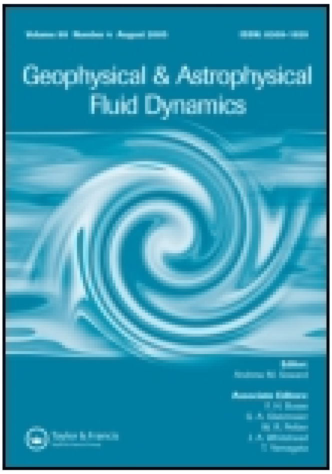

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ON THE GENERATION OF NON-AXISYMMETRIC MAGNETIC FIELDS IN MEAN-FIELD DYNAMOS

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The magnetic fields of the Earth and other planets deviate in varying degrees from symmetry about the rotational axis. While, for example, the field of Saturn is highly symmetric, that of Uranus shows a striking asymmetry. With these observations in mind we investigate excitation conditions of axisymmetric and non-axisymmetric field modes in spherical mean-field dynamo models. In models of α^2-type the marginal dynamo numbers for modes with different azimuthal dependences are close together if the α-effect is concentrated in a thin layer. Preference of non-axisymmetric modes over axisymmetric ones occurs if we include weak differential rotation, anisotropies of the α-effect or the γ-effect, the last one corresponding to a radial transport of magnetic flux. We discuss consequences of these results for planetary dynamos.

KEY WORDS: Magnetohydrodynamics, mean-field dynamos, non-axisymmetric magnetic fields.

1. INTRODUCTION

A great number of spherical kinematic mean-field dynamo models have been investigated which proved to be helpful for understanding the processes responsible for the magnetic fields of the Earth, the planets, the Sun and solar-type stars, the magnetic stars and other objects; e.g., Krause and Rädler (1980). In these models the distributions of the electric conductivity and the fluid motions are assumed to be symmetric with respect to both the rotation axis and the equatorial plane. It turned out that the modes of the mean magnetic field occurring in these models are not necessarily symmetric but may well be non-axisymmetric with respect to the rotation axis. In addition they can be symmetric or antisymmetric about the equatorial plane. The possibility of non-axisymmetric modes has first been investigated for $\alpha\omega$-dynamos by Stix (1971), Krause (1971), and Roberts and Stix (1972); further results have been presented by Ivanova and Ruzmaikin (1985), Rädler (1986a), and Ruzmaikin *et al.* (1988). Non-axisymmetric modes in α^2-dynamos were first found by Rädler (1975) and have been investigated for a variety of models by Rüdiger (1980) and Rädler (1986a). For a wide range of assumptions the axisymmetric modes are favored over the non-axisymmetric ones, i.e. an axisymmetric mode is most easily excited. In some $\alpha\omega$-models a slight preference of non-axisymmetric modes has been found for moderate magnitudes of differential rotation. For sufficiently large differential rotation there are good reasons to assume that it is always an axisymmetric mode which is easiest to excite (Rädler, 1986b). In α^2-models a clear preference for non-axisymmetric modes occurs if particular anisotropies of the α-effect or related effects are taken into account.

The possibility of both axisymmetric and non-axisymmetric modes of the mean magnetic field in the models envisaged is of interest for understanding the observed fields of the objects mentioned. The magnetic field of the Earth clearly deviates from axisymmetry, as also, to varying degrees, do the fields of other planets. Only the Saturnian field shows a surprisingly high degree of axisymmetry, whereas the Uranian field exhibits an extremely high asymmetry. Deviations of the solar magnetic field from axisymmetry are indicated by its well-known sectorial structure (e.g. Bai, 1988). Likewise the observations of magnetic stars can only be understood by assuming non-axisymmetric structures of the magnetic fields, often discussed in terms of the "oblique rotator" model. (However the magnetic fields of the early type stars may possibly be of "fossil", not of dynamo, origin.)

