dynamo models with respect to reasonable changes in the functional form of α-quenching was considered in Tavakol et al. (1995), where it was shown that the dynamics was qualitatively robust with respect to such changes. The forms of α considered there were, however, algebraic. One potential problem with the algebraic forms of α-quenching is that they act instantaneously and this may have a bearing on the occurrence of the more complicated modes of behaviour, such as chaos and intermittency, observed in such models.

A possible way of remedying this latter shortcoming is to relax the instantaneous feature by employing a dynamic (explicitly time dependent) α effect. The underlying physics of this type of effect has been given by Kleeorin & Ruzmaikin (1982) and Zeldovich et al. (1983) (see also Kleeorin et al. (1995)), where the existence of non-instantaneous quenching was shown to be a consequence of the fact that the magnetic helicity is conserved in the absence of diffusion or boundary effects.

Truncated one-dimensional models with dynamic α effect have recently been investigated by Schmalz & Stix (1991), Covas et al. (1997a,b) and Covas & Tavakol (1997). There is also a

two dimensional truncated analogue of such results studied by Schlichenmaier & Stix (1995).

Our aim here is to make a detailed study of the dynamics of axisymmetric mean field dynamo models with a dynamic α-quenching in order to find out how they compare with models involving algebraic α-quenching and in particular whether the novel features discovered in such settings, such as chaotic and intermittent-type behaviour survive as α-quenching is made dynamic. To make this study comparative, it was necessary to extend the previous results of Brandenburg et al. (1989a,b) and Tavakol et al. (1995) which employed an algebraic α-quenching.

The structure of the paper is as follows. In Sect. 2 we briefly introduce the equations for the axisymmetric mean field dynamo models with both dynamic and algebraic α-quenchings. Sect. 3 contains our results for each case, with Sects. 3.1 and 3.2 summarising our results for spherical and spherical shell dynamo models respectively. In Sect. 4 we briefly discuss the occurrence of multiple attractor regimes for these models and in Sect. 5 we report on the presence of intermittent types of behaviour in the shell models. Finally we present in Sect. 6 our conclusions.

2. The model

The standard mean field dynamo equation (cf. Krause & Rädler 1980) is of the form

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B} - \eta_t \nabla \times \boldsymbol{B}), \qquad (1)$$

where $\boldsymbol{B}$ and $\boldsymbol{u}$ are the mean magnetic field and the mean velocity respectively and η_t is the turbulent magnetic diffusivity. For simplicity, and to facilitate comparison with previous work, α and η_t are assumed to be scalars. The magnitudes of the α and ω effects are given by the dynamo parameters C_α and C_ω. Even though our main aim here is to study the effects of making α dynamical, nevertheless, for the sake of comparison, it is necessary to consider analogous models with algebraic α-quenching. We therefore consider both cases.

In the algebraic case we take the functional form of algebraic α-quenching to be the usual one, namely

$$\alpha_a = \frac{\alpha_0 \cos \theta}{1 + \boldsymbol{B}^2}, \qquad (2)$$

an expression that has been adopted in numerous studies since Jepps (1975). In the dynamical case, α can, according to Zeldovich et al. (1983) and Kleeorin & Ruzmaikin (1982) (see also Kleeorin et al. 1995), be divided into a hydrodynamic and a magnetic part, thus

$$\alpha = \alpha_h + \alpha_m, \qquad (3)$$

where $\alpha_h = \alpha_a$ is given by Eq. (2) and the magnetic part satisfies an explicitly time dependent diffusion type equation with a nonlinear forcing in the form

$$\frac{\partial \alpha_m}{\partial t} = \frac{1}{\mu_0 \rho} (\boldsymbol{J} \cdot \boldsymbol{B} - \frac{\alpha \boldsymbol{B}^2}{\eta_t}) + \nu_\alpha \nabla^2 \alpha_m, \qquad (4)$$

where ν_α is a physical constant parameter taken to be half times the value of η_t, ρ the density of the medium, μ_0 the induction constant and in the $\alpha \boldsymbol{B}^2$ term the full α from Eq. (3) is used. This equation is slightly different from that given by Kleeorin & Ruzmaikin (1982), where the damping term has the form $-\alpha_m/\tau_\alpha$ (where τ_α is a typical relaxation time) instead of $+\nu_\alpha \nabla^2 \alpha_m$. For a detailed comparison between different variations of this equation see Covas et al. (1997a,b).

The models we shall consider here are in the forms of sphere and spherical shells, with the outer boundary in both cases being denoted by R and in the case of spherical shell models, the fractional radius of the inner boundary of the shell being denoted by r_0. As is customary, we shall in the following, discuss the behaviour of the dynamos considered by monitoring the total magnetic energy, $E = \frac{1}{2\mu_0} \int \boldsymbol{B}^2 dV$, in $r \leq R$. We split E into two parts, $E = E^{(A)} + E^{(S)}$, where $E^{(A)}$ and $E^{(S)}$ are respectively the energies of those parts of the field whose toroidal field is antisymmetric and symmetric about the equator. The overall parity P given by $P = (E^{(S)} - E^{(A)})/E$. In this way, $P = -1$ denotes a antisymmetric (dipole-like) pure parity solution and $P = +1$ a symmetric (quadrupole-like) pure parity solution.

The physically interesting case is when $\nu_\alpha < \eta_t$ corresponding to an adjustment time of the magnetic α effect that is longer than the magnetic diffusion time (cf. Zeldovich et al. 1983). For $\boldsymbol{B}$ we assume vacuum boundary conditions and for α_m we use

