The Solar Cycle
ASP Conference Series, Vol. 27, 1992
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TURBULENT PUMPING IN THE SOLAR DYNAMO

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ABSTRACT We investigate a mean field $\alpha\Omega$-type dynamo where differential rotation is generated by nondiffusive contributions to the Reynolds stress. A passive overshoot layer is included beneath the convection zone. Magnetic fields are continuously transported downward by a turbulent pumping mechanism. This can lead to an equatorward migration of magnetic flux, even though $\alpha\partial\Omega/\partial r > 0$ at lower latitudes.

INTRODUCTION

It has long been recognized that the solar dynamo might not operate in the convection zone (CZ) proper, but rather in a thin interface (overshoot layer) between the CZ and the radiative interior. This has led to a number of models in which the thin interface layer is represented by a one-dimensional $\alpha\Omega$-dynamo with only latitudinal extent (Schmitt and Schüssler 1989, Choudhuri 1990, Jennings 1991, but see also Jennings *et al.* 1990).

Direct three-dimensional simulations of dynamo action in a convective layer with overshoot (Brandenburg *et al.* 1991a, Nordlund *et al.* 1992) suggest that a dynamo actually operates in the entire CZ and not only in the interface. The local growth rate is large in all layers in the CZ, but declines rapidly to zero in the lower stably stratified overshoot layer. Moreover, there is a significant transport of magnetic flux downwards from the unstably stratified interior (second panel in Figure 3 in Nordlund *et al.*), which causes accumulation of the magnetic energy at the interface (first panel in Figure 3 in Nordlund *et al.*). In this sense the dynamo might appear to be located at the interface, although dynamo action occurs all the way through the CZ.

With these results in mind we consider here an $\alpha\Omega$-type dynamo including, however, a parametrization of the effects of turbulent pumping of magnetic field into the overshoot layer from the overlying CZ.

THE MODEL

The interface between the CZ and the radiative interior has the property that the effective (turbulent) magnetic diffusivity rapidly falls to zero there. As shown by Roberts and Stix (1972), this has the effect of preventing an oscillatory magnetic field from significantly penetrating into the radiative interior due to the skin effect.

The model presented here is an extension of axisymmetric $\alpha\Lambda$-dynamos presented earlier (Brandenburg *et al.* 1990, 1991b). In addition to the induction equation for the mean magnetic field $\langle B \rangle$ we also solve the angular momentum equation for the angular velocity Ω to describe the generation of differential rotation by the Reynolds stresses, which is represented by the Λ-effect (e.g. Rüdiger 1989). We concentrate here on models where the meridional motions and density stratification are omitted, but we also present a result with both the equations for the meridional flow and the pressure included. The main modifications to our previous work are that we now take depth-dependent profiles for the turbulent magnetic diffusivity η_t and the turbulent kinematic viscosity ν_t, in such a way that below the interface η_t and ν_t decrease smoothly to values about 10-20 times smaller than in the CZ. (Larger contrasts lead to a significant separation of time scales which is prohibitive in terms of computer time.) For the α- and Λ-effects we assume similar profiles which, however, go to zero in the radiative interior. The turbulent pumping effect is assumed to give an additional contribution $-\gamma_0 g(r)\hat{\mathbf{r}}$ to the radial velocity component, where $\gamma_0 > 0$ and $g(r)$ is a smooth profile normalized to a maximum value of unity in the lower half of the CZ ($0.7 < r < 0.85$) and vanishing elsewhere.

The coefficients for the Λ-effect are chosen such that the equatorial angular velocity increases outwards, ie $\partial\Omega/\partial r > 0$ for latitude $0 - 30°$ and Ω decreases outwards in higher latitudes, ie $\partial\Omega/\partial r < 0$ for latitude $30 - 90°$. This general behaviour is suggested from helioseismology (eg, Libbrecht 1988) and has recently also been used in dynamo models (Belvedere *et al.* 1991). We use the following coefficients for the Λ-effect (for definition see Paper I): $V^{(0)} = -1$, $V^{(1)} = H^{(1)} = 5/4$; these values were suggested by Rüdiger and Tuominen (1990).

