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CURRENT SHEET FORMATION IN THE INTERSTELLAR MEDIUM

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ABSTRACT

There is phenomenological evidence that magnetic reconnection operates in the interstellar medium, and magnetic reconnection is also necessary for the operation of a galactic dynamo. The extremely long ohmic diffusion times of magnetic fields in typical interstellar structures suggest that reconnection occurs in two stages, with thin current layers that have relatively short resistive decay times forming by magnetohydrodynamical processes first, followed by reconnection of the fields in the layers. We propose that ambipolar drift can lead to the formation of these thin sheets in weakly ionized interstellar gas and can delineate the parameter regime in which this occurs by means of a numerical model: we find that the magnetic field cannot be too large and the medium cannot be too diffusive. Both limits are imposed by the requirement that the field be wound up about 1 time by the eddy.
Subject headings: diffusion — ISM: magnetic fields — MHD

1. INTRODUCTION

A host of interstellar magnetohydrodynamic (MHD) problems appear to demand rapid reconnection of magnetic field lines, but theories of reconnection predict that the process is actually quite slow, by virtue of the high conductivity of interstellar gas and the large size of interstellar structures. The magnetic Reynolds number R_M, which is the ratio of the ohmic diffusion time to the advection time, can be estimated for interstellar gas to be approximately $R_M \approx 3 \times 10^{10} L_{pc} v_{km} T^{3/2}$, where L_{pc} is the magnetic scale length in parsecs, v_{km} is the characteristic velocity of the medium in km s^{-1}, and T is the electron temperature in degrees kelvin. In order to convey some flavor for the issues, we briefly discuss three interstellar reconnection problems.

One concerns operation of a galactic dynamo. Galactic magnetic fields display both large-scale order and small-scale randomness, with the ordered component about 30%–100% as strong as the random component (Heiles 1996). The relative sizes of the ordered and random components bear on theories of the origin and maintenance of galactic magnetic fields. A variety of theoretical arguments and numerical experiments suggest that magnetic fields generated by dynamos in weakly resistive media are chaotic, with the ratio of random to mean field proportional to a fractional power of R_M (e.g., Cattaneo et al. 1995; Gilbert, Soward, & Childress 1996). This follows from the basic result that amplifying a magnetic field by motions within a fixed volume of infinitely conducting fluid also lengthens the field lines, with some tangling expected. Therefore, the fact that galactic magnetic fields are relatively well ordered implies either that the effective resistivity of interstellar gas exceeds the plasma value by many orders of magnitude or that, as suggested by Kulsrud & Anderson (1992), dynamos do not operate in galaxies at the present epoch. Dynamo models that employ enhanced resistivity predict successfully some of the large-scale features of galactic magnetic fields (e.g., Beck et al. 1996) and their correlation with other galaxy properties, such as rotation curve. Resistivity is assumed to be provided by turbulence, which distorts the field by advection until it develops structure on a scale at which resistive effects can play a role. Both the large- and the small-scale fields then decay rapidly through ohmic processes or, more likely, through magnetic reconnection, which is a hybrid of resistive and dynamical processes. Whatever reduction in scale length is required to achieve reconnection is accomplished at the expense of amplifying the turbulent component of the field by stretching, requiring a large ratio of rms to mean field unless the reconnection rate is essentially independent of plasma resistivity.

Magnetic reconnection may also be important in the star formation problem. Calculations of the structure and collapse of magnetized, self-gravitating disks show extreme curvature of the magnetic field lines across the midplane of the disk (Galli & Shu 1993, Fig. 4; Li & Shu 1997, Fig. 3), in the manner discussed by Mestel (1966). Reconnection of the field lines across the equator would reduce the magnetic flux through the cloud and would reduce its magnetic support since material in the reconnection region, isolated from the galactic cosmic-ray flux, would quickly recombine and be unable to retain its magnetic field.

Finally, the filaments seen in nonthermal radiation near the Galactic center appear to originate from an interaction between the poloidally oriented galactic magnetic field and the molecular clouds that carry a nearly orthogonal field (Morris 1996; Morris et al. 1997). In this interpretation, the filaments are bundles of magnetic field lines illuminated by radiation from electrons accelerated to relativistic energies in this interaction. Serabyn & Morris (1994) suggested that reconnection of sheared magnetic field lines at the cloud boundaries accelerates the particles.