$$\alpha_m = 0 \quad \text{on} \quad r = R. \qquad (5)$$

For the numerical studies reported in the following sections, we used a modified version of the axisymmetric dynamo code of Brandenburg et al. (1989a). We considered full sphere and spherical shell models and used a grid size of 41 $\times$ 81 mesh points and employed single precision arithmetic, ie 4 byte floating point real numbers. To test the robustness of the code we verified that no qualitative changes were produced by employing a finer grid, different temporal step length (we used a step length of $10^{-4} R^2/\eta_t$ in the results presented in this paper) and/or by increasing the machine precision. Our results are presented in the following sections.
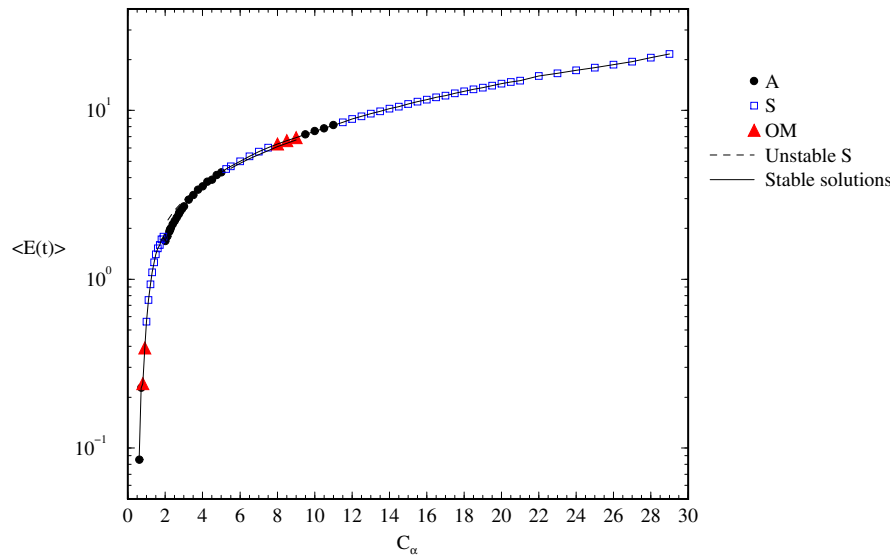
3. Results

We studied the two cases, with algebraic and dynamic α-quenchings separately. For the dynamical case we solved equations (1) and (4) and for the case with the algebraic α-quenching we solved equations (1) and (2). The latter extends previous studies by Brandenburg et al. (1989a,b) and Tavakol et al. (1995).

We should emphasise that all our conclusions presented here (such as transitions between different modes of behaviour) are subject to the necessarily finite resolution of the parameter space chosen. In other words, it is possible that we have missed out certain types of solutions either within our relatively coarse parameter mesh, or outside.

Furthermore, an important point regarding the comparison of these models is that the parameters C_α and C_ω do not play
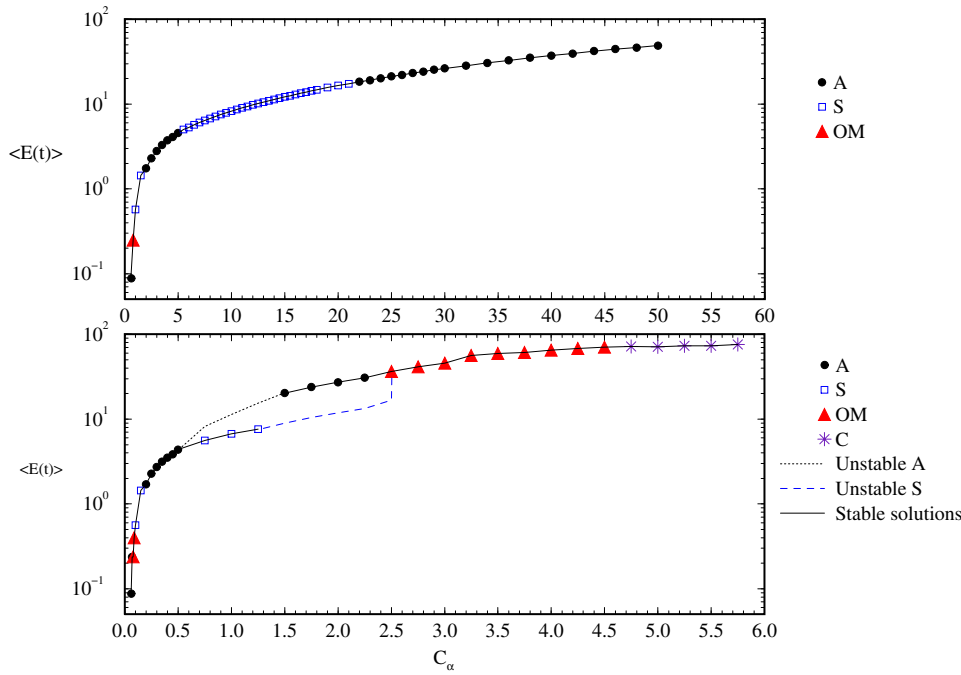
Fig. 1. Energy diagram for the full sphere ($r_0 = 0$) with algebraic α. The value of C_ω used was -10^4 in the top panel and -10^5 in the bottom one.

Fig. 2. Energy diagram for the full sphere ($r_0 = 0$) with dynamic α. The value of C_ω used was -10^4.

identical roles in these models and as a result the comparison of these models, using these parameters, is problematic. Since our main aim is to study the types of possible behaviour allowable in the supercritical regimes, we therefore took the following strategy. Firstly, we chose C_ω values which are effectively the highest values numerically allowed by our code, which turned out to be -10^4 in the dynamic case and -10^5 in the algebraic case. However, we also studied the algebraic case at $C_\omega = -10^4$, which is the same value used also in the dynamic case.

The notation used in the figures throughout the paper is as follows: "A" represents fixed antisymmetric solutions about the equator, "S" fixed symmetric, "OM" oscillatory mixed, "M" non-oscillatory mixed and "C" chaotic solutions. We also intro-

duce unstable branches in the pictures when these are distinct enough.