For the turbulent magnetic diffusivity we assume $\eta_t = \frac{1}{3}u_t\ell \approx 10^{13}\mathrm{cm}^2/\mathrm{s}$, where u_t is the rms-velocity of the turbulent motions, and ℓ is the density scale height. In the models presented below we take $\ell = 0.04R$ (corresponding to 30 Mm for the Sun), so that $u_t \approx 3\eta_t/\ell$ is about 100 m/s. The convective turnover time $\tau = \ell/u_t$ is then about three days. We adopt the traditional estimate $\alpha \approx \Omega\ell\cos\theta$ (eg, Krause and Rädler 1980), where θ is colatitude. As pointed out by Parker (1979) and Zeldovich *et al.* (1983), α saturates for rapid rotation if α approaches u_t. This is taken into account by replacing α by $u_t\tanh(\alpha/u_t)$. The magnetic field strength is limited by nonlinear feedbacks, both from the angular velocity and from a quenching of the α-effect. This α-quenching gives rise to an additional multiplicative factor in α, which is taken to be of the form $1/(1 + \alpha_B\langle B \rangle^2)$, where $\alpha_B = (\mu_0\rho u_t^2)^{-1}$. We emphasize that, in all cases considered below, $\alpha > 0$ in the northern hemisphere.

As discussed in Paper I, cylindrical Ω-contours occur for large Taylor numbers (greater than about 10^4-10^5). Since this is incompatible with the solar Ω-contours we simply take Ta$=10^4$, which corresponds to a turbulent magnetic Prandtl number $\mathrm{Pr}_M = \nu_t/\eta_t$ of about 30. This value, which significantly deviates from unity, is somewhat unsatisfactory. However, it is perhaps not too surprising if the assumption of an isotropic turbulent viscosity is inappropriate. This "Taylor number puzzle" is one of the current problems of solar mean field dynamos and deserves a separate investigation.

RESULTS

Synthetic butterfly diagrams for the poloidal and toroidal magnetic fields are computed by measuring the B_r and B_ϕ components immediately below the surface. In the absence of a turbulent transport term ($\gamma_0 = 0$) there is no significant dynamo wave migration, see Figure 1. Note that the B_r field is strongest at the poles. This is partly because α is also strongest there.

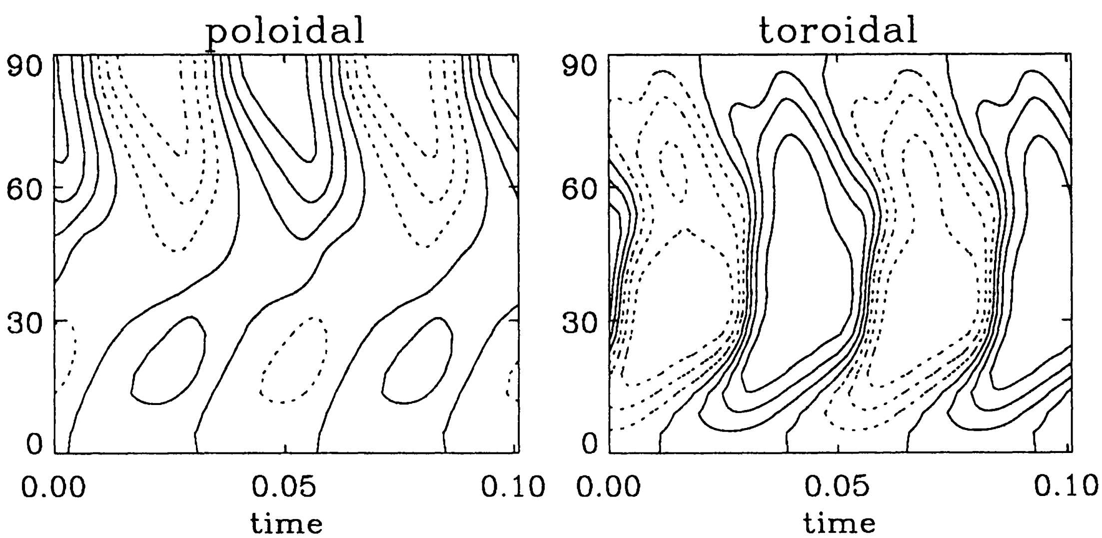

Fig. 1. Butterfly diagrams for the poloidal (left) and toroidal (right) magnetic fields for a model with $\gamma_0 = 0$. Time is measured in magnetic diffusion times.

