Supplemental Material – Efficient quasi-kinematic large-scale dynamo as the small-scale dynamo saturates

Pallavi Bhat,* Kandaswamy Subramanian, and Axel Brandenburg
(Dated: March 27, 2022)

Shear helical dynamo: large-scale flows and late time behavior

We mentioned in the main paper that a large-scale component arises in the velocity field. This is shown in Fig. 1(a). The large-scale flow is seen to arise early on,

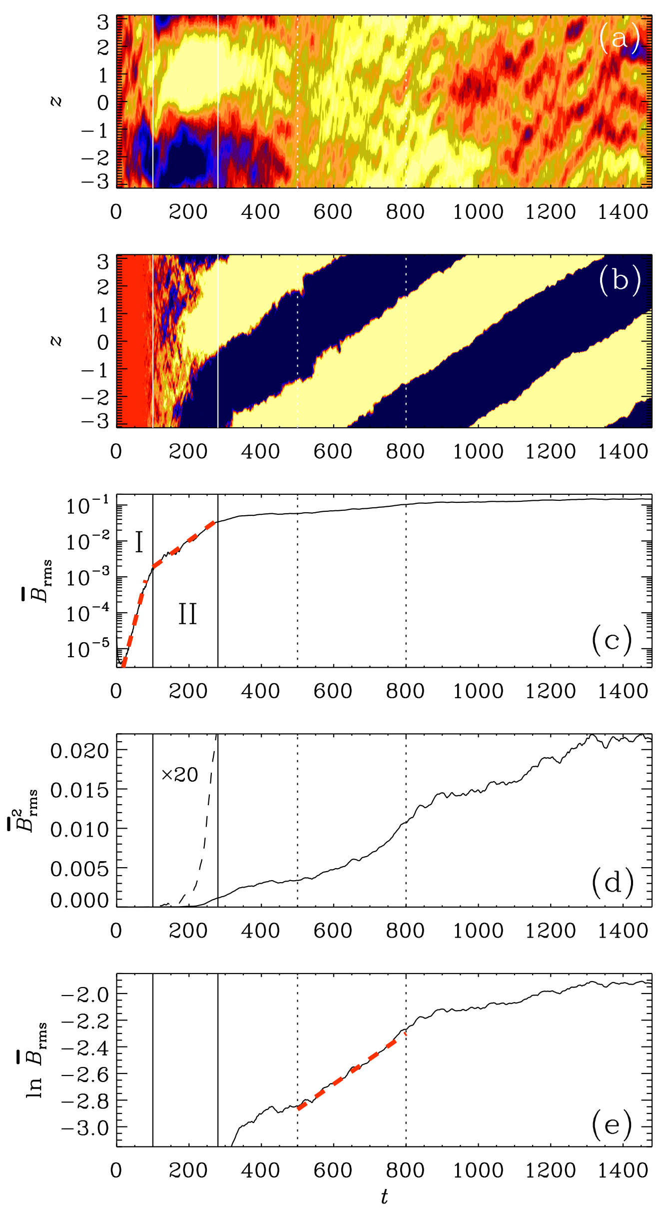

FIG. 1. Comparison of $\overline{u}_y$, $\overline{B}_y$, and three representations of $\overline{\boldsymbol{B}}_{\rm rms}$ for Run A. The dotted line is scaled by $\times 20$.