3.1. Spherical dynamo models

The results of our computations for the spherical case with algebraic α-quenching are summarised in Fig. 1, in which the top and the bottom panels are for C_ω values -10^4 and -10^5 respectively. An important new feature of these results is the presence of chaotic behaviour in the case where $C_\omega = -10^5$. As far as we are aware, this is the first time chaos has been observed in a full sphere (as opposed to spherical shell) model of this type. The sequence of transitions in these two cases are different at high C_α values. The corresponding results with dynamic α are
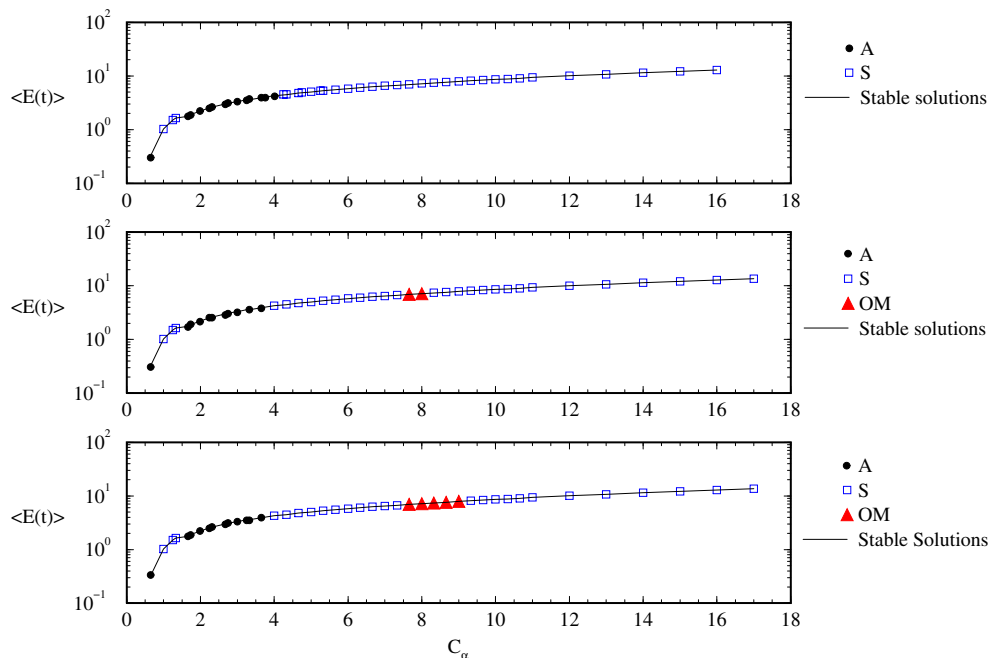
Fig. 3. Energy diagrams for the thick shell dynamo ($r_0 = 0.2$) with dynamic α and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^4.
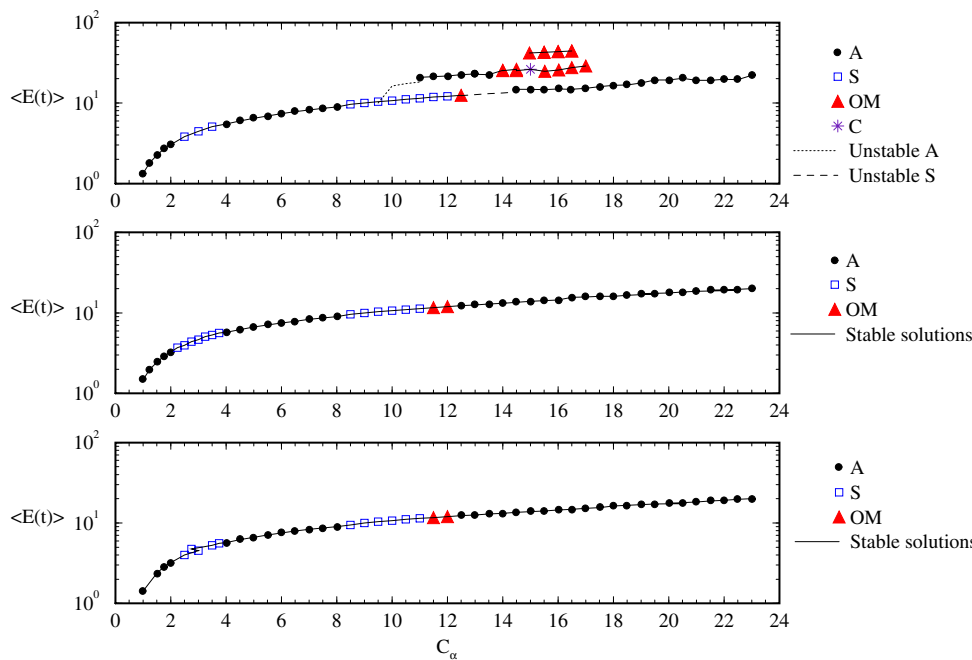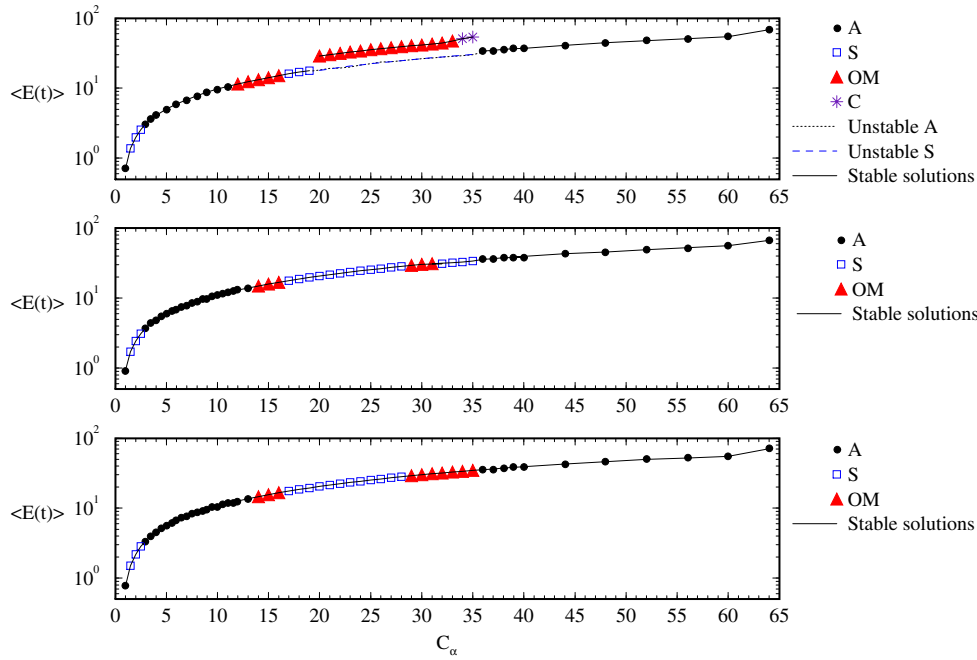

Fig. 4. Energy diagrams for a medium shell dynamo ($r_0 = 0.5$) with dynamic α and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^4.

presented in Fig. 2. The sequence of transitions in this case is antisymmetric $\rightarrow$ symmetric $\rightarrow$ mixed $\rightarrow$ antisymmetric $\rightarrow$ symmetric, which is more complicated than the antisymmetric $\rightarrow$ symmetric $\rightarrow$ antisymmetric sequence for the algebraic case.