The inclusion of a modest turbulent pumping, $\gamma_0 = 20\eta_t/R = 0.3u_t$, results in a poleward migration in lower latitudes ($< 30°$) and an equatorward migration in higher latitudes – exactly the opposite to that in the Sun, see Figure 2. This is, in principle, not so surprising since it is known that the sign of $\alpha\partial\Omega/\partial r$ determines to some extent the migration direction (e.g., Parker 1979). The role of the turbulent pumping seems to emphasize this.

A stronger turbulent pumping $\gamma_0 = 0.6u_t$ leads to a more dominant equatorward migration, which completely removes the poleward migration feature close to the equator, that was stronger seen in the previous plot, see Figure 3. With significantly stronger pumping the dynamo becomes steady. As γ_0 increases from zero to a value of about u_t, the contours of the butterfly diagram change from being vertical (oscillatory field with no migration) to horizontal (steady fields).

The pumping mechanism leads to an accumulation of toroidal field immediately below the convection zone. The poloidal field lines rise almost

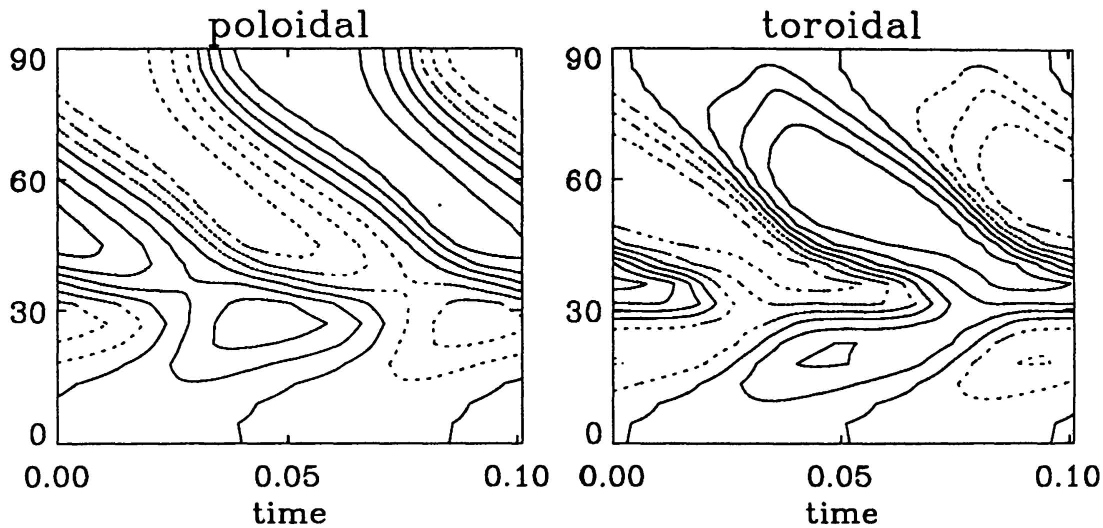

Fig. 2. Butterfly diagrams for the poloidal (left) and toroidal (right) magnetic fields for a model with $\gamma_0 = 0.3u_t$.