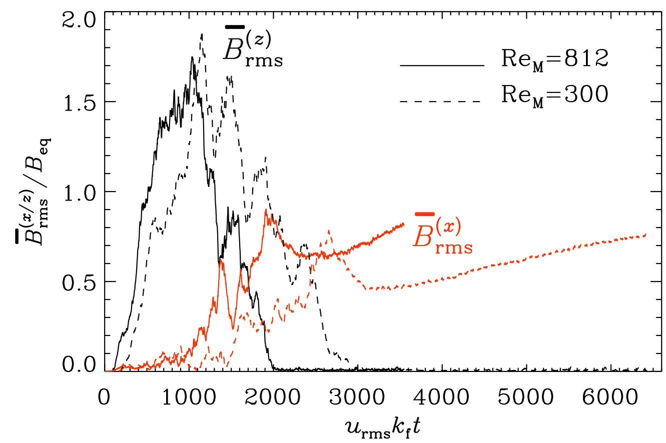

FIG. 2. Comparison of the rms values of the z dependent (xy-averaged, black lines) and x dependent (yz-averaged, red lines) mean fields, $\overline{\boldsymbol{B}}^{(z)}_{\rm rms}$ and $\overline{\boldsymbol{B}}^{(x)}_{\rm rms}$, respectively, for Run A (solid line) and a similar one at $\mathrm{Re_M} = 300$ (dotted lines).

just after the end of phase I, at $t = 100$. This is due to a vorticity dynamo [1] and arises also without magnetic field [2]. Note that, because the large-scale flow varies only along the z direction, the relevant terms in the mean-field dynamo equation basically go to zero and thus this large-scale flow is not responsible for any magnetic field generation. At $t = 400$, the large-scale flow weakens, changes its form, and disappears by $t = 1000$. This suppression is due to the magnetic field [3, 4]. Figure 1(b) shows that a dynamo wave begins to emerge at $t = 100$, but it has initially a larger phase speed than at later times. In Fig. 1(c), we see that there is exponential growth in phases I and II with different growth rates. Figure 1(d) shows that this growth of $\overline{\boldsymbol{B}}^2_{\rm rms}$ does not grow linearly in time. There is also evidence for another period of exponential growth at later times after phase II; see Fig. 1(e). The growth rate however is very slow and this perhaps arises due to a temporary fluctuation during the resistive phase.

As mentioned in the main paper, a fourth stage involves fratricide of this $\alpha\Omega$ dynamo by its α^2 sibling [5]. This can be seen by plotting the x dependent (yz-averaged) mean field, which we denote by $\overline{\boldsymbol{B}}^{(x)}_{\rm rms}$ with superscript (x). The usual z dependent (xy-averaged) mean field is now denoted by $\overline{\boldsymbol{B}}^{(z)}_{\rm rms}$ with superscript (z). This fratricide happens at much later times during $u_{\rm rms}k_{\rm f}t = 1000$–$2000$; see Fig. 2, where the black solid curve of $\overline{\boldsymbol{B}}^{(z)}_{\rm rms}$ shows decay and simultaneously, $\overline{\boldsymbol{B}}^{(x)}_{\rm rms}$ in

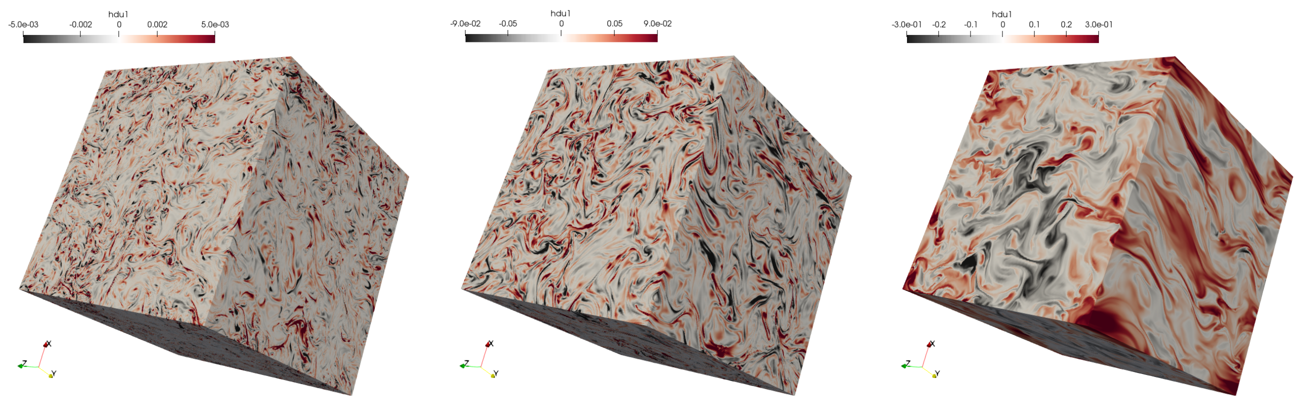

FIG. 3. B_y for Run A at $t = 50$, 200 and 400.