Comparing Figs. 1 and 2 we observe both similarities and differences between them. Qualitative similarities include the occurrence of symmetric, antisymmetric and mixed modes and the absence of intermittent types of behaviour. The differences lie in the details of transitions and more importantly, the way the introduction of dynamic α appears to remove the possibility of the occurrence of chaotic behaviour in such models (at least when comparing with the algebraic case for $C_\omega = -10^5$), whilst

at the same time increasing the likelihood of the occurrence of the oscillatory mixed modes at $C_\omega = -10^4$. We should note that the reason why such chaotic behaviour was not discovered in Tavakol et al. (1995) was that these authors only considered a value of -10^4 for C_ω. Also important is the stronger increase in the time averaged energy of the solutions for the algebraic case as C_α increases. As we shall see latter on this feature is quite frequent in the algebraic models. To summarise, the effect of making α dynamical is therefore substantial, with both qualitative and quantitative changes.

Fig. 5. Energy diagrams for a thin shell dynamo ($r_0 = 0.7$) with dynamic α and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^4.
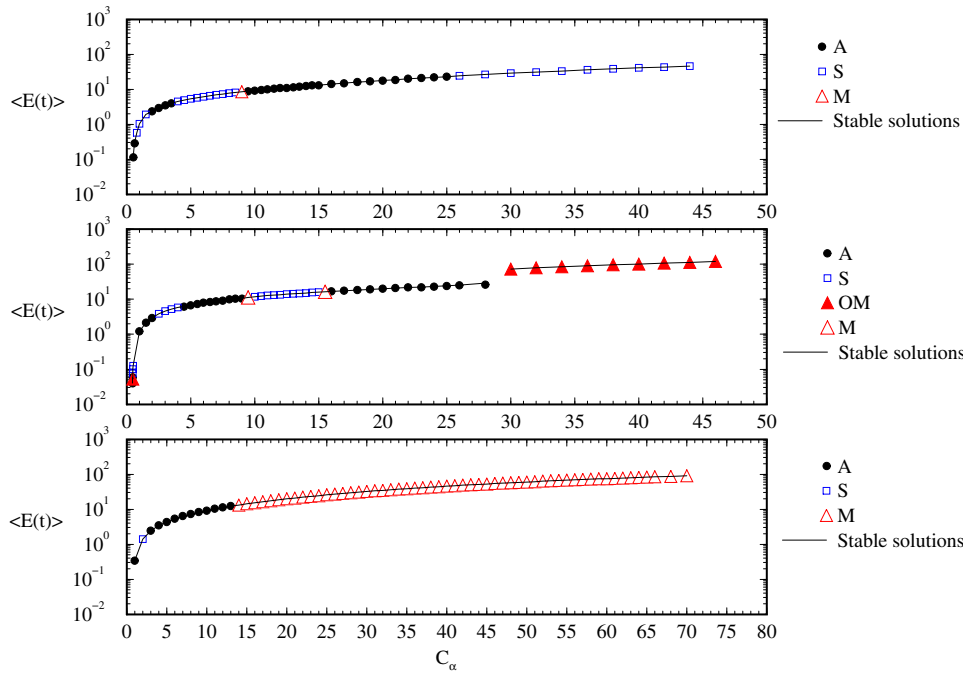
Fig. 6. Energy diagram for algebraic α. The panels from top to bottom correspond to r_0 values 0.2, 0.5 and 0.7 respectively. The value of C_ω used was -10^4 and $F = 0$.

3.2. Spherical shell dynamo models

In the case of the shell dynamos, we assumed in both the dynamic and algebraic cases that the $\alpha^2\omega$ dynamos were situated in a conducting fluid with a vacuum outer boundary condition. The inner boundary condition was assumed to be a superposition of perfectly conducting and penetrative boundary conditions in the forms (Tavakol et al. 1995):

$$(1 - F)a + F\left(\frac{\partial a}{\partial r} - \frac{a}{\delta}\right) = 0, \qquad (6)$$

and

$$(1 - F)\left(\frac{1}{r}\frac{\partial(rb)}{\partial r} - \alpha\frac{1}{r}\frac{\partial(ra)}{\partial r}\right) + F\left(\frac{\partial b}{\partial r} - \frac{b}{\delta}\right) = 0. \qquad (7)$$

