Magnetohydrodynamics of Accretion Disks

An ACCRETION DISK is a flat formation of gas and dust rotating about a central object and accreting matter inwards by transporting angular momentum outwards, so that the centrifugal support is gradually removed from the fluid parcels. There are three main classes of accretion disks: (i) disks around accreting compact stars (white dwarfs, neutron stars or black holes) in binary systems, (ii) disks around protostars and (iii) disks around active galactic nuclei. In several cases the existence of disks has been verified observationally with the Hubble Space Telescope. Images can be found at http://www.stsci.edu/pubinfo/pictures.html. An example of an image of a disk around a young stellar object together with an associated jet is given in figure 1.

Accretion disks form because matter generally has angular momentum and therefore cannot fall directly onto the central object. In the disk midplane the gravitational acceleration, GM/R^2, at distance R from the central object of mass M is balanced mainly by the centrifugal acceleration, $\Omega^2 R$. Here G is the gravitational constant and Ω is the angular velocity. Except in a few systems the disk mass is negligible and so this balance leads to the rotation law

$$\Omega(R) = (GM/R^3)^{1/2} \tag{1}$$

which is essentially Kepler's third law. In the vertical direction, i.e. parallel to the rotation axis, gravity is balanced by a pressure gradient, so that the mass is concentrated towards the midplane. There is usually some energy dissipation caused by turbulence and magnetic fields. The heat released during this process can be radiated away at the two disk surfaces. The corresponding removal of rotational energy causes matter gradually to spiral towards the central object. Large-scale fields and winds from the disk surfaces can also remove angular momentum from the disk, and would thus contribute to the accretion mechanism directly.

Accretion disks around young stellar objects are only transient phenomena because they provide essentially just a waiting queue for matter before it can fall onto the central object. The disk will disappear once all the available material has been consumed, which could be after about 10^7 yr in the case of protostellar disks.

Energy conversion in disks

The kinetic and potential energy of accreted matter in the disk is constantly being converted into heat and then radiation by dissipative processes. In disks around neutron stars or white dwarfs the luminosity of the disk can by far exceed the luminosity of the central object. The total orbital energy per unit mass is $\frac{1}{2}(R\Omega)^2 - GM/R = -\frac{1}{2}GM/R$. The mass accretion rate $\dot{M}$ is controlled by the rate of dissipation, giving a disk luminosity of

$$L = \frac{GM\dot{M}}{2R}. \tag{2}$$

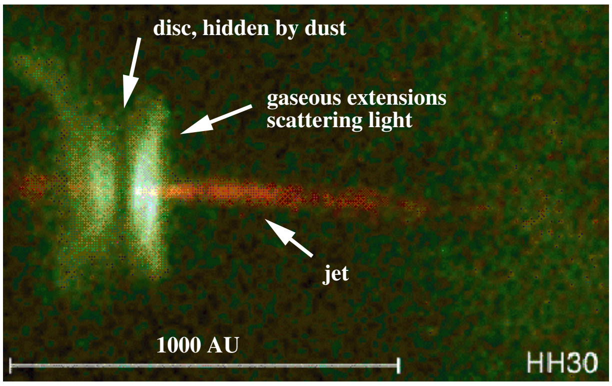

Figure 1. Image of a disk around a young stellar object together with a jet emanating from the disk center along the rotation axis. The length of the horizontal bar is 1.5×10^{11} km. (Adapted from Burrows *et al* (1996).)

This formula is essentially independent of the efficiency of the mechanism that accomplishes the energy conversion. If the central object is a white dwarf or a neutron star there will be a boundary layer at the star's surface, where as much energy can be dissipated as in the disk itself.

Equation (2) assumes that all the heat is radiated away instantaneously. In recent years it has been found that in some disks the luminosity is much lower, because significant amounts of energy can be advected radially towards the center. If the central object is a black hole the advected energy may never appear as radiation.

In the case of a black hole at the center the disk can extend down to three Schwarzschild radii, i.e. $r_{\rm min} = 6GM/c^2$, or less in the case of rotating black holes (c is the speed of light). The energy that is released by an accreted mass m is $\frac{1}{2}GMm/r_{\rm min}$. Using for $r_{\rm min}$ the expression above, this becomes ηmc^2, where $\eta \approx 0.1$ is an efficiency factor relative to the maximum possible value permitted by Einstein's famous formula $E = mc^2$. Note that the efficiency of hydrogen fusion in stars is only $\eta = 0.007$. For this reason the rate of energy release by accretion, $\eta\dot{M}c^2$, of a disk around a supermassive black hole of 10^8 solar masses with $\dot{M}$ of a few solar masses per year can be as large as 10^{40} W. This is believed to be the mechanism that powers quasars.