In all three of these cases, we face a situation in which reconnection must compete with dynamical processes: stirring by turbulence, gravitational collapse, or collisions between clouds and flux tubes. There are two ways in which this could occur, either the reconnection rate is virtually independent of R_M or the magnetic scale length becomes so small that R_M is locally of order 1. In molecular gas with typical velocities of 1 km s^{-1} and temperatures of 10° K, for example, this requires $L_{\rm pc} \approx 10^{-12}$. How could such small magnetic length scales form in a medium in which the magnetic field is large enough to be quite stiff?

Recently, we proposed that ion-neutral drift provides a mechanism for the formation of sharp structures in the weakly ionized component of the interstellar medium (Brandenburg & Zweibel 1994, 1995, hereafter BZ94 and BZ95, respectively). The drift arises because magnetic forces act on the charged component of the medium only. On timescales long compared with the ion-neutral collision time, the drift speed is such that the Lorentz force balances the frictional force exerted by collisions with neutrals. Ion-neutral drift was discovered, and named ambipolar diffusion, by Mestel & Spitzer (1956), who proposed that it is the primary mechanism for removing magnetic flux from weakly ionized interstellar clouds. This idea is still favored today. Ambipolar drift also plays an important role in damping small-amplitude hydromagnetic waves in the interstellar medium and in the structure of interstellar shocks, both processes releasing significant amounts of heat (McKee et al. 1993; Draine & McKee 1993).

Although ion-neutral drift is always dissipative, it is not always diffusive in the classical sense of the term. As the ions drift in response to the Lorentz force, dragging the field lines with them, the magnetic profile can actually steepen. This has been recognized in recent studies in addition to our own (Mac Low et al. 1995; Suzuki & Sakai 1996). It can occur in magnetic fields once the uniform and random components are of comparable strength.

The formation of these steep magnetic profiles, or current sheets, extends the magnetic power spectrum close to the resistive scale, and reconnection of the magnetic field within the current sheets appears unavoidable. We therefore suggested (BZ94; BZ95) that reconnection occurs readily in weakly ionized fluids, *even without winding up the field to the point that the random field dominates the mean field*. The purpose of this paper is to pursue that suggestion.

In § 2 of this paper, we describe a parameter study of current sheet formation in two-dimensional eddies, varying the ambipolar diffusivity and magnetic field strength while taking Lorentz forces into account. We show that the initial magnetic energy density must be less than the initial kinetic energy density and that the initial ambipolar drift time must be longer than the dynamical time in order for the field to wind up enough to form singularities. We show that in situations where current sheets form, the structure of the current is very intermittent, being almost zero outside the sheets. The generalization to three dimensions is that the magnetic field becomes nearly force-free (Brandenburg et al. 1995). This reduces the efficiency of dynamo action. This situation is reminiscent of the solar corona, where current sheets form as a result of footpoint motion of fields anchored in the convection zone. In the corona too, the field is almost force-free, because of the low plasma beta. The problem of coronal heating is still controversial, but the suggestion of E. N. Parker (as reviewed in his book; Parker 1993) that rapid energy conversion is possible because of the formation of tangential discontinuities and a multitude of nanoflares appears favorable (Galsgaard & Nordlund 1996). We appeal here to the possibility that a similar mechanism might also work in the interstellar medium, although the mechanism of current sheet formation is quite different.

In § 3, we apply our results to interstellar gas, primarily to the cold molecular phase. Section 4 is a summary and conclusion.

2. CURRENT SHEET FORMATION IN A ROTATING EDDY

2.1. *Formulation of the Problem*

We consider a weakly ionized gas with nearly infinite electrical conductivity and almost all of the mass residing in the neutrals. We are concerned with timescales longer than the ion-neutral collision time, so that the relative drift between ions and neutrals is determined by balancing Lorentz and frictional forces,

$$\boldsymbol{v}_i - \boldsymbol{v}_n = \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho_i \nu_{in} c} , \tag{2.1}$$

where $\boldsymbol{v}_i$ and $\boldsymbol{v}_n$ are the ion and neutral fluid velocities, $\boldsymbol{B}$ and $\boldsymbol{J}$ are the magnetic field and current density, $\boldsymbol{J} = (c/4\pi)\nabla \times \boldsymbol{B}$, c is the speed of light, ρ_i is the mass density of ions, and ν_{in} is the ion-neutral collision frequency.