In this way the boundary conditions may be changed by varying F, with $F = 0$ corresponding to the perfect conductor case and $F = 1$ to the case where the magnetic field goes to zero at some distance δ below the inner boundary (Brandenburg et al. 1992). In our numerical computations we considered shells of different thickness, quantified by the parameter r_0 (we took $r_0 = 0.2, 0.5, 0.7$ in units of the outer shell radius) and in each
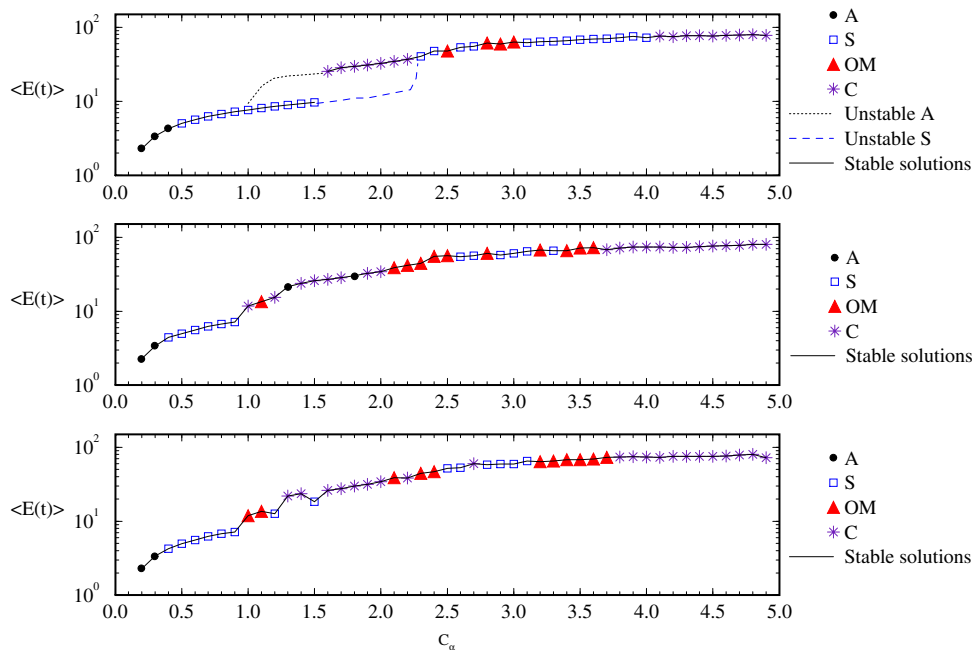
Fig. 7. Energy diagrams for a thin shell with algebraic α dynamo ($r_0 = 0.2$) and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^5.
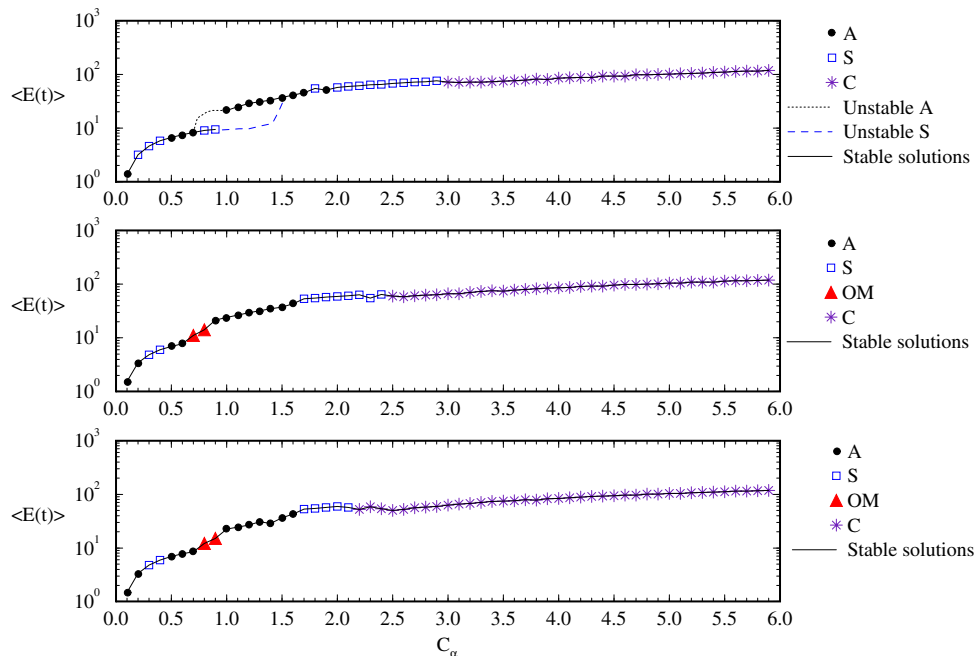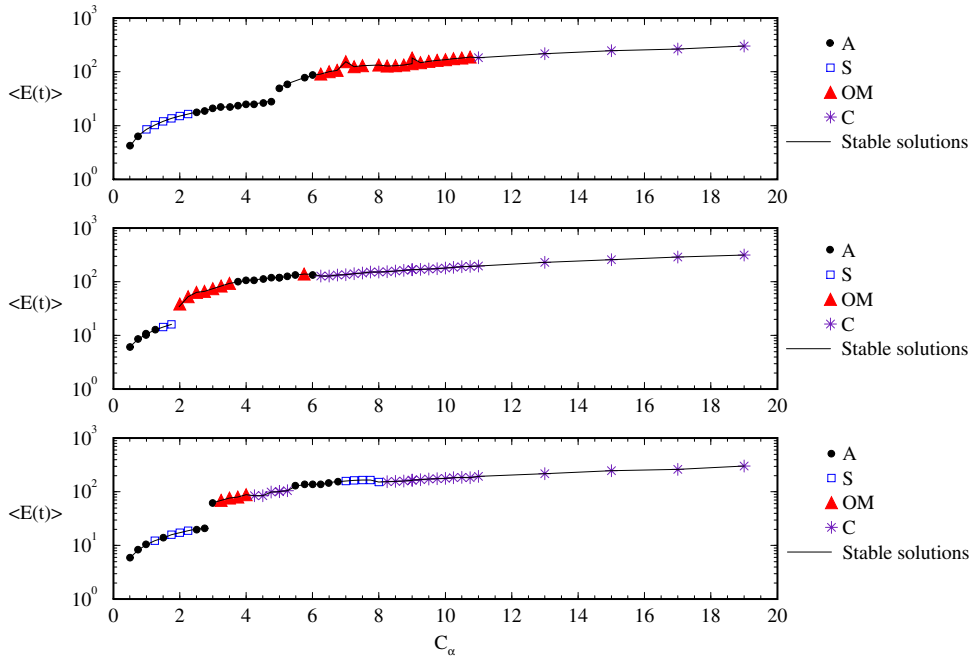

Fig. 8. Energy diagrams for a medium shell with algebraic α dynamo ($r_0 = 0.5$) and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^5.

case took different values of F ($F = 0, 0.5, 1$). In this way we were able to study the robustness of spherical shell dynamo models with respect to changes in both thickness and boundary conditions.

In the dynamical case, the results of our computations in shells of different thickness, and in each case with the three values of F, are depicted in Figs. 3-5 respectively. As can be seen, the behaviour of the thicker shell dynamo model ($r_0 = 0.2$) consists of an initial sequence of antisymmetric $\rightarrow$ symmetric $\rightarrow$ antisymmetric $\rightarrow$ symmetric sequence, with the $F > 0$ models having their asymptotic symmetric states being slightly inter-

rupted by a mixed regime. An important feature of these thicker shell models is that the symmetric modes appear to be more likely in the C_α measure sense. The behaviour of the thinner shells ($r_0 = 0.5$ and $r_0 = 0.7$) seem more varied dynamically. Important new features here are the occurrence of multiple attractors at intermediate values of C_α, for models with $F = 0$, as well as intermittent and chaotic modes of behaviour. Also these models seem to have an antisymmetric asymptotic state, with the antisymmetric modes dominating in the C_α measure sense, in contrast to the symmetric states in the $r_0 = 0.2$ case. The comparison of these figures also shows that, overall, thicker shells

Fig. 9. Energy diagrams for a thin shell with algebraic α dynamo ($r_0 = 0.7$) and with $F = 0, 0.5$ and 1. The value of C_ω used was -10^5.

are more robust to changes in F, at least for $F > 0$, which seems to indicate that $F = 0$ models (perfect conductor) are rather special in this case, in the F measure sense.

In the above we have compared models with dynamic and algebraic α for different values of C_ω, because we felt C_ω should be as large as numerically permissible. However, we also considered models with algebraic α-quenching and the same value of C_ω as in the dynamic case. Those results are shown in Fig. 6 and, as can be seen, there is no evidence for complicated or chaotic behaviour. We also note that most of the mixed mode solutions possess non-oscillatory mixed parity and are therefore in this sense simpler. We add that we confined ourselves to the $F = 0$ case in this picture, since this produced the most interesting dynamical behaviour in the dynamic α case, as we shall see below.

