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Structural stability of axisymmetric dynamo models

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Abstract. We examine the stability of the dynamical behaviour of axisymmetric $\alpha^2\omega$ dynamo models in rotating spherical shells as well as in spheres. Overall, our results show that the spherical dynamo models are more stable in the following senses: spherical models (i) do not seem to allow chaotic behaviour and (ii) are robust with respect to changes in the functional form of α. On the other hand, spherical shell models (i) are capable of producing chaotic behaviour for certain ranges of parameter values and (ii) possess, in the combined "space" of parameters and boundary conditions, regions of complicated behaviours, in the sense that there are regimes in which small changes in either the dynamo parameters or the boundary conditions can drastically change the qualitative behaviour of the model. Finally, we discuss briefly the physical relevance of our results.

Key words: Sun: magnetic fields – stars: magnetic fields – MHD – instabilities – chaotic phenomena – Sun: magnetic fields

1. Introduction

A great deal of effort has gone into the understanding of the solar and stellar dynamos in terms of mean field dynamo models in both complete spheres and in spherical shells. In particular, a number of detailed studies have been made of the properties of mean field dynamos in which the nonlinearity is introduced through the so called α-quenching mechanism (see for example, Brandenburg et al. 1989a,b; Moss et al. 1991). These studies have revealed novel features, such as solutions with mixed parities, seen in spherical $\alpha^2\omega$ dynamo models. Such results are of particular interest because they can suggest observations which could in turn enhance or diminish our faith in the models employed. An important shortcoming of these models is that they invariably involve approximations. These fall into two categories: (i) those involving convenience, as for example is the case with imposition of exact symmetries which allow the models to be formulated in terms of two spatial dimensions only, and (ii) those involving ignorance, such as those that arise from uncertainties regarding the precise nature of turbulent processes that operate in dynamo active regions. For example, in standard mean field theories, the procedures for parameterizing the second order correlations (those of higher order being usually ignored in such treatments) between the fluctuating fields $\boldsymbol{u}'$ and $\boldsymbol{B}'$ are bound to be complicated and not well-determined in the solar and stellar settings, where the turbulence is highly inhomogeneous and anisotropic. In addition, solving the model equations involves the specification of the boundary conditions which are unlikely to be known precisely. Now, given that such simplifications can never be truly justified, the question arises as to the extent to which the dynamical behaviour of such models is stable with respect to slight changes in various features of the models that are not well known.

There are two reasons why this is of relevance. Firstly, there are important results from nonlinear dynamical systems theory which indicate that the appropriate theoretical framework within which mathematical models are constructed and observational data are analysed might be that of structural fragility (Smale 1966; Tavakol & Ellis 1988; Coley & Tavakol 1992). Put briefly, this amounts to the possibility that small changes in the models under consideration could produce qualitatively important changes in their behaviours. More precisely, these results show that the set of all structurally stable systems is not everywhere dense in the space of all dynamical systems. As a result the stability of dynamical systems cannot be assumed *a priori*, but need to be established concretely in each case under study and subject to the specific perturbations under consideration. Secondly, small changes in the systems (or their parameters) can radically change the basins of attraction of their underlying attractors and thus their qualitative behaviour. Consequently, the dependence on boundary conditions can be sensitive to small changes in the models or their control parameters.

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Clearly such fragility, if present in real dynamo regimes (and their corresponding realistic mathematical models), could have important observational and (theoretical) interpretational consequences in that it may allow diverse modes of behaviour within a population of stars of similar type, say, to be understood within the same theoretical setting, without the need to invoke other mechanisms. On the other hand the absence of observational evidence in real dynamo regimes for a particular type of fragility exhibited by the models under consideration (with respect to plausible perturbations) would indicate the unsuitability of the approximations and/or the parameterizations employed in such models. Thus, the presence or absence of similar types of fragility in both the models and in real dynamo settings would enhance our trust in the approximations and parameterizations employed.