Equation (2.1) implies instantaneous equilibration of the ion-neutral drift and neglects ion inertia, Reynolds stress, and pressure. We have tested these approximations with a full two-fluid treatment of a one-dimensional problem (BZ95) in which equation (2.1) is known to lead to singularity formation (BZ94). We found that although two-fluid and resistive effects combine to remove the singularity, under interstellar conditions the current density becomes so large as to be singular for all practical purposes. Therefore, we retain the one-fluid treatment embodied in equation (2.1) because of its simplicity.

The magnetic induction equation in an infinitely conducting medium,

$$\frac{\partial B}{\partial t} = \nabla \times (\boldsymbol{v}_i \times \boldsymbol{B}) , \tag{2.2}$$

can be written as

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v}_i \times \boldsymbol{B}) + \nabla \times \frac{(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{\rho_i \nu_{in} c} , \tag{2.3}$$

where $\boldsymbol{v}$, the center-of-mass velocity, is an approximation to $\boldsymbol{v}_n$, and we have used equation (2.1). The equation for $\boldsymbol{v}$ is the usual ideal fluid equation

$$\rho \frac{D\boldsymbol{v}}{dt} = -\nabla P + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c} + \boldsymbol{F}_{\rm ext} , \tag{2.4}$$

where $D/Dt \equiv \partial/\partial t + \boldsymbol{v} \cdot \nabla$ is the convective derivative, P and ρ are the total pressure and density, and $\boldsymbol{F}_{\rm ext}$ represents gravity or any other force that acts on the single fluid. In particular, ion-neutral friction does not appear in $\boldsymbol{F}_{\rm ext}$, only in the individual equations for the charged and neutral components, because friction does not change the combined momentum of the two species.

We now specialize to the problem at hand: an unforced ($\boldsymbol{F}_{\rm ext} = 0$) two-dimensional configuration in which none of the vectors have $\hat{z}$-components and all quantities are independent of z. We assume that the flow is incompressible, a condition that demands highly subsonic motions. We are aware that the effects of compressibility can be important in the interstellar medium, but the incompressible approximation allows us to isolate the effects of ambipolar drift in a simple manner. We discuss compressibility effects further in § 4.

We introduce magnetic and velocity stream functions A and ψ such that $\boldsymbol{B} = \nabla \times \hat{z}A$ and $\boldsymbol{v} = \nabla \times \hat{z}\psi$. We then take the curl of equation (2.4) (assuming ρ is constant, consistent with the incompressibility condition) and uncurl equation (2.3). This leads to the two dynamical equations

$$\frac{D\omega}{Dt} = \frac{\boldsymbol{B}}{4\pi} \cdot \nabla\nabla^2 A \ , \qquad (2.5a)$$

$$\frac{DA}{Dt} = \frac{B^2}{4\pi\rho_i \nu_{in}} \nabla^2 A \ , \qquad (2.5b)$$

where ω is the $\hat{z}$-component of vorticity, $\nabla \times \boldsymbol{v} = -\nabla^2\psi\hat{z}$.

We have in mind an array of differentially rotating eddies, so we model a single one by embedding the flow in a domain with periodic boundary conditions. The flow initially vanishes outside a circle with radius slightly smaller (by three mesh widths) than the half-width of the computational domain. We solve an initial-value problem, with $A(x, y, t)$ and $\psi(x, y, t)$ taken to be

$$A(x, y, 0) = B_o y; \quad \psi(x, y, 0) = U_o R \cos^4 \frac{\pi r}{2R} H(1 - r) \ , \qquad (2.6)$$

where $r \equiv (x^2 + y^2)^{1/2}$ and H is the Heaviside (step) function. We take $R = 1$ and $U_o = 1$, and carry out the computation in a square box that is slightly larger than the eddy; we choose the half-length L to be 1.04, where $-L \leq (x, y) \leq L$. Although the velocity vanishes outside $r = 1$ initially, magnetic torques set the entire domain into motion shortly after the computation begins. The initial rms velocity $v_{\rm rms}$ within the unit circle is 1.05 U_o, which we round off to U_o in making estimates involving $v_{\rm rms}$.

