Review of "Collision fluctuations of lucky droplets with superdroplets" by Xiang-Yu Li et al.

This manuscript could be considered for publication after a major revision.

This is an interesting study assessing the collision fluctuation of the Monte Carlo algorithm of super-droplet method through a unique approach. Their results suggest that super-droplet method can faithfully capture the behavior of lucky droplets if “jumps” (artificially enhanced coalescence between lucky droplets) do not occur. The condition tested in this study is based on the so-called lucky droplet model and very idealized. Some more future works have to be done to clarify the consequence and relevance of their findings to more realistic simulations in cloud scales, but the results of this study is very insightful and encouraging to the cloud modeling community.

However, very unfortunately, the manuscript is not at all well organized and not clearly written. A lot more elaboration is required to make it into its final form. For example, the notations are not consistent and confusing. The numerical setup is not thoroughly explained.

I have to say I had a hard time reading this manuscript. It is like an early draft not ready for a review. Nevertheless, recognizing that this is a cutting-edge study, I look forward to reading the revision of this manuscript.

## Major Comments

1) [request] Table 1
   The definition of $N_{\mathrm{p/s}}$ is not clear and confusing. The number of droplets in a superdroplet can differ in each superdroplet $i$, and it varies in time. The definition of $N_{\mathrm{p}}$ is also confusing. The total number of droplets varies in time, and $N_{\mathrm{p}} = N_{\mathrm{p/s}} N_{\mathrm{s}}$ does not hold all the time. Please use appropriate notations and symbols throughout the manuscript.

2) [request] P. 8, ll. 133--134
   To reduce the computational cost, Shima et al. (2009) introduced two techniques; multiple coalescence trick and sample reduction trick. Dziekan and Pawlowska (2017) and Unterstrasser et al. (2020) confirmed that these techniques work efficiently. Please explicitly mention that these are not adopted in this study. Note also that when comparing the results with Dziekan and Pawlowska (2017), you have to take this difference into account.

3) [request] P. 8, l. 135, “which pairs of droplets collide.”
   This should be “which pairs of superdroplets collide and coalesce.” It should be also mentioned that all pairs in $\delta x^3$ have the possibility to collide and coalesce.

4) [request] P. 8, ll. 138--139, “To avoid a probability ...”

Is $\delta t$ a fixed constant? Or, do you adjust it adaptively? Please clarify this point.

5) [request] Pp. 8--9, ll. 139--141, Eq. (4)
It is explained that "$N_{\mathrm{p/s}}$ is the largest initial number of droplet per superdroplet $i$ or $j$ (Table 1)". However, this has to be "the largest ~~initial~~ number of droplet per superdroplet $i$ or $j$". Further, the above definition of $N_{\mathrm{p/s}}$ conflicts with that of Table 1. Please resolve this issue.

Please also clarify how $\delta x^3$ is assigned in this study.

Around here or elsewhere, what about explaining explicitly that background droplets do not coalesce each other? It must be informative to the readers.

6) [request] P. 9, Eq. (7)
Please clarify if $N^i_{\mathrm{p/s}}$ is an integer or a real number in this study. In Shima et al. (2009), it is defined as an integer, and they use Eq. (16) in their paper when splitting a superdroplet to guarantee that they remain integeres.

7) [request] P. 10, Sec. 2b. "Numerical setup"
Please explain the numerical setup in more detail. How big is the domain? How many grids do you have in the domain? What is the boundary condition? How the superdroplets are initialized? How do you solve the equation of motion (1)? How big is the time step? What is the difference between 1-D and 3-D superdoplet simulations?

8) [request] P. 11, ll. 196--197, "In 3-D, however, the number density ..."
Please elaborate. I do not understand why there is no fluctuation of number density in 1-D.