The relation between the magnetic field modes defined here and the magnetic fields generated by real dynamos is still a matter of debate. In contrast to the kinematic, i.e. linear dynamo, real dynamos are subject to the back-reaction of the magnetic field on the motions, that is, they operate in a nonlinear regime. Some symmetric dynamo models in the above sense have been studied with simple assumptions about the back-reaction. There are results which suggest that it is the most easily excitable mode only which determines the magnetic field of a steady state in the nonlinear regime, and that the other modes are then no longer of interest; see Krause and Meinel (1988) and Rädler and Wiedemann (1989). There are, however, other results which show that the magnetic field in the steady state may depend on the initial conditions and that an oscillatory field may consist of several parts which are related to different modes; see Brandenburg *et al.* (1989a, b). Moreover, it seems reasonable to admit even in the kinematic theory, slight asymmetries of the models, e.g. asymmetries in the distribution of the motions. Then it can be expected that the magnetic field generated, in addition to a part related to the most easily excitable mode of the corresponding symmetric model, contains also parts related to other modes with comparable excitation conditions; see Rädler (1989).

We present here a few more results concerning excitation conditions of axisymmetric and non-axisymmetric magnetic field modes in dynamo models of α^2-type, considering also influences of anisotropies of the α-effect, of related induction effects of the fluctuating motions and of differential rotation. Our investigations have been partly motivated by a comparative study of the magnetic fields observed on the Earth, Jupiter, Saturn and Uranus presented by Rädler and Ness (1988), from which, in the sense explained above, several questions arose concerning the circumstances under which the excitation conditions of the axisymmetric and certain non-axisymmetric modes lie closely together.

2. THE MODEL AND THE NUMERICAL METHOD

We consider dynamo processes in a spherical rotating body of electrically conducting fluid surrounded by free space. The mean magnetic flux density, **B**, is supposed to obey the induction equation

$$\frac{\partial \mathbf{B}}{\partial t}=\eta\,\Delta\mathbf{B}+\operatorname{curl}(\mathbf{u}\times\mathbf{B}+\mathscr{E}) \tag{1}$$

inside the fluid body and to continue as an irrotational field outside. Here η is the magnetic diffusivity of the fluid, which is considered to be constant, **u** the velocity of the mean motion of the fluid and $\mathscr{E}$ the mean electromotive force caused by the fluctuations of the motions and of the magnetic field.

As mentioned above, we assume symmetries of the mean motion and the distribution of the fluctuating motions about both the rotation axis and the equatorial plane. More precisely, we assume that all mean quantities are invariant under rotation of the total velocity field about the rotation axis and under reflection about the equatorial plane. We also assume that the mean motions and, on average, also the fluctuating motions are steady, that is, all mean quantities are also invariant under translations of the velocity field along the time axis. Under these circumstances the general solution of the equations governing the **B**-field can be understood as a superposition of **B**-modes having the form of

$$\mathbf{B}=\mathscr{R}e(\hat{\mathbf{B}}\exp\{im\phi+(\lambda-i\omega)t\}). \tag{2}$$

Here $\hat{\mathbf{B}}$ is a vector field which is symmetric about the rotation axis, either symmetric or antisymmetric about the equatorial plane, and steady. m is a non-negative integer indicating the azimuthal variation of the mode, ϕ the azimuthal coordinate, λ and ω are real constants, the first of which describes the growth rate of the mode. As usual, we denote these modes by Am or Sm, according to their antisymmetry or symmetry about the equatorial plane and to their azimuthal variation, that is, we speak of A0, S0, A1, S1,... modes. We restrict our attention to the most easily excitable mode of each type, that is, to that with the largest value of λ. We furthermore consider only marginal cases, that is, we determine for a given mode the magnitude of induction effects such that $\lambda=0$. In the marginal case the axisymmetric modes, $m=0$, are steady if $\omega=0$ or, if $\omega\neq 0$, they have the form of oscillations with a steady amplitude and the frequency ω. The marginal non-axisymmetric modes, $m\neq 0$, are waves with steady amplitudes and migrating in azimuthal direction with an angular velocity ω/m.