Here, as a start, we make a preliminary comparative study of the behaviour of the axisymmetric $\alpha^2\omega$ dynamo models, with respect to changes in the value of the dynamo parameter C_α, different $\alpha(\boldsymbol{B})$-profiles and the inner boundary conditions in the case of the spherical shell dynamos, and with respect to the value of C_α and different functional forms of $\alpha(\boldsymbol{B})$ for the spherical case.

The structure of the paper is as follows. In Sect. 2 we briefly introduce the axisymmetric mean field dynamos, Sect. 3 contains our results concerning the stability of spherical dynamos, and in Sect. 4 we present our results concerning spherical shell dynamos. In Sect. 5 we draw some brief conclusions.

2. Axisymmetric mean field dynamos

The standard mean field dynamo equation (cf. Krause & Rädler 1980) is of the form

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B}) - \nabla \times (\eta_t \nabla \times \boldsymbol{B}), \qquad (1)$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the mean velocity and the mean magnetic field. The quantities α (giving rise to the α effect) and η_t (the turbulent magnetic diffusivity) appear in the process of parameterization of the second order correlations $\langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$ between the fluctuations $\boldsymbol{u}'$ and $\boldsymbol{B}'$ by

$$\langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle = \alpha \boldsymbol{B} - \eta_t \nabla \times \boldsymbol{B}. \qquad (2)$$

In the following, as in a large number of such studies, we shall confine ourselves to the isotropic case where both α and η_t may be treated as scalar functions depending only upon $\boldsymbol{B}$. What is usually done in practice in the study of $\alpha^2\omega$ models is to take a prescribed nonlinear functional form for $\alpha(\boldsymbol{B})$ and to study the dynamical consequences of the resulting nonlinear dynamo model as a function of the dynamo parameters subject to some boundary conditions. The problem, however, is that in addition to the uncertainty in the dynamo parameters and the boundary conditions, the exact functional (and in general precise tensorial) forms of α, and in principle also of η_t, are complicated and not well understood in the solar and stellar settings. Further, it is possible that the effective functional form of α as well as the effective dynamo parameters and the boundary conditions may vary on long time scales due to the inhomogeneity and the non-steadiness of the turbulent regimes in the solar and stellar convective zones. In order to have a clear picture of the possible range of the dynamical types of behaviour which such dynamo models are capable of, it would therefore be necessary to study the stability of the dynamo dynamics with respect to changes in (i) the functional form of α (and η) (ii) the effective dynamo parameter(s) and (iii) the boundary conditions.

The results presented in this paper were obtained using a modified version of the axisymmetric code of Brandenburg et al. (1989a,b), employing single precision arithmetic, i.e. using 4 byte floating point real numbers. We verified that no qualitative changes were produced by employing a finer spatial grid, different temporal step and/or by increasing the machine precision. In the following, we will discuss the behaviour of the dynamos considered by employing the total magnetic energy, E, in $r \leq R$, given by $E = E^{(A)} + E^{(S)}$, where $E^{(A)}$ and $E^{(S)}$ are the energies of the antisymmetric and symmetric part of the magnetic field respectively, and the overall parity P given by $P = (E^{(S)} - E^{(A)})/E$. Thus $P = -1$ denotes an antisymmetric (odd) pure parity solution and $P = +1$ a symmetric (even) pure parity solution (Brandenburg et al. 1989a,b). For orientation, note that pure dipole and pure quadrupole fields have $P = -1$ and $P = +1$ respectively.