The results of our computations for the corresponding models with algebraic α-quenching, for $C_\omega = -10^5$, are shown in Figs. 7-9 respectively. As can be seen, there are important differences to the models with dynamic α-quenching. The crucial difference being the very much enhanced likelihood of complicated (chaotic) modes of behaviour in these models, and this increases with the decreasing thickness of the shells. Furthermore, models with $F = 0$ are again somewhat special, because of the occurrence of jumps in energy levels, which indicates the possibility of hysteresis. Also, the thick shell case ($r_0 = 0.2$) is more diverse in its transition patterns with the uniformity of the dynamical modes of behaviour growing as the shell becomes thinner.

To summarise then, comparing the shell models with the algebraic and dynamic α-quenchings, we observe that there are again important qualitative and quantitative differences. The introduction of a dynamic α can have different effects depending upon the region of the parameter space considered. In particular,

at $C_\omega = -10^4$ the dynamic α models show more variety. However, when comparing models with numerically allowed upper limits of C_ω the case of algebraic α-quenching can exhibit more complicated modes of behaviour.

4. Multiple attractor regimes

An interesting feature of the models with dynamic α is the presence of parameter intervals over which the system possesses multiple attractors, ie the occurrence of different dynamical solutions possessing different energies at the same value of the control parameter C_α, but depending upon the different initial parities, P_i, chosen. Multiple attractors have previously been found for axisymmetric two-dimensional dynamo models both with α-quenching (Brandenburg et al. 1989b) and with feedback from the large scale motions (Muhli et al. 1995). Tworkowski et al. (1997) have also found evidence for the existence of multiple attractors, with algebraic α-quenching, but that the types of attractors are different and the likelihood seems to be less[1]. Here we found this mode of behaviour to occur in the intermediate shell models ($r_0 = 0.5$) in the neighbourhood of $C_\alpha \approx 15$ (see Fig. 4). To demonstrate this phenomenon more clearly, we have plotted in Fig. 10 two dimensional phase space projections of different solutions, with the different attractors being distinguished by their phase space locations. This distinction is further amplified by comparing the corresponding plots of the parity, also shown in this figure, which shows three different types of behaviour. Despite the appearance of behaviours depicted in this figure, the attractors are nonetheless periodic.

Another issue of interest is how sensitive the behaviour of such models is with respect to changes in the control parame-

---

[1]   We do not distinguish within chaotic signals those that may be interpreted as slightly intermittent.
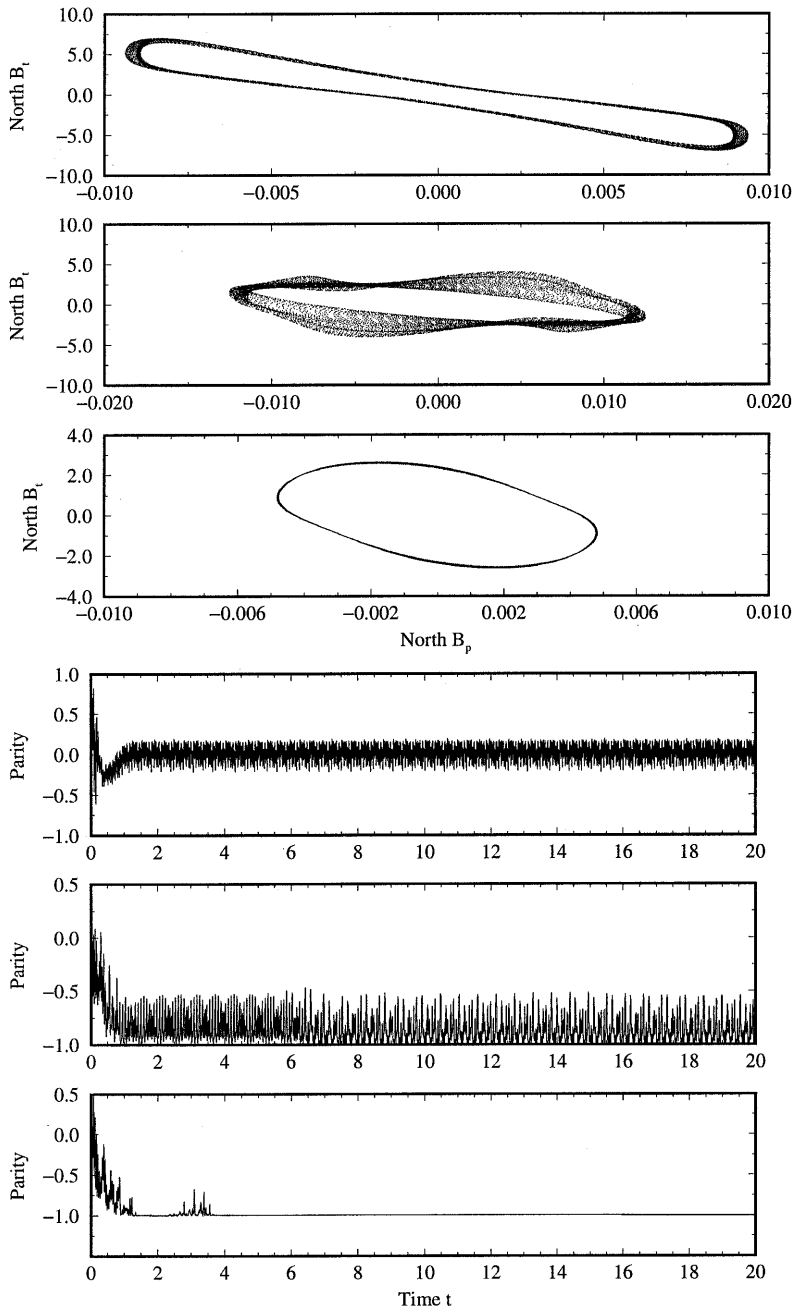
Fig. 10. Phase space projections and time series of the three (6 due to the North-South symmetry) types of attractors found for the dynamic α model with $r_0 = 0.5$ and $C_\alpha = 14.6$ and $C_\omega = -10^4$. For clarity, the projections are drawn as dots, rather as continuous lines. B_p and B_t are respectively the poloidal and toroidal parts of the magnetic field at $r_0 = 0.6875$ and $\pm 83°$ latitude.

ters of the system, namely C_α and the initial parity P_i (which can be treated as an initial condition). To study this, we looked at the effects of changing these parameters on the asymptotic properties of the dynamo models. The results of our calculations are shown in Fig. 11. As can be seen, there is a region of $C_\alpha - P_i$ space in which small changes can drastically qualitatively change the solution. Such final state sensitivity (fragility) was also found in (Tavakol et al. 1995) for analogous models with algebraic α-quenching and also in a more precise way in finite mode truncated models considered by Covas and Tavakol (1997), where it was shown that the basins of attraction were in fact fractal.