We parameterize the strength of ambipolar drift through the ambipolar diffusivity at the initial time

$$\lambda_{\rm AD0} \equiv \frac{B_o^2}{4\pi\rho_i \nu_{in}} \ . \qquad (2.7)$$

We will sometimes refer to $\lambda_{\rm AD}$, which is related to $\lambda_{\rm AD0}$ by $\lambda_{\rm AD} \equiv \lambda_{\rm AD0} B^2/B_o^2$ and is the actual ambipolar diffusivity as a function of space and time.

We use a numerical scheme that is third order in time and spectral in space, with 128 × 128 collocation points used in most of the runs. Repetition with 256 × 256 points shows little change in the solution throughout most of the domain, except that when current sheets start to form, their development can be followed for somewhat longer times before they become underresolved and subject to strong numerical diffusion.

2.2. *Nature of the Solutions*

Our problem is described by two parameters: the initial diffusivity, $\lambda_{\rm AD0}$, and the initial magnetic field strength, B_o. The first appears in the magnetic induction equation only (eq. [2.5b]). The second appears in the equation of motion only (eq. [2.5a]) and scales the magnetic forces that act on the flow.

The state with weak B_o and no diffusivity (i.e., perfect coupling between ions and neutrals) can be considered a reference state. In this case, equation (2.5a) drops out of the problem, and equation (2.5b) can be integrated to yield the Cauchy solution, which can be written in polar coordinates as

$$A(r, \theta, t) = A[r, \theta - \Omega(r)t, 0] \ ; \quad \Omega \equiv v/r \ . \qquad (2.8)$$

It is easy to show from equation (2.8) that after about one rotation time, B_θ grows like t, $\hat{z}J$ grows like t^2, the radial magnetic force grows like t^3, and the aximuthal magnetic force grows like t^2.[1] The azimuthal force tends to spin down the inner part of the eddy and spin up the outer part. The evolution of the rotation depends on the boundary conditions; if we had imposed a radiation boundary condition, appropriate to an isolated eddy, the spin-down would be different. However, the phenomena we are interested in tend to occur within a single rotation period if they occur at all, so the choice of boundary condition is probably not significant. The radial force is approximately a pressure gradient force, directed toward local minima of the magnetic field, which are located roughly where the dominant field component, B_θ, changes sign.

In the case of nonzero diffusivity, the ions drift with respect to the neutrals. According to equation (2.1), the ion drift velocity is proportional to the Lorentz force. Ions drift into the magnetic field minima, transporting the field lines with them. The drift velocity approaching each minimum first increases and then decreases, so field lines are pulled in at different rates and tend to accumulate near the magnetic valleys, steepening the magnetic profile and forming local peaks in the current, or current sheets. These effects occur through the nonlinear diffusion term on the right hand side of equation (2.5b). If the field is weak enough that its effect on the neutrals is negligible, the neutral velocity can be taken as given. This is the case we treated in BZ94.

If the diffusivity $\lambda_{\rm AD0}$ is sufficiently large, the magnetic field never winds up enough to form magnetic valleys, and we would expect that current sheets do not form in such a case. We could attempt to estimate the critical $\lambda_{\rm AD0}$ by assuming it to be such that the eddy rotation time and fiducial ambipolar drift time $L^2/\lambda_{\rm AD0}$ are equal. Aside from the ambiguity inherent in defining a turnover time for a differentially rotating structure, we note that the effective diffusivity grows as the field is amplified (compare eqs. [2.5b] and [2.7]) while the length scale over which the field changes shrinks as t^{-1}. These latter two effects mean that the critical $\lambda_{\rm AD0}$ should be less than that obtained from this simple estimate.

If one accounts for the ohmic diffusivity, there must also be a minimum value of $\lambda_{\rm AD0}$ below which ordinary diffusion dominates and current sheets therefore cannot form. Although the ohmic diffusivity is so small in the interstellar medium that this limit is irrelevant, it is important in our simulations, which have numerical resistivity.

Since Lorentz forces oppose rotation, we expect there to be a maximum initial field strength, of order the equipartition field $(4\pi\rho)^{1/2}v$, above which current sheets do not form. The critical value of B_o should be below equipartition because the field is amplified by stretching and because local forces near the current sheets can become quite strong.

[1] Although the magnetic field grows algebraically in a steady shear flow, this is not generally true in turbulent flow. Rather, the length of the field lines, and the field strength, grow exponentially with a time constant of order the eddy turnover time (Parker 1979, chap. 17).