2.1. Spherical $\alpha^2\omega$ dynamo models

In order to study what happens to the behaviour of the spherical $\alpha^2\omega$ dynamo models as these functional forms of α are allowed to change, we took as our reference model the detailed study of such models by Brandenburg et al. (1989a,b) in which they assumed a functional form for α given by

$$\alpha = \frac{\alpha_0 \cos\theta}{1 + \boldsymbol{B}^2}, \qquad (3)$$

with constant α_0 and η_t. We use the radius R of the sphere as the unit of length and the global diffusion time R^2/η_t as the unit of time. The magnitudes of the α and ω effects are given by the usual parameters $C_\alpha = \alpha_0 R/\eta_t$ and $C_\omega = \Omega_0' R^2/\eta_t$, where Ω_0' is the radial derivative of the angular velocity, here assumed to be a constant. Treating C_α as the control parameter and using $C_\omega = -10^4$, Brandenburg et al. (1989a) showed that the $\alpha^2\omega$ dynamos are capable of a range of dynamical modes of behaviour, including (symmetric and antisymmetric) pure parity modes as well as mixed parity solutions.

To study the effects of such changes, we took three other functional forms for α:

$$\alpha = \frac{\alpha_0 \cos\theta}{\boldsymbol{B}^4} \left(\frac{3 + \boldsymbol{B}^2}{1 + \boldsymbol{B}^2} + \frac{\boldsymbol{B}^2 - 3}{\boldsymbol{B}} \tan^{-1} \boldsymbol{B} \right), \qquad (4)$$

given by Kitchatinov (1987), the empirical

$$\alpha = \frac{\alpha_0 \cos\theta}{(1 + \boldsymbol{B}^2)(1 + 0.05|\boldsymbol{B}|)}, \qquad (5)$$

and the isotropic part of the more recent form due to Rüdiger & Kitchatinov (1993)

$$\alpha = \frac{15\alpha_0 \cos\theta}{32\boldsymbol{B}^2}\left(1 - \frac{4\boldsymbol{B}^2}{3(1+\boldsymbol{B}^2)^2} - \frac{(1-\boldsymbol{B}^2)\tan^{-1}\boldsymbol{B}}{\boldsymbol{B}}\right). \quad (6)$$

Equations (4) and (6) have qualitatively similar asymptotic behaviour for small and large values of $\boldsymbol{B}$, but only Eq. (6) is derived consistently in the framework of first order smoothing theory. Equation (4) was derived in the theory of stellar differential rotation to describe the dependence on the inverse Rossby number (Kitchatinov 1987). Equation (5) is an *ad hoc* modification of Eq. (3) in order to obtain the correct $|\boldsymbol{B}|^{-3}$ behaviour for large values of $\boldsymbol{B}$ (Moffatt 1972).

In the calculations discussed in Sect. 3, we replaced the functional form of $\alpha(\boldsymbol{B})$ given by (3) as employed in our reference model (Brandenburg et al. 1989a,b) by one of the forms given in (4), (5) and (6), and then studied the dynamical details of the resulting $\alpha^2\omega$ models.

2.2. $\alpha^2\omega$ dynamo models in a spherical shell

When studying the stability of $\alpha^2\omega$ spherical shell dynamo models, we first took the functional form of α to be that given by Eq. (3) and studied the effects of changes in the value of C_α and the boundary conditions. For the sake of comparison, we then looked at the effects of changing the functional form of α as in Sect. 2.1.

In our calculations, we assumed constant α_0 and η_t, and again we took the outer radius R of the sphere as the unit of length, the global diffusion time as the unit of time and allowed the magnitudes of the α and ω effects to be given by the dynamo parameters C_α and C_ω, which we fixed at a value -10^5. Our $\alpha^2\omega$ dynamo is situated in a rotating spherical shell of electrically conducting fluid. We split the axisymmetric field $\boldsymbol{B}(r,\theta)$ into poloidal and toroidal parts, by writing

$$\boldsymbol{B} = \nabla \times (a\hat{\phi}) + b\hat{\phi}, \quad (7)$$

where $\hat{\phi}$ is the unit azimuthal vector. On the outer boundary the magnetic field is matched to a potential field representing the solution in a vacuum. If we assume the inner boundary to be a perfect conductor, then the boundary conditions for the poloidal and toroidal fields are respectively

$$a = 0 \quad \text{and} \quad \frac{1}{r}\frac{\partial(rb)}{\partial r} - \alpha\frac{1}{r}\frac{\partial(ra)}{\partial r} = 0. \quad (8)$$