5. Intermittency

As was mentioned in Sect. 1, spherical shell dynamo models with algebraic α-quenching have been shown to be capable of producing various forms of intermittent-type of behaviour (Tworkowski et al. 1997), ie dynamical modes of behaviour for which the statistics taken over different time intervals are different. Now given that the presence of a dynamic α may drastically reduce the likelihood of complicated behaviour, the question arises as to whether one can still have intermittent-type behaviour for models with dynamic α. We found that the presence of dynamic α-quenching greatly reduced this possibility and despite the relatively extensive numerical studies of these
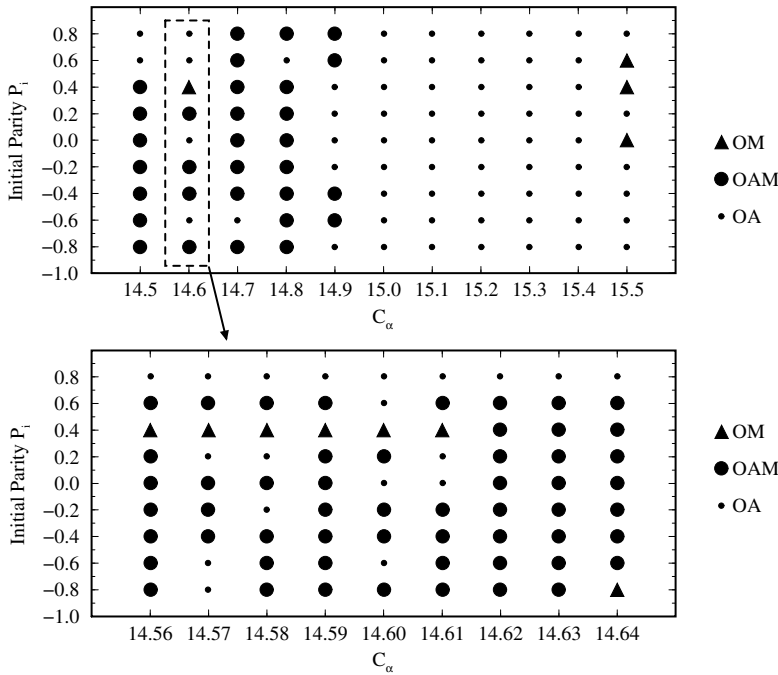
Fig. 11. Fragility with respect to initial parity P_i and initial C_α. The symbols A, OAM and OM mean respectively: a periodic (in energy) pure antisymmetric solution; a periodic (both in energy and parity) almost pure antisymmetric solution and a periodic (both in energy and parity) mixed parity solution.

models, reported above, the only examples of this type of behaviour that we were able to find were in a shell model with $r_0 = 0.5$ in which the algebraic part of the α-quenching was taken to be the form given in Kitchatinov (1987). Fig. 12 shows an example of such a behaviour, whereby intervals of parity being very nearly antisymmetric are punctuated by migrations towards zero parity and then a sudden drop. This mode of behaviour is an example of the type of intermittency referred to as *icicle* intermittency (Brooke and Moss 1995, Brooke 1997, Brooke et al. 1997). To distinguish between this intermittent mode of behaviour with chaotic behaviour, we show in Fig. 13 an example of the latter kind.

The point here is that overall, the employment of a dynamic α seems to decrease the likelihood of intermittent-type behaviour relative to the cases where algebraic α-quenching is used.

6. Conclusion

We have made a study of axisymmetric mean field spherical and spherical shell dynamo models, with both dynamic and algebraic α-quenchings.

In the full sphere models, the main similarities are the occurrence of symmetric, antisymmetric, mixed modes and absence of intermittent modes of behaviour. The differences are in the details of the transitions between the different modes of behaviour and, more significantly, the presence of chaotic behaviour in the algebraic α-quenching case. As far as we are aware, this is the first time chaotic behaviour has been observed in full sphere models of this type.

For the spherical shell models, we again observe important qualitative and quantitative differences. In particular, the effect of introducing dynamic α-quenching depends crucially on the

region of the parameter space considered. For $C_\omega = -10^4$, the dynamic α models seem to produce more varied modes of behaviour. On the other hand, considering the numerical upper bound of C_ω in each case shows that the introduction of dynamic α-quenching drastically reduces the likelihood of the occurrence of chaotic behaviour, which was observed in models with algebraic α-quenching. These models also show multi-attractor regimes with the possibility of the final state sensitivity (fragility) with respect to small changes in C_α and the initial parity, as well as intermittent modes of behaviour. We also observe that, in the highly nonlinear regimes (with the extreme C_ω values), the symmetric modes are preferred in the full sphere models and thick shells while the reverse appears to be the case for intermediate and thinner shells. This is in contrast to kinematic theory, where the most preferred mode is antisymmetric for the full sphere and thick shell cases, and symmetric for thinner shells (Roberts 1972).

It might have been expected that since making the α effect dynamical adds more complexity to the system, that it should have uniformly enhanced the complexity of its possible modes of behaviour. Indeed previous experiments with a different form of time dependent α suggest that the system could then exhibit more complicated temporal behaviour. For example, Yoshimura (1978) investigated systems with a time-delay built into the α effect. Our present results show that the picture is likely to be complicated with the outcome depending on the region of parameter space considered, rather than a simplistic decrease or increase in complexity. To establish the true measure of this change requires an extensive study of the behaviour of these models in the C_α-C_ω parameter space, to which we shall return in a future publication.