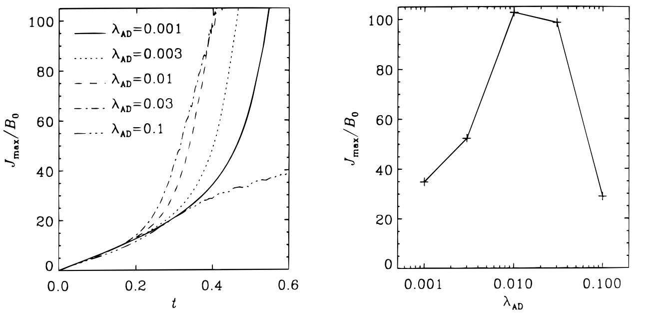

FIG. 1.—Evolution of the maximum current density, normalized to B_o, in a rotating eddy for five different diffusivities and $B_o = 0.1$. *Left panel*: growth of the current density with time. *Right panel*: maximum current at time $t = 0.4$.

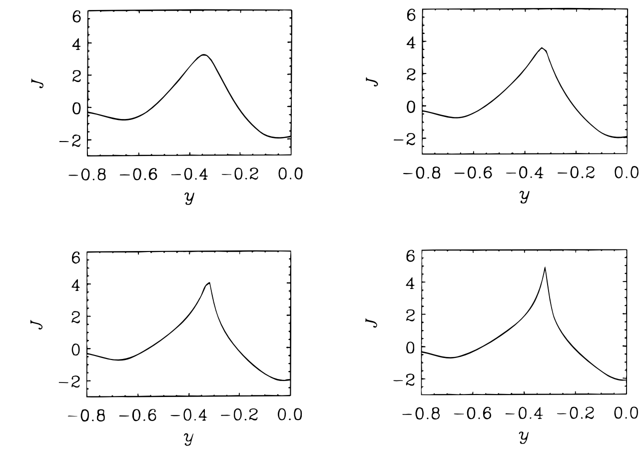

FIG. 3.—Profile of J as a function of y (at fixed x) at four different times, showing the formation of a cusp. *Upper left panel*: $t = 0.46$; *upper right panel*: $t = 0.52$; *lower left panel*: $t = 0.58$; *lower right panel*: $t = 0.65$.

Thus, if we sketch on the ($\lambda_{\rm AD0}$, B_o)-plane the region in which current sheets can form, we expect a peninsular shape: $\lambda_{\rm AD0}$ must lie between two limits, and B_o must lie below an upper limit. This is indeed what we see.

Our diagnostic for the formation of current sheets is the behavior of $J_{\rm max}/B_o$, the maximum value of the current within the domain, normalized by B_o, as a function of time. Figure 1 (*left panel*) is a plot of $J_{\rm max}/B_o$ versus time for five values of $\lambda_{\rm AD0}$, all with $B_o = 0.1$. The value of $J_{\rm max}/B_o$ at time $t = 0.4$ is shown as a function of $\lambda_{\rm AD0}$ in all five cases in Figure 1 (*right panel*). In four of these cases, $J_{\rm max}$ is clearly increasing much faster than any global dynamical rate in the problem, and certainly much faster than the t^2 behavior expected in the ideal theory. In the case $\lambda_{\rm AD0} = 0.1$, the largest diffusivity represented on the plot, $J_{\rm max}$ is increasing, but quite slowly. The rotation period at the center of the eddy is $2/\pi$ in the units used here. Thus, current sheet information is occurring in less than one rotation time, before the field has wound up to the point that the random field dominates the mean field. This behavior may be contrasted with Figure 2, which has similar plots for $B_o = 1$. For all six values of $\lambda_{\rm AD0}$, $J_{\rm max}$ peaks at a rather low value and subsequently declines as the field brakes the fluid and the field lines straighten out. The field is too strong to form current sheets in this case.

Figure 1 shows that the current sheet formation is rapid. It also turns out to be essentially independent of velocity once the flow has acted to shear the field. We have shown this by abruptly setting the velocity to zero in simulations in which the current has begun to turn up. The subsequent evolution is much the same as when the velocity field is retained.