In reality, however, the magnetic fields will penetrate some distance into the interior of the star, and a strict perfect conductor inner boundary condition might therefore be too restrictive (or even quite inappropriate!). A less restrictive condition can be formulated by assuming that the field falls to zero at some distance δ below the boundary, approximated by

$$\frac{\partial a}{\partial r} - \frac{a}{\delta} = 0, \quad \text{and} \quad \frac{\partial b}{\partial r} - \frac{b}{\delta} = 0. \quad (9)$$

This might approximate the behaviour of an oscillating field in the shell that is rapidly damped by the skin effect in the external region. These two different boundary conditions can give rather different behaviours (Brandenburg et al. 1992), which motivated us to consider intermediate cases by using a continuous "interpolation" between perfectly conducting and penetrative boundary conditions in the forms

$$(1-F)a + F\left(\frac{\partial a}{\partial r} - \frac{a}{\delta}\right) = 0, \quad (10)$$

for the poloidal field, and

$$(1-F)\left(\frac{1}{r}\frac{\partial(rb)}{\partial r} - \alpha\frac{1}{r}\frac{\partial(ra)}{\partial r}\right) + F\left(\frac{\partial b}{\partial r} - \frac{b}{\delta}\right) = 0, \quad (11)$$

for the toroidal field, where $F \in [0,1]$. The results of our calculations are discussed in Sect. 4. (We would stress that, in reality, the region below a convectively unstable shell in a star is physically ill-understood with, for example, substantial convective overshoot occurring. Our point is that, given such uncertainties, there is no unambiguously 'correct' boundary condition. We have just identified two more-or-less plausible alternatives.)

3. Spherical dynamo results

For the spherical $\alpha^2\omega$ dynamo calculations, we used a grid size of 21×41 mesh points, uniformly distributed over $0 \leq r \leq R$, $0 \leq \theta \leq \pi$ respectively, with a time step of 2×10^{-4}. We studied the range of dynamical behaviour as a function of C_α as the form of α was varied. In order to facilitate comparison of our results, the analogues of the bifurcation diagrams given in Brandenburg et al. (1989a,b) for functional forms of α given by (4), (5) and (6) are shown in Fig. 1, with $C_\omega = -10^4$. As can be seen, Fig. 1 is in good qualitative agreement with Figs. 8a of Brandenburg et al. (1989a) and 11 of Brandenburg et al. (1989b). This clearly shows that the $\alpha^2\omega$ models considered by Brandenburg et al. (1989a,b) are stable with respect to reasonable changes in the functional form of α.

Furthermore, our numerical results show that increasing the value of C_α, whilst keeping C_ω fixed, did not produce any chaotic behaviour in the resulting dynamos.

4. Spherical shell dynamo results

In our calculations of the spherical shell dynamo, the shell was characterized by setting the bottom of the convective zone at radius $r_0 = 0.6$, the penetration depth $\delta = 0.05$ and $C_\omega = -10^5$. In view of the more complicated behaviour exhibited by the solutions of the shell models, a finer mesh of 41×81 was used, together with a time step of 10^{-4}.

We now take the $\alpha(\boldsymbol{B})$ profile given by Eq. (3). Our results show that for all values of $C_\alpha \leq 3$, the behaviour is periodic, independently of the inner boundary conditions, i.e. for all values of $F \in [0,1]$. Similarly, for $C_\alpha \geq 15$, the behaviour seems to be chaotic and insensitive to the inner boundary conditions. In a dynamical sense these two regimes appear to be underpinned by the presence of strong attractors, with basins of attraction that seem to cover the whole of $F \in [0,1]$. On the other hand in