As far as the mean electromotive force $\mathscr{E}$ is concerned we accept the usual assumption that this quantity, at a given point in space and time, can be represented by **B** and its first spatial derivatives in this point. Even under this assumption, $\mathscr{E}$ has in general a complex structure; e.g. Rädler (1980). We restrict our attention here, with some arbitrariness, to a few particular contributions to $\mathscr{E}$ which are given by

$$\mathscr{E} = -\alpha_1(\hat{\mathbf{z}}\cdot\hat{\mathbf{r}})\mathbf{B} - \alpha_4[(\hat{\mathbf{z}}\cdot\mathbf{B})\hat{\mathbf{r}} + (\hat{\mathbf{r}}\cdot\mathbf{B})\hat{\mathbf{z}}] - \gamma_1\hat{\mathbf{r}}\times\mathbf{B}. \tag{3}$$

The coefficients α_1, α_4, and γ_1 are determined by the assumptions about the fluid motions; $\hat{\mathbf{z}}$ and $\hat{\mathbf{r}}$ are unit vectors parallel to the rotation axis and in the radial direction, respectively. Due to the assumed symmetry of the fluid motions, these coefficients depend only on radius and latitude but not on the azimuthal coordinate, and they are symmetric about the equatorial plane.

As for the mean motion we include only a differential rotation, that is,

$$\mathbf{u} = \Omega r\hat{\mathbf{z}}\times\hat{\mathbf{r}}, \tag{4}$$

where Ω and r are the mean angular velocity and the radial coordinate, respectively. For the coefficients describing the induction effects we now write

$$\alpha_1 = C_{\alpha_1}(\eta/R)f_\alpha(x), \tag{5}$$

$$\alpha_4 = C_{\alpha_4}(\eta/R)f_\alpha(x), \tag{6}$$

$$\gamma_1 = C_{\gamma_1}(\eta/R)f_\gamma(x), \tag{7}$$

$$\Omega = C_\Omega(\eta/R^2)g(x), \tag{8}$$

where the dimensionless parameters C_{α_1}, C_{α_4}, C_{γ_1}, and C_Ω describe the magnitudes of these effects. R is the radius of the fluid body, and $x = r/R$. We further write

$$f_\alpha = \begin{cases} -\frac{15}{16d_a}(1-\xi_a^2)^2 & \text{for } |\xi_a| \leqq 1 \\ 0 & \text{for } |\xi_a| \geqq 1 \end{cases} \qquad \xi_a = \frac{x - x_a}{d_a}, \tag{9}$$

$$g = \begin{cases} -1 & \text{for } \xi_\Omega \leqq -1 \\ \frac{1}{2}(1 - \frac{3}{2}\xi_\Omega + \frac{1}{2}\xi_\Omega^3) & \text{for } |\xi_\Omega| \leqq 1 \\ 0 & \text{for } \xi_\Omega \geqq 1 \end{cases} \qquad \xi_\Omega = \frac{x - x_\Omega}{d_\Omega}, \tag{10}$$

where a stands for α or γ, and x_a, d_a, x_Ω, and d_Ω are constants. These specifications ignore any latitudinal dependence of α_1, α_4, γ_1, and Ω; they coincide with specifications used by Rädler (1986a). Note that $\int_0^1 f_a(x)dx = 1$ if $x_a - d_a \geqq 0$ and $x_a + d_a \leqq 1$.