In the cases in which $J_{\rm max}/B_o$ shows runaway growth, the current profile is similar to that found earlier (BZ94). Figure 3 shows a slice of J along the x-axis at a fixed value of y. The structure of J is highly intermittent, very large within the current sheets and nearly zero outside them. There is a heuristic explanation for why ambipolar drift has this effect. One can use the equations of motion and the magnetic induction equation to derive an equation for energy flow

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{B^2}{8\pi}\right) + \boldsymbol{\nabla}\cdot\left[\frac{1}{2}\rho v^2 \boldsymbol{v} - \frac{(\boldsymbol{v}_i \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi}\right] = -\frac{(\boldsymbol{J} \times \boldsymbol{B})^2}{\rho_i \nu_{in} c^2}\ . \quad (2.9)$$

In a fully ionized medium, with $\rho = \rho_i$, $\boldsymbol{v} = \boldsymbol{v}_i$, and the right-hand side equal to zero, equation (2.9) is the usual energy conservation law for a cold fluid, stating that the rate of change of the total energy density equals minus the divergence of the kinetic and electromagnetic energy fluxes. Ion-neutral friction is clearly an energy sink. If the fluid relaxes in a way that minimizes its energy-loss rate, then $\boldsymbol{J}$ must become aligned with $\boldsymbol{B}$ (in three dimensions) or vanish (in two dimensions). But this cannot generally occur everywhere, because of dynamical forcing or topological constraints. This leads to an intermittent current structure, in which the energy losses are confined to thin sheets or filaments. Evidence for this type of relaxation in a three-dimensional situation is shown clearly in Figure 13 of Brandenburg et al. (1995).

Figure 4 shows the region of the ($\lambda_{\rm AD0}$, B_o)-plane within which current sheets form. It has the peninsular shape we deduced from physical arguments. The limiting field strength B_o is approximately 0.5, and the upper limit to $\lambda_{\rm AD0}$ is approximately 0.06. The lower limit to $\lambda_{\rm AD0}$, which in our computations is determined purely by numerical diffusion, is less than 10^{-3}.

Finally, we make a correspondence between the parameterization used here and the interaction of a magnetic field strength with eddies of variable size and velocity in a medium with fixed density and collision frequency. The diffusivity $\lambda_{\rm AD0}$ is measured in units of the eddy size and speed, LU_o. If we define a global ambipolar Reynolds number $R_{\rm AD}$ as the ratio of the ambipolar drift time to the dynamical time, and use the global length scale for both, then in our units $\lambda_{\rm AD0}$ is just $R_{\rm AD}^{-1}$. The magnetic field is measured in units of the equipartition field, $(4\pi\rho U_o^2)^{1/2}$. If we assume that

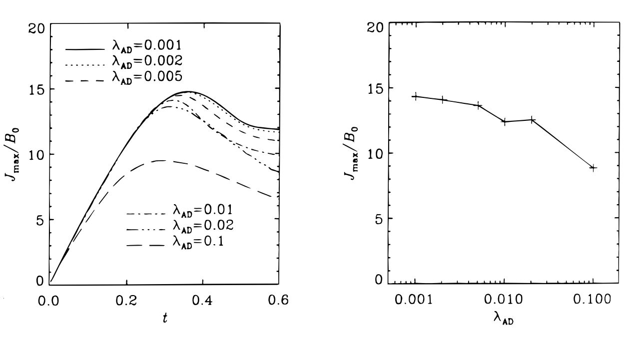

FIG. 2.—Same as Fig. 1, but six different diffusivities and $B_o = 1.0$

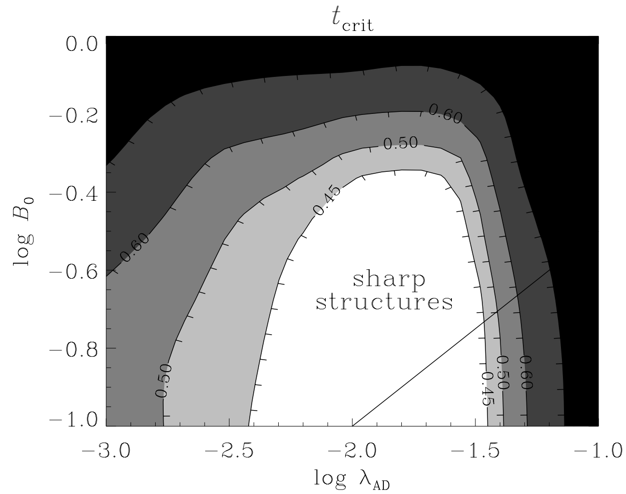

FIG. 4.—The (λ_{AD0}, B_o)-plane, showing the region in which current sheets form. The contours are labeled by the logarithm of the time at which the current runs away. The part of the plane in which the current remains smooth is shown in black. The line segment is a fit to the kinematic results of BZ94.