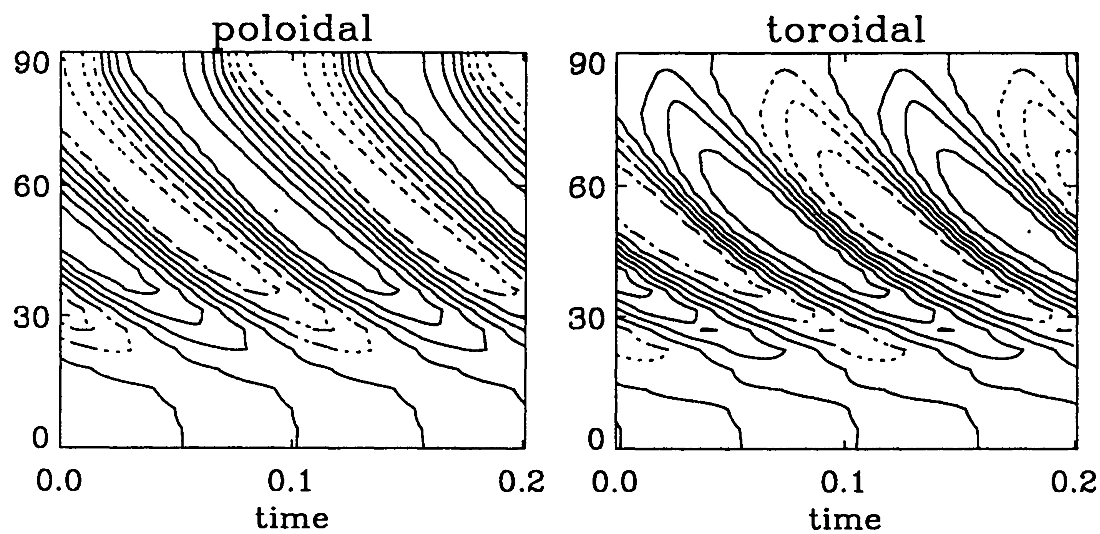

Fig. 3. Butterfly diagrams for the poloidal (left) and toroidal (right) magnetic fields for a model with $\gamma_0 = 0.6u_t$. Note that the time axis covers a range twice as long as in Figs. 1 and 2.

vertically through the CZ. Meridional cross sections of poloidal and toroidal field are shown in Figure 4.

We also performed some calculations without an overshoot layer. In this case similar butterfly diagrams as for the case with overshoot are obtained. However, the model without overshoot is more sensitive to the γ_0 parameter. A value around 40 leads to steady solutions without migration. For $\gamma_0 = 30$ we found butterfly diagrams roughly comparable with the case with overshoot and

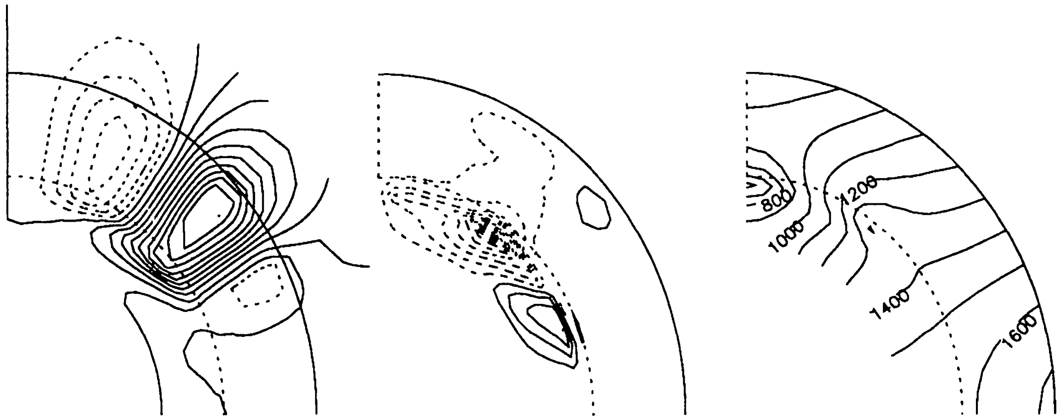

Fig. 4. Meridional cross sections of poloidal field lines (left), and contours of toroidal field (middle) and of angular velocity Ω for the model of Fig. 3. Dotted contours denote the opposite field orientation. Note the strong concentration of toroidal magnetic field at the bottom of the convection zone. The Ω contours are somewhat affected by the magnetic field, especially close to the poles.