The equations governing the **B**-field pose an eigenvalue problem for the complex parameter $\lambda - i\omega$ or, if we put $\lambda = 0$, for a pair of parameters consisting of one of the quantities C_{α_1}, C_{α_4}, C_{γ_1}, or C_Ω and of ω. Representing **B** by

$$\mathbf{B} = -\operatorname{curl}(\mathbf{r}\times\nabla S) - \mathbf{r}\times\nabla T, \tag{11}$$

Table 1 Marginal values of C_{α_1} and the corresponding frequencies ω (in units of η/R^2) of Am modes in a model with α_1-effect versus d_α, for $x_\alpha = 0.8$. Note that for very thin α-layers the differences of C_{α_1} for different m are smaller. The truncation level was $N_l = 18$ and $N_r = 150$

d_α	A0	A1	A2	A3	A4
			C_{α_1}		
0.10	2.57	2.63	2.79	3.00	3.23
0.08	2.43	2.48	2.61	2.78	2.97
0.06	2.28	2.32	2.43	2.56	2.70
0.05	2.20	2.24	2.33	2.44	2.56
			ω		
0.10	0	1.20	0.94	0.64	0.42
0.08	0	1.02	0.81	0.55	0.37
0.06	0	0.82	0.67	0.45	0.29
0.05	0	0.71	0.59	0.40	0.25

by the defining scalars S and T and expanding them into series of spherical harmonics we can reduce this problem to an eigenvalue problem for an infinite set of ordinary differential equations for the coefficients of these expansions, which depend on the radial coordinate only. It is assumed that the solutions of this problem are approximated by the solution of the corresponding problem for a finite set of equations, which results from the infinite one by omitting all coefficients belonging to spherical harmonics the order l of which exceeds a sufficiently large bound, N_l. This finite set of equations is subject to a discretization with respect to the radial coordinate x, using a sufficiently large number of gridpoints, N_r. In this way the eigenvalue problem for differential equations is reduced to a matrix eigenvalue problem, and this is solved numerically by standard methods.

3. NUMERICAL RESULTS

3.1 *Models with Isotropic α-effect*

Let us first deal with models where only the α_1-effect is involved. Earlier investigations show that the marginal dynamo numbers of the A0, S0, A1, and S1 modes are always rather close together, whereas the higher modes A2, S2, A3, S3,... are less easily excited. There are some reasons to believe that the discrimination of the modes with higher m disappears if the α-layer becomes thinner, and if it is shifted closer to the surface. In Tables 1 and 2 and in Figure 1 the results of the present computations for models considered earlier by Rädler (1986a) are summarized. The differences of C_{α_1} for different m become smaller if d_α decreases or x_α increases, but the changes are rather small in all cases. Note that $\int_0^1 f_\alpha(x)dx < 1$ for $x_\alpha > 0.9$. This is the reason for the sharp increase of C_{α_1} when the

Table 2 Marginal values of C_{α_1} and the corresponding frequencies ω (in units of η/R^2) of Am modes in a model with α_1-effects versus x_α, for $d_\alpha=0.1$. The truncation level was $N_t=18$ and $N_r=150$

x_α	A0	A1	A2	A3
		C_{α_1}		
0.8	2.57	2.63	2.79	3.00
0.9	2.83	2.87	2.98	3.13
0.92	2.95	2.99	3.09	3.23
0.94	3.18	3.22	3.32	3.45
0.96	3.63	3.67	3.75	3.88
		ω		
0.8	0	1.21	1.50	1.23
0.9	0	1.91	2.28	2.12
0.92	0	2.19	2.84	2.80
0.94	0	2.50	3.50	3.68
0.96	0	2.77	4.17	4.67

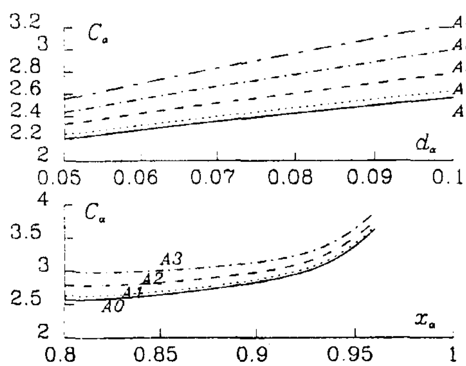

Figure 1 Marginal values of C_{α_1} for Am modes in a model with α_1-effect versus d_α, for $x_\alpha=0.8$ (upper panel), and versus x_α, for $d_\alpha=0.1$ (lower panel).