solid red is growing. Importantly, it appears that the phase II is not $\mathrm{Re_M}$ dependent as seen by comparing the near parallel solid and dashed black lines in Fig. 2 between $t = 100$ to 400. Thus, there is no evidence for catastrophic quenching of this phase in this range of $\mathrm{Re_M}$ explored.

Spatial organization of the field

To understand how the magnetic field changes in different stages of growth, we show in Fig. 3 the magnetic field component B_y of Run A at times $t = 50$, 200 and 400. At $t = 50$, the system is in kinematic stage and thus the fields are of small-scale nature. At $t = 200$, the large-scale dynamo (LSD) is active and one finds that the fields have started ordering themselves on larger and larger scales. In the third stage of resistive decay of helicity, the fields become increasingly organized and at $t = 400$, there is a coherent field on the largest scale in the box.

Magnetic helicity evolution in the dynamo with shear and helical forcing

In the main paper we mentioned that the build up of small-scale helicity during the second stage is expected to eventually quench the LSD. Initially, the LSD due to α-effect results in a helical polarization of the field, which is of opposite signs on small and large scales. The Lorentz force associated with this small scale helical field can back react to then quench the α-effect [6–9]. Moreover, the large-scale field growth, which as discussed in the main paper is similar in helical and non-helical runs during the first stage when the SSD dominates, can however be differentiated at later stages by its helicity properties. Thus, it is of interest to examine the magnetic helicity power spectrum $H(k)$ for this standard signature of the LSD and study how it evolves in the first two stages of large-scale field growth discussed in the main paper.

In the kinematic stage, when the SSD is dominant, there is no clear separation between the positively helical and the negatively helical fields as shown by $H(k)$ in Fig. 4 for Run A. However, towards the end of the second stage (the curve at $t = 250$), a clear separation in scales based on helicity develops, i.e., the helicity on smaller k, $k < k_\mathrm{f}$ is one sign represented by blue diamonds and the helicity on larger k is the opposite sign represented by red squares. It is this accumulation of small-scale helicity that could induce a magnetic back reaction to the initial kinetic α-effect and quench the LSD such that the exponential growth of the large-scale field transits to a resistively limited growth in the third stage.

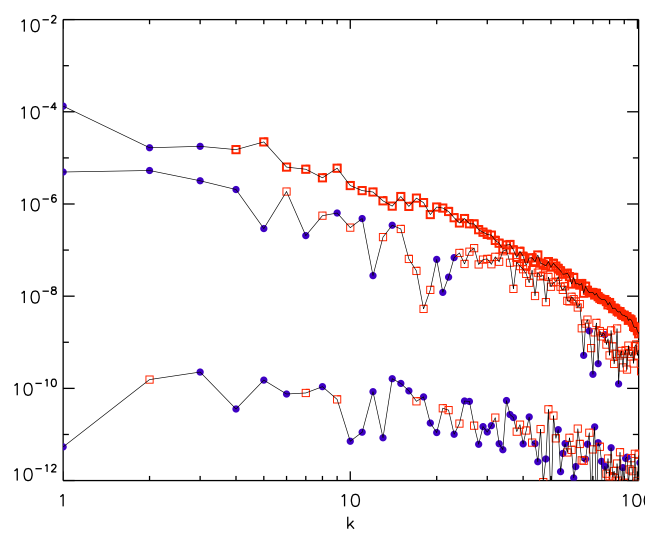

FIG. 4. The magnetic helicity power spectrum is shown at three times, $t = 100, 150$ & 250 for Run A. The blue diamonds represent negative helicity and red squares represent positive helicity.