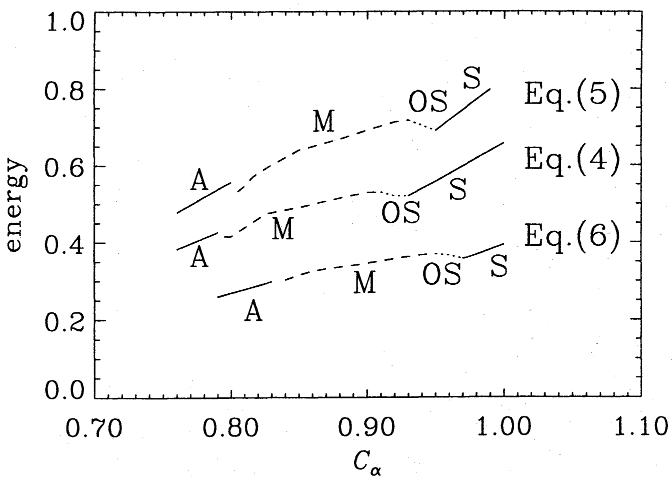

Fig. 1. Bifurcation diagram for a spherical dynamo with α given by Eqs. (4), (5), and (6). Only stable solutions are shown. A, M and S indicate that the dynamo is in an antisymmetric, mixed or symmetric regime, respectively. OS refers to an oscillating solution with positive parity

the intermediate region $3 \lesssim C_\alpha < 15$, the behaviour becomes complicated and sensitive to the boundary conditions, with windows of chaos and periodicity interspersed. For example, for $C_\alpha = 10$, periodic behaviour was observed for $0.3 \leq F \leq 0.5$, and chaotic behaviour for other values of F. However, we stress that because of the high computational cost of running the code for particular values of the parameters (especially in view of the presence of extremely long transients in some cases), we only evaluated the dynamo models for values of F running from 0 through to 1 in increments of 0.1. We note that from a dynamical systems point of view, it is plausible that taking a finer mesh of values of F would show still more complexity in smaller scales (such as the interspersed regions of periodicity and chaos in the case of the Logistic map (Jacobson 1981; Farmer 1985; Tavakol & Ellis 1988) and the Lorenz system (Sparrow 1982). This, however, does not change the main point of our results which indicates the presence of multiple attractors. More precisely, our numerical results indicate that to this precision there are in the range $C_\alpha \in [(3, 15)]$, at least two attractors present with their basins of attraction interspersed. The results are tabulated in Table 1. We have concentrated on C_α in the range 8 to 15 since this is the more interesting transition region which highlights the point that is being made here. We should add here that for spherical shell dynamos, transients can sometimes last longer than 50 diffusion time units. As a result, only after about 80 time units can a decision be made regarding the asymptotic state of the system in such cases. To be on the safe side, all runs (unless they settled down to a periodic state) were made with at least 100 diffusion time units.

To check the sensitivity of the shell dynamo models with respect to changes in the α profile, we briefly examined the behaviour of such dynamos at $C_\alpha = 2$ and $C_\omega = -10^5$, but with variable F, with the functional form of $\alpha(\boldsymbol{B})$ given by Eqs. (3), (4) and (6). The results are summarized in Table 2. We chose C_α to be 2 in order to emphasize the dramatic change in qualitative behaviour that can be produced by using different $\alpha(\boldsymbol{B})$ profiles.

Table 1. Summary of solutions for a range of values of F and C_α for Eq. (3). 'S' indicates a solution having a symmetric parity whilst 'C' stands for one whose parity and energy is chaotic. 'N' indicates a solution with a noisy parity and energy behaviour, but whose power spectrum is not noisy, as is the case with a truly chaotic solution. In all cases $C_\omega = -10^5$

	F										
C_α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
8	C	S	N	N	C	C	C	C	C	C	C
9	C	C	C	S	S	C	C	C	C	C	C
10	C	C	C	S	S	S	C	C	C	C	C
11	C	C	C	C	C	S	S	C	C	C	C
12	C	C	C	C	C	S	S	S	C	C	C
13	C	C	C	C	C	C	S	S	S	S	C
14	C	C	C	C	C	C	C	C	C	S	S
15	C	C	C	C	C	C	C	C	C	C	C