α-layer is shifted closer to the surface (lower panel in Figure 1). The magnetic field structure of different modes is presented in Figure 2 (for $d_\alpha=0.1$) and in Figure 3 (for $d_\alpha=0.05$). The main effect of a thinner α-layer is an enhanced concentration of the B_ϕ-field in the radial direction whereas the latitudinal structure remains more-or-less unchanged.

However, adding a *weak* differential rotation can change the sequence of the marginal C_{α_1} values such that A1 and/or S1 are slightly preferred over the A0 and S0 modes. This can be seen from Table 3 showing values of C_{α_1} and ω for the A0, S0, A1 and S1 modes with and without differential rotation. As Figure 4 shows, the field structure seems to be more complex in the presence of differential rotation.

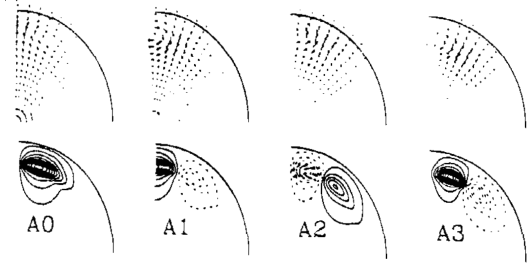

Figure 2 The magnetic field configuration of the Am model with α_1-effect, $x_\alpha=0.8$ and $d_\alpha=0.1$. (The values for C_{α_1} and ω are given in Table 1.) In the first row vectors (B_r, B_θ) are plotted in a quadrant of an arbitrarily chosen meridional plane. The second row shows contours of constant B_ϕ in the same plane. Solid lines refer to positive values of B_ϕ, dashed lines to negative ones.

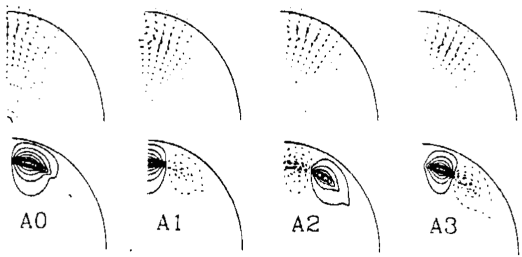

Figure 3 Sames as Figure 2 but $d_\alpha=0.05$. Note the concentration of the B_ϕ-field. The field vectors (B_r, B_θ) are merely unaffected.

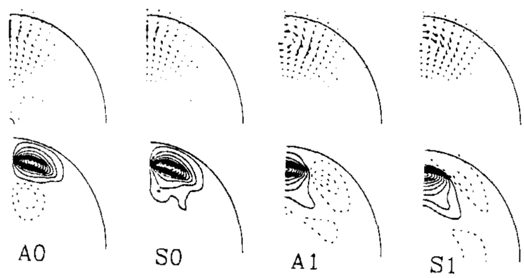

Figure 4 The magnetic field configuration of the A0, S0, A1, and S1 modes in a model with α_1-effect and weak differential rotation, $x_\alpha=0.8$, $d_\alpha=0.1$, $x_\Omega=0.5$, $d_\Omega=0.1$, and $C_\Omega=-300$. (The values for C_{α_1} and ω are given in Table 3.)

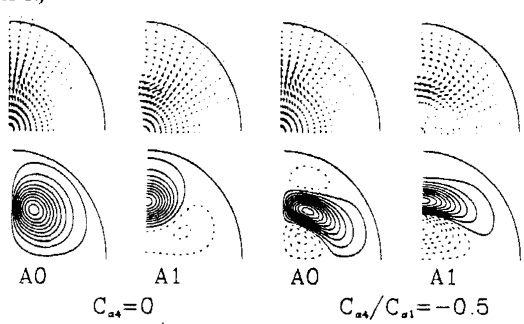

Figure 5 The magnetic field configuration of the A0 and A1 modes in a model with α_1- and α_4-effect, $x_\alpha=0.5$, $d_\alpha=0.4$. The profile in this case is smoother than in the previous ones and α has its maximum at half the radius. (The values of C_{α_1} and ω are given in Table 4.)