9) [request] P. 12, ll. 215--218, "The rate $\lambda_k$ ..."
The explanation here is incorrect and misleading. Please revise it. If I understand correctly, $\lambda_k$ is the coalescence rate that the lucky droplet coalesce with any one background droplet. And the definition of $\lambda_{k1}$ is very unclear; in Eq. (4) $i$ and $j$ are used for superdrplet indices, but $k$ here represents the $k$-th coalescence of the lucky droplet, and the second subscript 1 seems to be representing the background droplet. Therefore, by any means, the statement "$\lambda_k \equiv \lambda_{k1}$" is wrong. Perhaps this is what you mean: Let $N'$ be the number of droplets in $\delta x^3$, then

$$\begin{aligned}\lambda_k &= N'\pi(r_k+r_1)^2|\vec{v}_k-\vec{v}_1|E(r_k,r_1)/\delta x^3\\ &\approx \pi(r_k+r_1)^2|\vec{v}_k-\vec{v}_1|E(r_k,r_1)n.\end{aligned}$$

10) [suggestion] P. 13, ll. 226--227, "The actual time until ..."
It should be informative to point out that the variance is $1/\lambda_k^2$.

11) [request and question] P. 14, Fig. 4

Could you explain how you calculated $P(T)$ of LDM? Is it possible to derive the analytic form? Or, did you plot it numerically?

Is $\langle T \rangle$ equal to $T_{125}^{\mathrm{MFT}}$?

12) [request] P. 17, Eq. (16)

Again, the meaning of the subscript is different in Eq. (16) and in Eq. (2). Please clarify.

13) [question and request] Approaches I--IV

Let me confirm: approach I = LDM; approach II = explicit collision model; approach III = Monte-Carlo model described in Sec. 3e; approach IV = superdroplet method.

In approaches I (LDM) and III (Monte-Carlo 3e), background droplets are not considered explicitly.

In approaches I, II, and III, superdroplets are not used, i.e., all $N_{\mathrm{p/s}}^{i} = 1$.

Are these correct? Please explain these points more clearly in the manuscript.

It seems a tall box domain is used for approach II. Please specify the size. Please also clarify the boundary condition. Was this domain also used for approach IV (superdroplet method)? Or, was some different geometry used for approach IV?

14) [question] P. 17, l. 318, “LDM”

Do you mean “approach III”?

15) [request] P. 18, l.327, “$N_{\mathrm{p/s}} = 1$”

Please clarify. I suppose you set the initial multiplicity of all the background superdroplets and the lucky superdroplet equal to 1, i.e., for all $i$, $N_{\mathrm{p/s}}^{i} = 1$.

16) [request] Caption of Figure 6

The configuration of the superdroplet simulation is partly explained for the first time in the caption, but not in the main text. Please describe all the detailed information necessary to reproduce the result in the main text, such as the domain size, boundary conditions, and time steps.

In the caption, it is explained that the number of superdroplets used for this simulation is $N_{\mathrm{s}} = 256$. I assume that 1 superdroplet is for lucky superdroplet. In the next sentence, it is explained that the mean number density of droplets is $n_0 = 2.28 \times 10^9 \mathrm{m}^{-3}$. This must be the INITIAL mean number density. Is lucky superdroplet included in $n_0$? I have estimated the size of the domain by $\delta x^3 = (255 \text{ or } 256)/n_0 \approx 1.1 \times 10^{-7}\mathrm{m}^3$. Is this correct?

If my interpretation above is correct, and also because the lucky superdroplet has to coalesce $124$ times to reach the size $50\mu\mathrm{m}$, the number density of droplets $n$ will be almost half of $n_0$ at the end of the simulation. I think this is not the situation that you want to simulate.

Please clarify all these points in the main text.

17) [request] Appendix A1 and Fig. 15
The numerical setup tested here is very unclear. Suddenly, $N_{\mathrm{grid}}$ and $N_{\mathrm{d}}(=?N_{\mathrm{p}})$ were introduced without any explanation. Please provide all the details so that the readers can reproduce the results.

Please add “(a)” and “(b)” to Fig. 15. Replace “Figure 7(a)” at the end of the caption of Fig. 15 by “Figure 7”.

18) [comment] P. 19 and the rest of the manuscript
Because the sufficient detail of the simulations conducted are not provided, it is difficult to understand and evaluate the rest of the manuscript accurately.