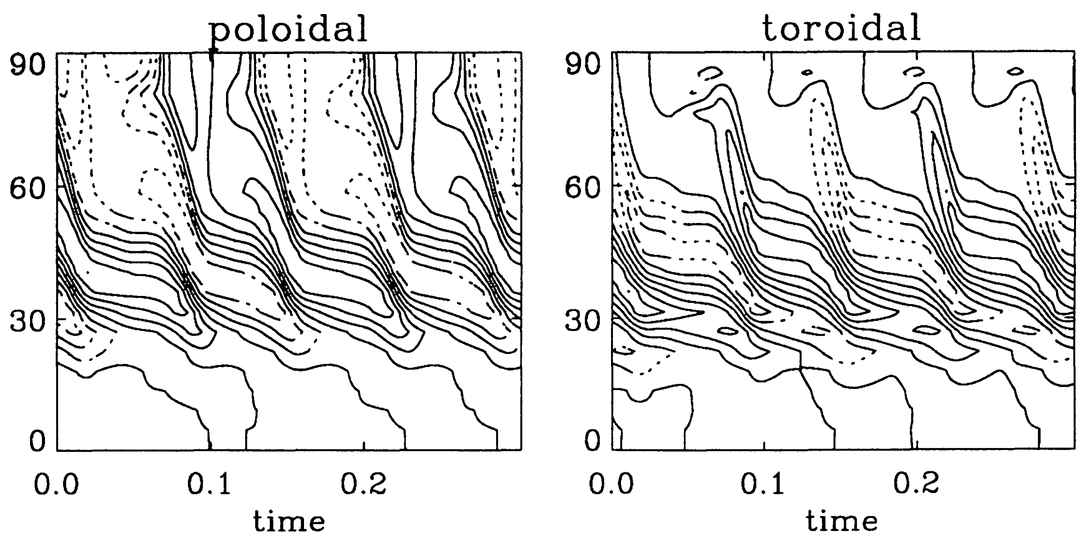

Fig. 5. Butterfly diagrams for the poloidal (left) and toroidal (right) magnetic fields for a model with $\gamma_0 = 0.6u_t$. Meridional motions and density stratification are included, but the overshoot layer is omitted.

$\gamma_0 = 40$. This is perhaps explainable by the resulting build up of field near the base of the CZ.

Finally, we present in Figure 5 the result where both meridional motions and a density stratification are included. Here we have neglected the overshoot layer, because otherwise a more careful treatment of the thermodynamics is required to avoid the circulation linking the convective and radiative regions. Again, we used $\gamma_0 = 0.6u_t$. There is still a poleward migration, but more quiescent-phases where migration is very slow also seem to occur.

DISCUSSION

Solar $\alpha\Omega$-type dynamos have been located either in the convection zone or at the interface between the convection zone and the radiative interior. Direct simulations of dynamo action have indicated that the role of the bulk of the convection zone as a major generator of magnetic fields cannot be neglected. Magnetic fields can be continuously pumped from the convection zone into the overshoot layer beneath it. The model presented here demonstrates that this can have important effects on the migration of dynamo waves. The usual criterion based on the sign of $\alpha\partial\Omega/\partial r$ (eg, Parker 1979) may not always apply in more realistic situations where turbulent transport is taken into account.

What is still missing is an explanation of the occurrence of sunspots at lower latitudes. There are strong magnetic fields also at higher latitudes, and their signatures are in fact observed in the Sun (e.g., Makarov, these proceedings). The distinct appearance of sunspots at low latitudes compared with the different nature of poloidal field indicators at high latitudes may be caused by different mechanisms transporting poloidal and toroidal fields. Recent investigations of Kichatinov (1991) indicate that the turbulent transport may be highly anisotropic. It would be desirable to confirm such detailed physics by means of direct simulations. The hope is that among the almost infinite amount of possible effects only a few are really important and that they will allow a reasonable description of solar and stellar dynamos.

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