Table 3 Marginal values of C_{α_1} and the corresponding frequencies ω (in units of η/R^2) for the A0, S0, A1, and S1 modes in a model with α_1-effect and differential rotation, $x_\alpha=0.8$, $d_\alpha=0.1$, $x_\Omega=0.5$, and $d_\Omega=0.1$. The truncation level was $N_l=18$ and $N_r=150$

C_Ω	A0	S0	A1	S1
		C_{α_1}		
0	2.57	2.57	2.63	2.63
−300	2.65	2.65	2.64	2.64
		ω		
0	0	0	1.21	1.29
−300	0	0	1.17	1.21

Table 4 Marginal values of C_{α_1} and the corresponding frequencies ω (in units of η/R^2) of the A0, A1, A2, and A3 modes in a model with α_1- and α_4-effect and differential rotation, $x_\alpha=0.5$, $d_\alpha=0.4$, $x_\Omega=0.8$, and $d_\Omega=0.1$. The first non-axisymmetric mode is easiest to excite for a wide range of C_Ω. The truncation level was $N_l=16$ and $N_r=100$

$C_{\alpha_4}/C_{\alpha_1}$	C_Ω	A0	A1	A2	A3
			C_{α_1}		
0	0	4.66	4.85	6.29	7.82
−0.5	0	8.31	7.05	10.08	13.18
−0.5	−100	8.99	7.36		
−0.5	−300	10.14	7.49	10.49	13.51
−0.5	−1000	4.84	7.57	10.58	13.61
			ω		
0	0	0	−0.20	−0.15	−0.08
−0.5	0	0	0.02	0.37	0.27
−0.5	−100	14.1	95.0		
−0.5	−300	5.4	297.1	597.1	896.8
−0.5	−1000	78.7	999.7	1998.0	2997.3

3.2 Models with Anisotropic α-effect and Related Effects

Anisotropies of the α-effect can, under certain circumstances, give rise to a preference of non-axisymmetric modes over axisymmetric ones. For example, in a model including the α_1- and α_4-effects, considered by Rädler (1986a), the A1 and S1 modes are favored over the A0 and S0 modes, if $C_{\alpha_4}/C_{\alpha_1}<-0.2$. We have studied the influence of a weak differential rotation in the same model. Table 4 gives results for A modes. (For S modes the marginal values of C_{α_1} are quite close to those for the A modes, S1 being slightly easier to excite than A1.) The preference of the A1 mode over the other Am modes is even more pronounced by

Table 5 Marginal values of C_{α_1} and the corresponding frequencies ω (in units of η/R^2) of the A0, A1, and A2 modes in a model with α_1- and γ_1-effect and differential rotation, $x_\alpha = x_\gamma = 0.5$, $d_\alpha = d_\gamma = 0.4$, $x_\Omega = 0.8$, and $d_\Omega = 0.1$. Note that now also the A0 mode becomes oscillatory even without differential rotation. The truncation level was $N_l = 16$ and $N_r = 100$

C_{γ_1}	C_Ω	A0	A1	A2
		C_{α_1}		
0	0	4.66	4.85	6.29
5	0	9.51	8.79	9.98
5	−50	9.53	8.82	
5	−100	9.51	8.84	
5	−200	8.95	8.85	
		ω		
0	0	0	−0.20	−0.15
5	0	50.7	66.5	57.2
5	−50	50.0	117.3	
5	−100	48.7	167.6	
5	−200	42.4	267.7	

differential rotation in a range of small C_Ω. Figure 5 gives examples of field structures. The reason why the axisymmetric modes are relatively hard to excite could be that the α_1-effect is partly cancelled by the α_4-effect. From (3) it is evident that this can happen for negative values of $C_{\alpha_4}/C_{\alpha_1}$ if **B** is directed parallel to $\hat{\mathbf{z}}$ or to $\hat{\mathbf{r}}$. Figure 5 shows that this is the case for the A0 mode ($\mathbf{B} \| \hat{\mathbf{z}}$ on the axis), but not so much for A1 ($\mathbf{B} \perp \hat{\mathbf{z}}$ on the axis).