The role of magnetic fields

In the absence of turbulence a purely laminar shear motion would be totally insufficient to explain the dissipation and corresponding heat release of real disks. Thus, turbulence is necessary to produce small enough scales where microscopic viscosity and ohmic diffusion can act to dissipate kinetic and magnetic energy into heat.

It has been a long-standing debate as to what causes turbulence in disks. In the absence of magnetic fields differentially rotating disks are unstable when the specific angular momentum, ΩR^2, increases inwards (Rayleigh's criterion). Indeed, purely hydrodynamic mechanisms such as (nonlinear) instabilities and convection have proved unsuccessful so far. Numerical work and theoretical arguments indicate that nonlinear instabilities do not operate in astrophysical disks and that convection produces accretion torques that are probably even of the wrong sign; furthermore, this process would not be self-sustained. For a comprehensive review of those issues see Balbus and Hawley (1998). The issue of purely hydrodynamical instabilities is not fully settled, however, because this would require direct simulations at Reynolds numbers of at least 10 000, which cannot be achieved with present computers.

In the presence of magnetic fields, however, there is a powerful linear instability, the magneto-rotational instability, which prevents the gas flow in the disk from being laminar. In 1991 Balbus and Hawley pointed out its importance in driving turbulence in accretion disks. Their papers have spawned a lot of work attempting to quantify the properties of the resulting turbulence and the related angular momentum transport. The magneto-rotational (or Balbus–Hawley) instability is a consequence of the shear, which causes a destabilization of the slow magnetosonic waves. The instability exists regardless of the orientation of the magnetic field. In the special case of a magnetic field parallel to the rotation axis the instability is axisymmetric.

In the absence of magnetic fields, or when the electrical conductivity is too low to make the magnetic field important, the disk would be purely hydrodynamic, in which case rotation would have a strongly stabilizing effect. Still, variation of the angular velocity in the vertical direction would lead to a linear hydrodynamic instability, but its growth rate is much smaller than that of the magneto-rotational instability. The vertical shear instability could be important if the conductivity is low.

The magneto-rotational instability

The magneto-rotational instability exists already in an incompressible, unstratified fluid that is differentially rotating provided that the shear parameter

$$q \equiv -\partial \ln \Omega / \partial \ln R \tag{3}$$

is positive. For thin accretion disks we have $q = +3/2$; see equation (1). In that case the dispersion relation is

$$\omega^4 - \omega^2(2v_A^2k^2 + \Omega^2) + v_A^2k^2(v_A^2k^2 - 3\Omega^2) = 0 \tag{4}$$

where $v_A = B/(\mu_0\rho)^{1/2}$ is the Alfvén speed, B is the vertical field strength, ρ is the unperturbed density, μ_0 is the permeability, k is the wavenumber along the magnetic field and ω is the frequency. There are two solutions for ω^2, an upper branch corresponding to Alfvén waves and a lower branch corresponding to slow magnetosonic waves. The fast magnetosonic waves have been filtered out by the assumption of incompressibility. On the lower branch ω^2 becomes negative when

$$v_A < \sqrt{3}\Omega/k. \tag{5}$$

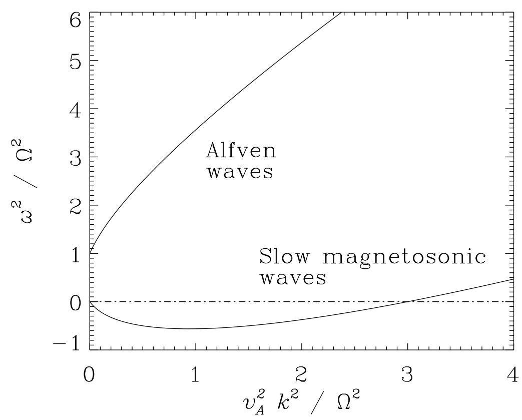

Figure 2. Dispersion relation for slow magnetosonic and Alfvén waves in an incompressible medium. For Alfvén waves ω^2 is always positive. However, for not too large values of k the slow magnetosonic waves become unstable when $v_A^2k^2 < 3\Omega^2$, so $\omega^2 < 0$, that is ω is imaginary.

In that case ω is purely imaginary and small perturbations grow exponentially with a maximum growth rate

$$\max(\mathrm{Im}\,\omega) = \tfrac{3}{4}\Omega \tag{6}$$

at scale $\ell = 2\pi/k_{max}$, where

$$k_{max} \approx \Omega/v_A. \tag{7}$$

It is clear from equation (5) that the instability works only if the field is not too strong. The largest possible field strength depends on the smallest admissible k, i.e. the largest scale available to the system. One such scale would be the disk height.

The disk is formally unstable even in the limit of vanishing magnetic field strength, $v_A \to 0$. However, maximum growth would then occur for perturbations whose scale would become progressively smaller, as given by equation (7). At larger scales the growth of the instability would be so slow that viscous effects would render the instability irrelevant for driving hydromagnetic turbulence.