19) [requet] P. 19, l. 349, “Figure 8 where $N_{\mathrm{p/s}}^{(\mathrm{luck})} = N_{\mathrm{p/s}}^{(\mathrm{back})} = 2$ ...”
First of all, you have to say that the INITIAL CONDITION OF MULTIPLICIY is $N_{\mathrm{p/s}}^{(\mathrm{luck})} = N_{\mathrm{p/s}}^{(\mathrm{back})} = 2$. You may consider it almost obvious, but such a small lack of explanation is piled up high in this manuscript. And, again, the numerical setup is unclear. What is the number of superdroplets used for this test? The same domain size as before? What are the time steps?

20) [question] P. 19, l. 359, “$N_{\mathrm{p}}^{(\mathrm{luck})} = 3$ superdroplets”
Do you mean “droplets”? If I understand correctly, approach III does not use superdroplets.

21) [suggestion] P. 19, Eq. (17)
It is better to give $\lambda_{ij}^{(\mathrm{luck})}$ simply by

$$\lambda_{ij} = \pi(r_i + r_j)^2|\vec{v}_i - \vec{v}_j|/\delta x^3.$$

The newly introduced variable $n_{\mathrm{luck}}$ satisfies $n_{\mathrm{luck}} = \epsilon n/N_{\mathrm{p}}^{(\mathrm{luck})} = 1/\delta x^3$. Further, more importantly, your definition of $n_{\mathrm{luck}}$ is confusing, because it does not correspond to the number density of lucky droplets, $N_{\mathrm{p}}^{(\mathrm{luck})}/\delta x^3$.

22) [request] P. 19, Eq. (18)
The definition of $\epsilon$ is also confusing. It seems to me that $\epsilon$ is defined by the initial ratio of lucky droplets and background droplets, $N_{\mathrm{p}}^{(\mathrm{luck})}(t=0)/N_{\mathrm{p}}^{(\mathrm{back})}(t=0)$. But, if so, we cannot apply this $\epsilon$ to Eq. (17).

If I understand correctly, in approach III (Monte-Carlo 3e), background droplets are not considered explicitly, hence the number density of background droplets is a fixed constant. Further, superdroplet is not used for the lucky droplets in approach III. Then, it is confusing to use $N_{\mathrm{s}}$ in Eq. (18)

Please clarify.

23) [request] P. 19, ll. 364--366, “we used $N_{\mathrm{s}} = 256...$”
This information must be explained much earlier.

24) [question] P. 20, ll. 367--373
In approach III, will you reduce the number of lucky droplets when they coalesce each other?

It is explained that Fig. 9 was produced by the approach II. Is this correct? I do not understand how multiple lucky droplets were introduced to the approach II.

I cannot find any results of approach III with multiple lucky droplets. Where is it?

25) [request] P. 20, ll. 374--385
It seems you suddenly switched the target and started talking about the superdroplet model. Please declare more explicitly which one of the four models you are currently talking about.

26) [request] P. 24, l. 470, “LDM (approaches I, II, and III)”
If I understand correctly, approach I = LDM; approach II = explicit collision model; approach III = Monte-Carlo model described in Sec. 3e. Please use the same definitions throughout the manuscript.

## Minor Comments

27) [suggestion] P. 5, ll. 64--69
Perhaps you can also cite Jaruga and Pawlowska (2018), Sato et al. (2018), Seifert et al. (2019), Shima et al. (2020), and Unterstrasser et al. (2020).

28) [typo] P. 8, l. 137, “$p_{ij} < \eta$” -> “$\eta < p_{ij}$”

29) [question] P. 15, ll. 259--261, “P(T) can be approximated by a lognormal ...”
How good is the approximation?

30) [question] P. 15, l. 271 and Table 2, "$T_k^{\mathrm{MFT}}$"
Do you mean $T_{125}^{\mathrm{MFT}}$?

31) [suggestion] Figure 7
Perhaps you had better label the vertical axis as $P(T/\langle T \rangle)$.

## References


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