Finally, in Tables 5 and 6 we present some results for the γ_1-effect. Earlier it has been found by Rädler (1986a) that a preference of the A1 and S1 modes is obtained in a model involving the α_1- and γ-effect when $|C_{\gamma_1}| > 3$. Our new interesting finding is the possibility of oscillatory A0 modes even in the absence of differential rotation. Furthermore, the effect of differential rotation is that the preference of the A1 mode becomes smaller and presumably disappears as C_Ω grows. Similarly, if α- and γ-layers are thin and situated close to the surface (Table 6), the preference of the A1 mode disappears. The behavior is opposite to that without γ-effect (Section 3.1). Figure 6 gives the field geometry. The field is now concentrated closer to the surface, demonstrating that γ_1-effect, with positive C_{γ_1}, corresponds to a transport of magnetic flux outwards in the radial direction.

4. CONCLUSIONS

Our numerical results confirm the suggestion that in the pure α^2-regime the differences in the excitation conditions between the modes with $m = 0$ or $m = 1$ and

Table 6 Marginal values of C_{α_1} of the A0, A1, and A2 modes in a model with α_1- and γ_1-effect, $x_\alpha = x_\gamma = 0.8$, $d_\alpha = d_\gamma = 0.1$, C_Ω. The profiles of α_1- and γ_1-effect are steeper and closer to the surface than in the case of Table 5. The truncation level was $N_l = 16$ and $N_r = 100$

C_{γ_1}	A0	A1	A2
		C_{α_1}	
0	2.57	2.63	2.79
1.0	3.01	3.05	3.15
1.5	3.53	3.55	3.61
		ω	
0	0	1.21	1.50
1.0	0	3.41	5.49
1.5	0	3.80	7.26

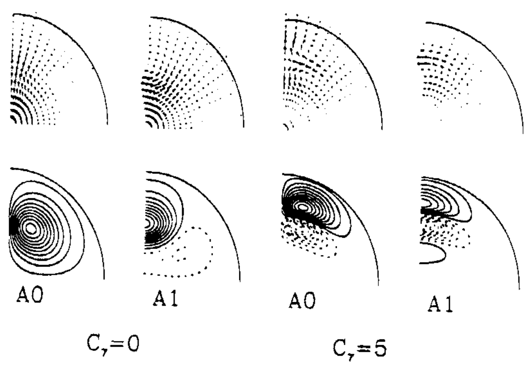

Figure 6 Same as Figure 5 but taking the γ_1-effect into account instead of the α_4-effect, $x_\alpha = x_\gamma = 0.5$, $d_\alpha = d_\gamma = 0.4$. Note the concentration of the B_ϕ-field to the surface when $C_\gamma \neq 0$. (The values of C_{α_1} and ω are given in Table 5.)

those with $m > 1$ becomes smaller if the α-layer becomes smaller or is shifted toward the surface of the fluid body. This result supports the suggestion made by Rädler and Ness (1988) to understand the relatively strong $m = 2$ contributions to the magnetic fields of Earth and of Jupiter as an indication that the dynamo works in a thin layer. Our results also show several possibilities to favor non-axisymmetric modes, in particular A1 and S1 modes, over axisymmetric ones. A weak differential rotation may act in this sense as well as anisotropies of the α-effect and related effects, in particular the α_4- and γ_1-effect. The different degrees of non-axisymmetry of the magnetic fields of the planets may be related to the different magnitudes of such induction effects. However, these effects can hardly explain the occurrence of modes with $m > 1$, since usually they then become more difficult to excite than those with $m=0$ or $m=1$. The relevance of these results for dynamos in the nonlinear regime remains to be investigated.

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