the eddy size and velocity are connected by a relation of the form $U_o(L) \propto L^{\gamma}$, where $\gamma > 0$, since scaling laws of this type occur in a number of turbulence models, then both the upper limit on λ_{AD} and the upper limit on B_o translate into lower limits on the sizes of eddies that can induce the formation of current sheets: the smallest eddies are both too diffusive and too slow. Thus, were it not for current sheet formation, ambipolar drift would impede the formation of structure on scales corresponding to $R_{AD} < 1$.

3. WINDUP AND STEEPENING IN THE INTERSTELLAR MEDIUM

In § 2, we modeled the interaction between an initially uniform magnetic field and a differentially rotating eddy. We parameterized the model by its initial field strength and ambipolar diffusivity. In order to discuss these effects in turbulent interstellar gas, however, it is more appropriate to fix the field strength and diffusivity, and consider a range of eddy sizes L and characteristic velocities v. We assume that eddies form, dissolve, and form again on times of order the eddy turnover time, L/v, and we do not consider processes that act on longer timescales.

We take the ion-neutral collision rate constant to be $\langle \sigma v \rangle = 2 \times 10^{-9}$ (Draine, Roberge, & Dalgarno 1983), take the neutral mass to be A_n times the proton mass, express B in microgauss, and assume the ions are much more massive than the neutrals. Using equation (2.7), this leads to the numerical value

$$\lambda_{AD} = 2.4 \times 10^{19} \frac{B_\mu^2}{A_n n_n n_i} \text{ cm}^2 \text{ s}^{-1} , \quad (3.1)$$

where n_i and n_n are the ion and neutral particle densities in units of cm^{-3}. According to the results of § 2.2, current sheets do not form unless $\lambda_{AD0}/Lv \leq 0.06$. Therefore, eddies in which sharp structures form satisfy the condition

$$L_{pc} v_{km} > 1.3 \times 10^{-3} \frac{B_\mu^2}{A_n n_n n_i} , \quad (3.2)$$

where L_{pc} and v_{km} are the eddy size and velocity in units of parsecs and km s^{-1}, respectively.

We found that current sheets can form only if $B_o \geq 0.5(4\pi\rho v^2)^{1/2}$, nearly independently of λ_{AD0}. This is equivalent to the requirement

$$B_\mu \leq 0.23(A_n n_H)^{1/2} v_{km} \quad (3.3)$$

for current sheet formation.

As an illustrative example, and as a way of reducing the number of free parameters in equations (3.2) and (3.3), we consider a molecular cloud environment in which v and L obey the line width–size relation

$$v_{km} \approx L_{pc}^{1/2} \quad (3.4)$$

(Solomon et al. 1987), and the ionization law is

$$n_i \approx 10^{-5} n_n^{1/2} \quad (3.5)$$

(McKee et al. 1993). We assume that all the hydrogen is molecular and 10% of atoms are He, so that $A_n = 2.4$. It is also convenient to introduce the column density $N_{pc} \equiv n_n L_{pc}$. Equation (3.2) can then be expressed as a limit on magnetic field strength,

$$B_\mu \leq 0.14 N_{pc}^{3/4} , \quad (3.6)$$

while the limit on field strength given by equation (3.3) is

$$B_\mu \leq 0.35 N_{pc} n_n^{1/2} . \quad (3.7)$$

Current sheet formation can occur as long as B_μ satisfies the more stringent of the inequalities in equations (3.6) and (3.7); the field is diffusivity limited for $N_{pc} < 48$ (eq. [3.6]) and dynamically limited (eq. [3.7]) otherwise. At column densities of 10^{21}, for example, the maximum field strength at which current sheets can form is dynamically limited, and is about 6.5 μG.

Equations (3.2) amd (3.4) can also be applied to diffuse gas, such as H I clouds. Eddies a few parsecs in size, with turbulent velocities of a few km s^{-1} and magnetic fields up to several microgauss, are in the regime of current sheet formation.