At late times, when the α^2 dynamo is operating, mag-

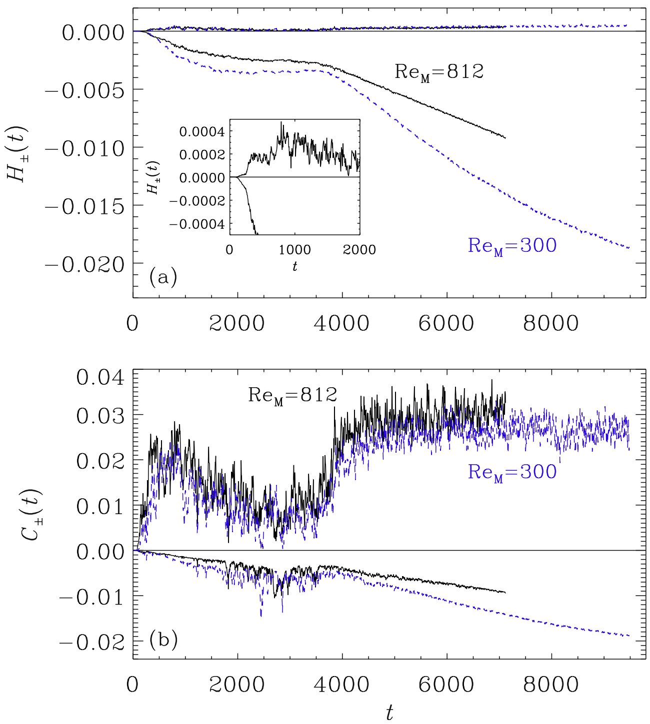

FIG. 5. Evolution of (a) $H_{\pm}(t)$ and (b) $C_{\pm}(t)$ for Run A and a similar run with $\mathrm{Re_M} = 300$. The inset highlights the early growth of $H_{+}(t)$.

netic helicity continues to build up at larger scales; see Fig. 5(a). Here, $H_{\pm}(t) = \int_{\pm} H(k,t)\,dk$, where $\int_{\pm}$ denote the integrals separately for positive and negative arguments, respectively. The gradual build-up of $H_{-}(t)$ happens by dissipating magnetic helicity of positive sign at small scales. The dissipation of $H_{+}(t)$ is proportional to the corresponding current helicity, $C_{+}(t)$, where $C_{\pm}(t) = \int_{\pm} k^2 H(k,t)\,dk$ are the contributions from positive and negative current helicity, respectively. Eventually, C_{+} and C_{-}, begin to cancel each other; see Fig. 5(b). This leads to the asymptotic steady state where the total current helicity goes to zero and the large-scale field goes to the box scale as in [6]. Note this late time behavior only occurs on the long resistive time scales.

* pallavi.bhat@icts.res.in

[1] T. Elperin, N. Kleeorin, and I. Rogachevskii, PRE **68**, 016311 (2003), astro-ph/0301264.
[2] P. J. Käpylä, D. Mitra, and A. Brandenburg, PRE **79**, 016302 (2009), arXiv:0810.0833.
[3] P. J. Käpylä and A. Brandenburg, ApJ **699**, 1059 (2009), arXiv:0810.2298.
[4] C. Guervilly, D. W. Hughes, and C. A. Jones, Journal of Fluid Mechanics **815**, 333 (2017), arXiv:1607.00824 [physics.flu-dyn].
[5] A. Hubbard, M. Rheinhardt, and A. Brandenburg, A&A **535**, A48 (2011), arXiv:1102.2617 [astro-ph.SR].
[6] A. Brandenburg, ApJ **550**, 824 (2001), astro-ph/0006186.
[7] A. Brandenburg, A. Bigazzi, and K. Subramanian, MNRAS **325**, 685 (2001), astro-ph/0011081.
[8] E. G. Blackman and A. Brandenburg, ApJ **579**, 359 (2002), astro-ph/0204497.
[9] A. Brandenburg and K. Subramanian, PhR **417**, 1 (2005a), arXiv:astro-ph/0405052.