Table 2. Summary of solutions for different values of F using Eqs. (3), (4) and (6) for the $\alpha(\boldsymbol{B})$ profile. 'S' indicates a solution having a symmetric parity, 'C' stands for one whose parity and energy is chaotic, and 'A' refers to an antisymmetric solution. The asterisk indicates the occurrence of very long chaotic transients and that the solution became periodic only after about 100 diffusion times. $C_\alpha = 2.0$, $C_\omega = -10^5$

	F						
	0.0	0.25	0.3	0.4	0.5	0.75	1.0
Eq. (3)	S	S	S	S	S	S	S
Eq. (4)	S	C	C	C	A*	A	A
Eq. (6)	A	C	C	C	A	A	A

It can clearly be seen that, even with fixed values for C_α and C_ω, spherical shell dynamos have different sensitivities with respect to the boundary conditions and the functional forms of α.

We verified that the decisive factor for the dynamical behaviour is the value of the product $D = C_\alpha C_\omega$. We found that changing C_α and C_ω, while keeping the product the same, did not produce any qualitatively different effects. This is consistent with these models being within the '$\alpha\omega$' regime of the $\alpha^2\omega$ dynamo model.

Finally, we should note that for spatially resolved mean field dynamos, solutions with temporal chaos have only recently been found in the context of accretion disc dynamos (Brooke & Moss 1994; Torkelsson & Brandenburg 1994), but to our knowledge not yet for spherical shell dynamos. We would like to emphasize that the spatial field distribution remains smooth, even though the dynamo number D is approximately 100 times above the marginal value for the onset of dynamo action. The fields shown in (Fig. 2) are quite typical. In Fig. 3 we present part of the time series of the energy and the corresponding power spectrum (using the full time series with a length of over 300 diffusion times), for a run with $F = 0.25$, $C_\alpha = 2$, and $C_\omega = -10^5$. The power spectrum shows clearly the basic cycle frequency and the next higher harmonic. At lower frequencies the signal

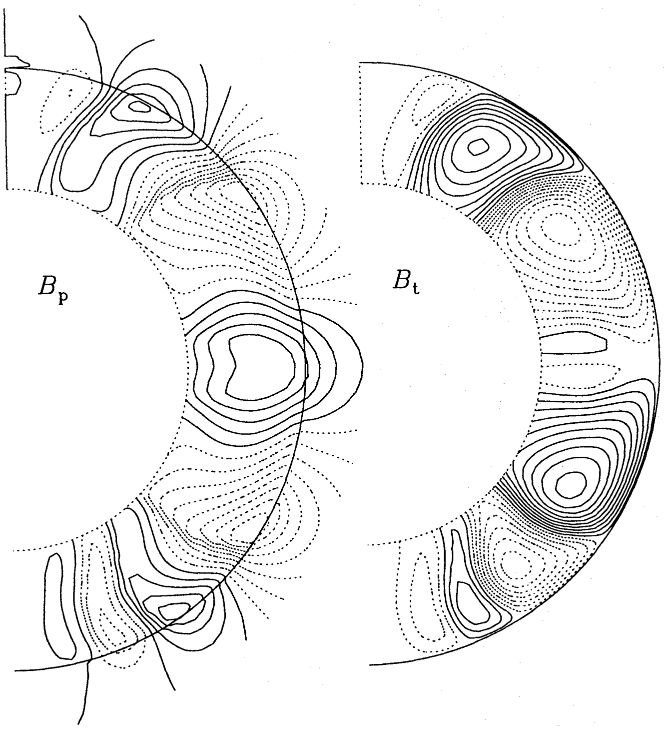

Fig. 2. Field lines of the poloidal field $\boldsymbol{B}_p$ and contours of the toroidal field $\boldsymbol{B}_t$ for a solution showing temporal chaos using Eq. (6). Dotted contours refer to negative values or opposite field line orientation. $F = 0.25$, $C_\alpha = 2$, and $C_\omega = -10^5$