The reader should keep in mind that equations (3.6) and (3.7) were derived from a study of current sheet formation in a particular family of eddies, which satisfy the initial conditions given by equation (2.6). The numerical coefficients in these equations undoubtedly depend on the turbulence model, so they should not be taken too seriously. We regard it as encouraging that our theory predicts current sheet formation under conditions that resemble the state of the interstellar medium.

4. SUMMARY AND CONCLUSIONS

Rapid reconnection of interstellar magnetic field lines appears to require a mechanism for making small-scale structure—or current sheets—in the field. In § 2 of this paper, we described a parameter study of current sheet formation caused by ambipolar drift in a differentially rotating eddy with an initially uniform background magnetic field. We extended our previous results on this problem (BZ94) by including the effect of Lorentz forces on the eddy dynamics, albeit in the incompressible approximation, and by considering a range of initial field strengths and diffusivities. We found that unless the field is too strong, or the ambipolar diffusivity too large, current sheets form in less than one eddy turnover time. Current sheets result from local minima in the strength of the wound-up field, toward which the Lorentz force is directed. The ion-drift velocity is proportional to the Lorentz force, and the magnetic field is frozen to the ions. Current sheets form at the stagnation points of the inflow, at which the magnetic field piles up.

The results of our parameter study can be expressed as necessary conditions on the size and velocity of the eddies that induce current sheet formation: it is generally the larger eddies, which carry more kinetic energy per unit mass. Eddies below a critical size tend to be too slow and too diffusive. The present calculation, of course, is no substitute for a study in which flows are present over a range of scales.

The model examined in this paper is highly restricted in the sense that both the magnetic field and the velocity field are two-dimensional and assumed to be coplanar. The addition of a uniform magnetic field perpendicular to the plane of the eddy tends to inhibit current sheet formation. The ion flow that forms the sheet is highly compressive and would build up a large gradient in the strength of the vertical field if it were not modified by Lorentz forces. For example, current sheets with the $J \propto x^{-2/3}$ structure found in BZ94 would be accompanied by vertical fields $B_z \propto x^{-1/3}$ if Lorentz forces did not act to remove the singularity. A rough argument suggests that under these conditions, current sheet formation by ambipolar drift is quenched when the current is amplified by a factor of order $(1 + B_h/B_{z0})$, where B_h is the strength of the horizontal field and B_{z0} is the strength of the initial field. The current is assumed to saturate at a value such that the opposing forces exerted by gradients of B_h and B_z cancel each other. Thus, the current can be amplified only if the relative value of B_{z0} is very small.

In a truly three-dimensional situation, however, with no globally dominant field component, the possibilities are far less limited. All three components of the field will be sheared, and we see no reason why locally converging magnetic forces should not lead to current sheets, just as they do in two dimensions. We expect ambipolar drift to drive the system into a bifurcated state in which the magnetic field is nearly force-free almost everywhere, except in thin current sheets. Of course, this holds only on the small scales at which ambipolar drift can compete with dynamical processes.

The incompressible limit in which we carried out our calculation requires high neutral gas pressure, while the turbulent pressure, and magnetic pressure, generally dominate in the interstellar medium. Suppose, then, that we did a compressible calculation. The neutrals too would then tend to stream into the magnetic valleys, as the ions do in our present calculation. This would lead to some radial readjustment, possibly through the formation of shocks. This radial flow would halt once the neutral pressure built up through compression, but the magnetic valleys would remain and would be even steeper. Ion drift would then bring the system the rest of the way to current sheet formation. Thus, restoration of compressibility would not prevent singularity formation. The crucial requirement, as we found in our fully self-consistent two-fluid study (BZ95), is that the ionization fraction be low enough, and the recombination rate high enough, to keep the region where ion pressure dominates magnetic pressure small. This requirement appears to be well satisfied in the interstellar medium.

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In the paper "Current Sheet Formation in the Interstellar Medium" by Ellen G. Zweibel and Axel Brandenburg (ApJ, 478, 563 [1997]), the address given for Dr. Brandenburg is incorrect. It should be Department of Mathematics, University of Newcastle, Newcastle upon Tyne, NE1 7RU, England, UK. His e-mail address is Axel.Brandenburg@newcastle.ac.uk.