In figure 2 we have plotted the two branches of the dispersion relation, equation (4), corresponding to slow magnetosonic and Alfvén waves. There are also the fast magnetosonic waves that result if the assumption of incompressibility is relaxed (see Balbus and Hawley 1998). However, when the sound speed c_s is much larger than v_A, the two lower branches are nearly independent of the neglect of the fast magnetosonic branch.

There is a mechanical analogue to the magneto-rotational instability, which can be helpful in understanding the nature of the instability. The following example applies to the nonaxisymmetric case that is relevant in the presence of a toroidal magnetic field. Consider two particles, A and B, on a gravitationally bound orbit around a central object (figure 3). Assume that the two particles have the same distance from the central object but

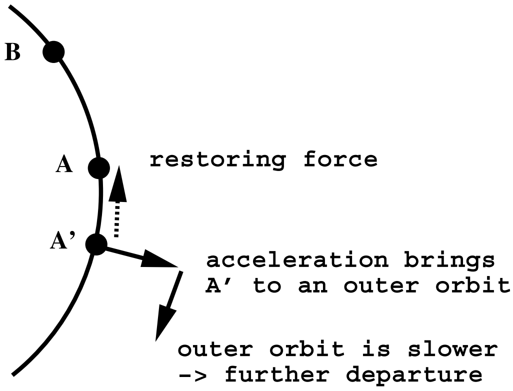

Figure 3. Mechanical analogue of the magneto-rotational instability.

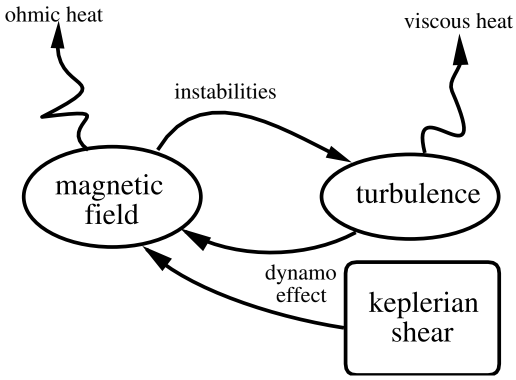

Figure 4. The energy in the Keplerian shear motion is converted into thermal energy via ohmic and viscous heating. Magnetic fields are needed as a catalyst to keep the system turbulent.

are separated azimuthally by a small amount. Interesting behavior occurs when the two particles are coupled elastically. Assume that the particle at position A is perturbed to position A′. The separation between the two particles is then increased and there is a restoring force trying to return the particle to its original position. This accelerates the particle from A′ back towards A. However, this leads to a gain in its angular momentum which then forces the particle onto an outer orbit. There, however, the orbital speed is slower, see equation (1), so the particle will separate from its original position A even further. This is the cause of an instability.

There are some parallels with the phenomenon of tidal disruption of a star passing near a black hole. In that example the restoring force is the gravitational attraction that holds the star together. In the case of the magneto-rotational instability the restoring force is the magnetic tension force. If the field is too strong, however, the fluid parcels stay in their original position, suppressing the instability.

If the conductivity in the disk is poor, there will be significant slippage between the field and the fluid. The magneto-rotational instability ceases when the collision frequency of neutral atoms with ions becomes less than the rotational frequency. This can happen in some parts of protostellar disks which have temperatures below 1000 K. At those temperatures the degree of ionization is very low and the fraction of charge carriers can be as small as 10^{-10}. In the protosolar nebula, out of which the solar system was formed, this may have been the case in a broad ring near the Earth's orbit.

Intrinsic magnetism of disks

It has long been suspected and now been confirmed by local simulations of rotating shear flow that the resulting turbulence is capable of maintaining the field by DYNAMO action. (See Schramkowski and Torkelsson (1996) for a review.) Dynamo action is a process by which kinetic energy can be converted into magnetic energy. In disks the energy comes from the kinetic energy in the shear. Most of it goes into the magnetic field which, in turn, keeps the instability going, thereby feeding the turbulence. All the energy that is removed from the shear is constantly being dissipated (see figure 4). In that sense the magnetic field acts effectively like a catalyst that enables kinetic energy to be tapped from the shear motion and then to be released as heat and radiation via viscous and ohmic heating.

The magnetic fields can contribute directly to exerting a torque that leads to an accretion flow and to angular momentum transport outwards. Any radial field component, B_R, will be sheared out by the differential rotation. This enhances the azimuthal field component, B_ϕ. The resulting torque, which is proportional to the product $B_R B_\phi$, is such that there is an inward mass flow in the disk and an outward transport of angular momentum. If the field was perfectly frozen into the gas (no magnetic diffusion) the field lines would eventually be parallel to the flow and would have no effect.