Finally, in view of the fact that the derivation of an α with dynamic quenching is more general (because it allows for explicit
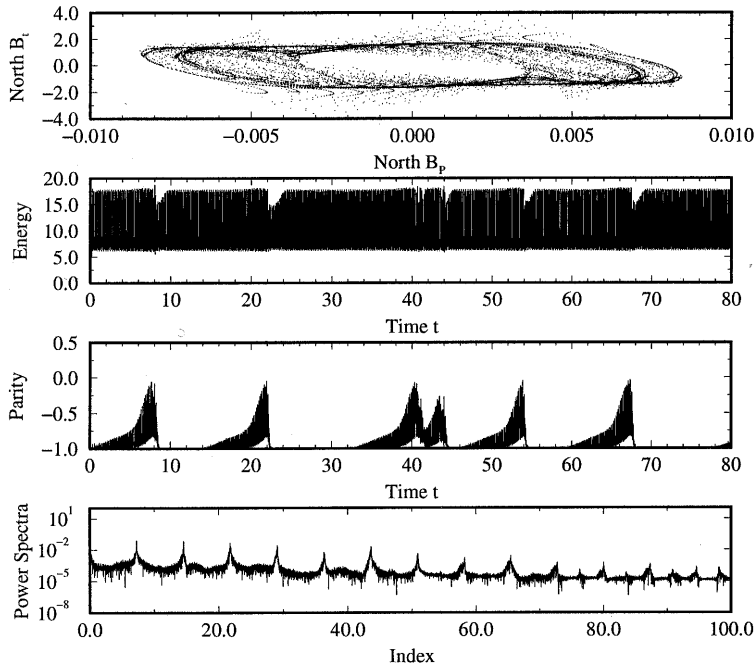
Fig. 12. Result for $r_0 = 0.5$ with the dynamic α model and a particular form of α_h (see text). The parameters are $C_\alpha = 9.35$, $C_\omega = -10^4$ and $F = 0$.
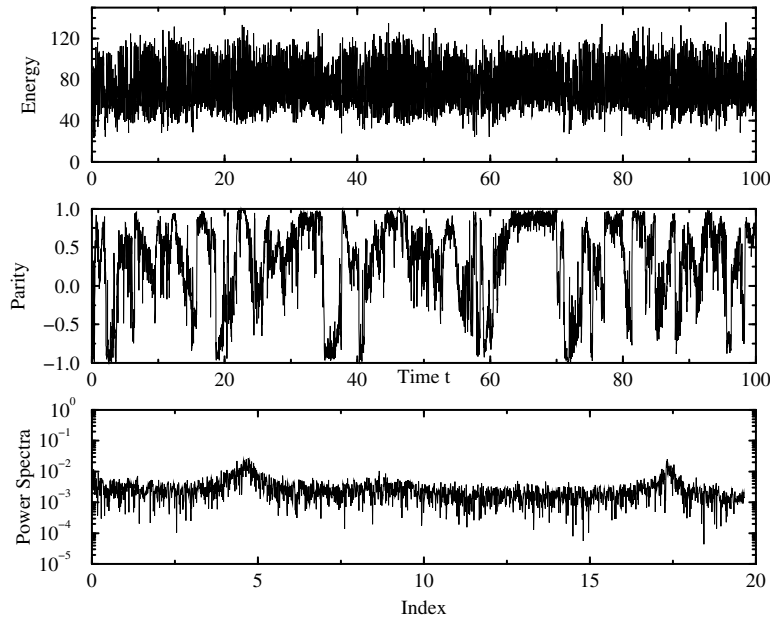
Fig. 13. Result for a full sphere model with an algebraic α-quenching. The parameters are $C_\alpha = 5.0$, $C_\omega = -10^5$ and $F = 0$.

time-dependence), the results thus obtained are more realistic than those obtained using the algebraic form.

References

Baliunas, S. L., Vaughan, A. H., 1985, Ann. Rev. Astron. & Astrophys., 23, 379

Brandenburg, A., Krause, F., Meinel, R., Moss, D., Tuominen, I., 1989a, A&A, 213, 411

Brandenburg, A., Moss, D., Tuominen, I., 1989b, Geophys. Astrophys. Fluid Dyn., 40, 129

Brandenburg, A., Moss, D., Tuominen, I., 1992, A&A, 265, 328

Brooke, J. M., Moss, D., 1995, A&A, 303, 307

Brooke, J. M., 1997, Europhysics Letters, 37, 171

Brooke, J. M., Pelt, J., Tavakol, R., Tworkowski, A., 1997, submitted to A&A

Covas, E., Tworkowski, A., Brandenburg, A., Tavakol, R., 1997, A&A, 317, 610

Covas, E., Tworkowski, A., Tavakol, R., Brandenburg, A., 1997, Solar Physics, in press

Covas, E. & Tavakol, R., 1997, Physical Review E, in press

Eddy J.A., 1976, Science, 192, 1189

Feudel, F., Jansen, W., Kurths, J., 1993, Int. J. Bifurc. Chaos, 3, 131

Gilman, P.A., 1983, ApJS, 53, 243

Jepps, S. A., 1975, J. Fluid Mech., 67, 625

Kitchatinov, L.L., 1984, Geophys. Astrophys. Fluid Dyn., 38, 273

Kleeorin, N. I., Ruzmaikin, A.A., 1982, Magnetohydrodynamica, N2, 17

Kleeorin, N. I., Rogachevskii, I., Ruzmaikin, A., 1995, A&A, 297, 159

Krause, F., Rädler, K.-H., 1980, *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Pergamon, Oxford

Muhli, P., Brandenburg, A., Moss, D., Tuominen, I., 1995, A&A, 296, 700

Ribes J.C., Nesme-Ribes E., 1993, A&A, 276, 549

Roberts, P. H., 1972, Phil. Trans. Roy. Soc., A272, 663

Schlichenmaier, R., Stix, M., 1995, A&A, 302, 264

Schmalz, S., Stix, M., 1991, A&A, 245, 654

Tavakol, R.K., Tworkowski, A. S., Brandenburg, A., Moss, D., Tuominen, I., 1995, A&A, 296, 269

Tobias, S.M., Weiss, N.O., Kirk, V., 1995, MNRAS, 273, 1150

Torkelsson, U., Brandenburg, A., 1994, A&A 292, 341

Torkelsson, U., Brandenburg, A., 1995, Chaos, Solitons and Fractals, 5, 1975

Tworkowski, A., Tavakol, R., Brandenburg, A., Moss, D., Tuominen, I., 1997, to be published.

Weiss, N. O., Cattaneo, F., Jones, C. A., 1984, Geophys. Astrophys. Fluid Dyn., 30, 305

Zeldovich, Ya.B., Ruzmaikin, A.A., Sokoloff, D.D., 1983, *Magnetic Fields in Astrophysics*, Gordon and Breach, New York

Yoshimura, H., Ap. J., 1978, 226, 706