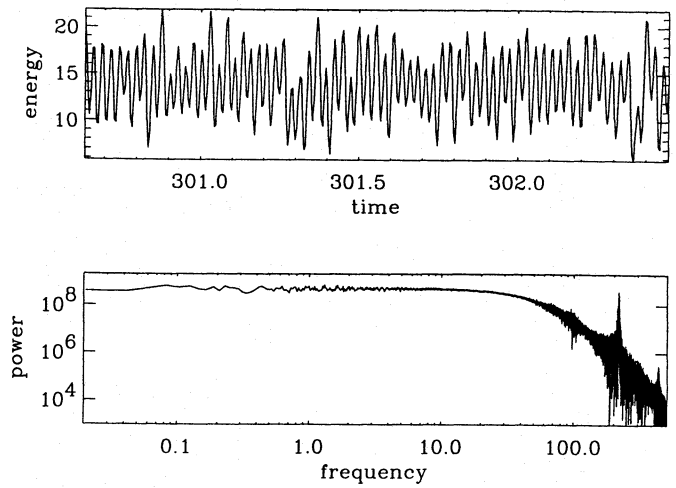

Fig. 3. A short piece of the time series of the energy and the power spectrum for the entire run covering over 300 diffusion times. Note the occurrence of white noise over three orders of magnitude in frequency. $F = 0.25$, $C_\alpha = 2$, and $C_\omega = -10^5$ using Eq. (6)

is essentially white noise. A detailed analysis of the properties of chaotic solutions for spherical shell dynamos will be given in a separate paper.

5. Conclusions

Our results concerning spherical shell dynamos with fixed outer boundary conditions, and the various functional forms of α investigated, seem to indicate that there are regions of the parameter space for which the behaviour is chaotic (or periodic) and at the same time stable to changes in the inner boundary conditions. These correspond to regimes with attractors possessing basins that attract all inner boundary conditions. On the other hand there are intermediate parameter regions for which the behaviour may be either periodic or chaotic, depending upon the inner boundary conditions. These regimes seem to indicate the presence of multiple attractors whose basins are interspersed in the interval $F \in [0, 1]$. Furthermore, changes in the functional form of α show that, even at fixed values of C_α, C_ω and boundary conditions, such dynamos are sensitive to the $\alpha(\boldsymbol{B})$ profile.

On the other hand, our study of spherical $\alpha^2\omega$ dynamos seem to show that they are not sensitive to changes in the functional form of α, and also they do not seem capable of producing chaotic behaviour.

Our results therefore seem to indicate that $\alpha^2\omega$ dynamos in spherical shells are much less robust with respect to changes in the functional form of α and of the boundary conditions than dynamos in full spheres. Dynamos in spherical shells are also capable of producing chaotic behaviour. This difference in the sensitivity of the spherical and spherical shell models is in principle decidable observationally and the enhanced variability of the shell dynamo models may be of potential importance in interpreting astrophysical observations.

We would like to emphasize that the uncertainty that we introduced into the inner shell boundary conditions corresponds to a real lack of understanding of the relevant physics: we do not know the boundary conditions that 'correctly' represent the true physical situation. It is also true that there is a similar uncertainty connected with the outer boundary conditions, at $r = R$. Studies of the solar photospheric and chromospheric magnetic fields clearly indicate that the commonly adopted simple vacuum boundary conditions are quite inaccurate. We have not explored this point, but it is plausible that fragile behaviour similar to that discussed above may also be present.

There is now a sample of about 90 late type stars (F-K) for which activity has been monitored over the last 28 years. This subject has been reviewed by Baliunas & Vaughan (1985). It does seem that stars of the same spectral type, rotational period, age, and composition can show quite different behaviour. Such variations in behaviour are consistent with a structural fragility of the underlying dynamo processes. Finally, we would like to add that the type of fragility found here may be of relevance to other astrophysical phenomena.

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