The strength of the dynamo-supported magnetic field is limited by various nonlinear feedbacks. On the one hand, when the field is too strong the magneto-rotational instability is suppressed, see equation (5), but there are other mechanisms, such as magnetic buoyancy, which can also limit the field strength.

A nondimensional measure for the total stress is

$$\alpha \equiv \langle \rho u_R u_\phi - B_R B_\phi/\mu_0 \rangle / \langle p \rangle \quad (8)$$

which is found from numerical simulations to be typically between 0.01 and 0.1. Here, $\boldsymbol{u}$ is the velocity, and p is the pressure. The precise value of α depends on the magnetic field strength, which may vary with time. Furthermore, since the gas pressure drops faster with height than the magnetic field, α can increase away from the midplane; see equation (8). However, there are at present no global simulations that include the region far outside the disk, where the field must eventually fall off.

Knowing the value of α the radial disk structure can be calculated in closed form, by neglecting the vertical disk structure. In terms of the accretion rate $\dot{M}$ the vertically integrated disk density, $\Sigma = \int \rho \, dz$, is given as a function of radius by

$$\Sigma \approx 2000\alpha_{-2}^{-4/5}\dot{M}_{13}^{7/10}M_1^{1/4}R_8^{-3/4} \text{ kg m}^{-2} \quad (9)$$

where $\alpha_{-2} = \alpha/10^{-2}$, $\dot{M}_{13} = \dot{M}/10^{13}$ kg s^{-1}, M_1 is the central mass in solar masses and $R_8 = R/10^8$ m (e.g. Campbell 1997).

The disk structure is not always steady. In fact, in some parameter regimes, for example where ionization and recombination become important, the disk can become viscously unstable and undergo limit cycle oscillations (CATACLYSMIC BINARIES). Similar phenomena are also known to occur in disks around young stars. In that case the steady solution would still be described by an equation similar to equation (9), but with different coefficient and exponents, because of the different radiative processes involved.

External magnetic fields

External magnetic fields are maintained by currents outside the disk, for example by currents in the central object (typically a neutron star or a white dwarf, but not a black hole), or in the environment in which the disk is embedded (molecular cloud or host galaxy).

If the magnetic field comes from a star at the center, angular momentum can be transferred to the star from those parts of the disk whose local angular velocity exceeds the stellar angular velocity. This is the case inside the corotation radius, i.e. the radius where the angular velocity of the disk coincides with that of the star. If the part of the disk inside the corotation radius contains sufficient angular momentum, this process can lead to a noticeable spin-up of the central star itself. Otherwise, if the disk does not extend sufficiently far inside the corotation radius most of the field lines of the star couple with the slowly rotating outer parts of the disk which then leads to spin-down of the star. It is often difficult to say whether spin-up or spin-down will occur; this depends on the field strength and field geometry that results from the presence of a central star with a magnetic field, all of which affect the precise location of the inner radius of the disk. Indeed, there are stars where spin-up and spin-down phases are observed to alternate on a timescale of months and years. This could be explained by changes of the location of the inner edge of the disk.

The star's magnetic field increases sharply towards the star (as r^{-3} for a dipole field) and the strong field in the inner parts causes the disk to disrupt. The precise disruption mechanism and hence the precise location of the inner disk radius are still controversial. Possible mechanisms include a viscous instability of the disk (Campbell 1997) or simply a loss of hydrostatic equilibrium. Somewhere near that radius the field will also become too strong for the magneto-rotational instability to operate; see equation (5). Near the inner edge of the disk gas is thought to be channelled along the stellar magnetic field lines towards the central star.

Outflows and jets

The outer layers, away from the disk midplane, are probably heated by MAGNETIC RECONNECTION (Joule dissipation). This plausibly leads to the formation of a hot CORONA. As in the Sun, this layer can then no longer be in hydrostatic equilibrium and therefore some gas must be blown off continuously in the form of a wind. Large-scale magnetic fields can also directly contribute to accelerating outflows. This may be possible if there is a large-scale poloidal field tilted away from the rotation axis by at least 30°. In that case the component of the centrifugal force along the field will dominate over the corresponding component of the gravity force and so gas can be driven along field lines away from the disk.

At larger distances from the disk surface these outflows are seen to be strongly collimated towards the rotation axis. There is at present no clear consensus as to what causes this collimation into jets (see ASTROPHYSICAL JETS). Perhaps the most plausible mechanism is based on magnetic forces, in particular the radially inward pointing component of the Lorentz force (i.e. the hoop stress), which results from the presence of a strong toroidal field. This toroidal field is partly advected from the disk by a slow wind and partly generated just outside the jet by shearing the poloidal field. Today most models assume an externally maintained magnetic field. It is still unclear whether dynamo-generated magnetic fields can be responsible for the launching and collimation of a jet. Other topics of current research concern the variability and knottedness of jets.

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