Detailed project description:
Astrophysical turbulence and dynamo action

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1 Overview

An important astrophysics activity at Nordita focuses on the calculation of the stochastic gravitational wave (GW) background. This activity is primarily being pursued by a PhD student at Nordita and a post-doc. For those calculations we use the PENCIL CODE. The strengths of the resulting GW field at the present time is calculated for a range of GW sources associated with turbulent stresses in the energy-momentum tensor. This activity is supported by a VR grant on "Stochastic Gravitational Wave Background from the Early Turbulent Universe" 2019-04234, January 2020 – December 2022, 4.00 MSEK. The allocation must also partially support the activity of the rest of the Nordita group, but I have also applied for a new VR grant for 2 post-docs. Another activity, carried out by a new post-doc, concerns the impact of primordial magnetic fields on the formation of first luminous sources.

In all cases, the computer-generated simulation data will be stored on the PDC and NSC machines for the duration of the project. To help the initiation of new projects, a much smaller amount of data will also be stored on the PDC machines for later usage. The total usage is currently around 300 TB and the number of files is 50,000,000.

2 Resource usage

For most of our simulations, we use the PENCIL CODE[1], which is hosted by Github[2]. This is an open-source code developed by Brandenburg, his current and former coworkers, some of whom are part of this project, as well as others that have been invited to join the effort. The performance of this code has been discussed at several international conferences; see, e.g., `http://www.nordita.org/~brandenb/talks/misc/PencilCode09.ppt`. The code has been optimized over the years and is still being improved in terms of performance and new features are also being added. All of the 36,072 revisions since 2001 are publicly available through our repository. We have adapted and optimized this code for spherical polar coordinate system. This addition to the code is used in several of the problems discussed here. The code runs well on all the different platforms.

On Dardel, we run production runs with up to 2048^3 mesh points on up to 32768 cores. A typical run requires at least 500,000 time steps, but it can sometimes be much more, depending on circumstances. With $4.2 \times 10^{-4} \mu$s per meshpoint and per timestep on Dardel. We anticipate that the time will be about 4 days of wall clock time at a cost of 600,000 CPU hours, while with $3.5 \times 10^{-3} \mu$s per meshpoint and per timestep, this means less than 3 days of wall clock time at a cost of 30,000 CPU hours per run.

To address properly the critical question of the dependence on the magnetic Reynolds number we have to use high resolution runs. As we move from 256^3 and 512^3 to 2048^3 mesh points (and correspondingly higher magnetic Reynolds numbers), we see the emergence of small-scale dynamo action at all depth. This does not yet affect the 512^3 runs, but for the 2048^3 run, small-scale dynamo action becomes critical. The last of these runs is for a deeper domain, so as to include more safely the deep parts where it is important to reach values of the mean field normalized to the equipartition value below 0.01, but this appears not to be possible due to small-scale dynamo action.

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[1] `http://www.nordita.org/software/pencil-code` (The Pencil Code Collaboration, 2020)
[2] `https://github.com/pencil-code`

To confirm our ideas and to understand the effects of small-scale dynamo action, we plan to perform about 2 big runs per month on Dardel, which requires at least 1,000,000 CPU hours, and about 4 intermediate ones, which requires 150,000 CPU hours on each of them. To shed light on some of the observational features of accretion disks, particularly those related to interactions between the inflow stream and the accretion disk, we must run high-resolution global simulations with radiation. This is a new activity, where each run takes about 300,000 CPU hours. We plan to run three of those. Our total time requirement is therefore 2,500,000 CPU hours on Dardel and Tetralith combined.

Computationally, all machines are comparable, but there can be unexpected future changes or outages on some machines that hamper scientific progress. Important is also the waiting time in the queue and occasional opportunities when jobs start immediately. Most of our activity will reside on Dardel. However, to maximize our scientific productivity, and not to be affected too much by outages and long waiting times, we also apply for time on Tetralith.

Regarding scaling tests, we have previously determined strong scaling of PENCIL code on Triolith for three mesh sizes. The time per time step and mesh point is given for different processor numbers and layouts. Generally, it is advantageous to keep the number of processors in the x direction small. The code is well adapted to modern computing platforms.
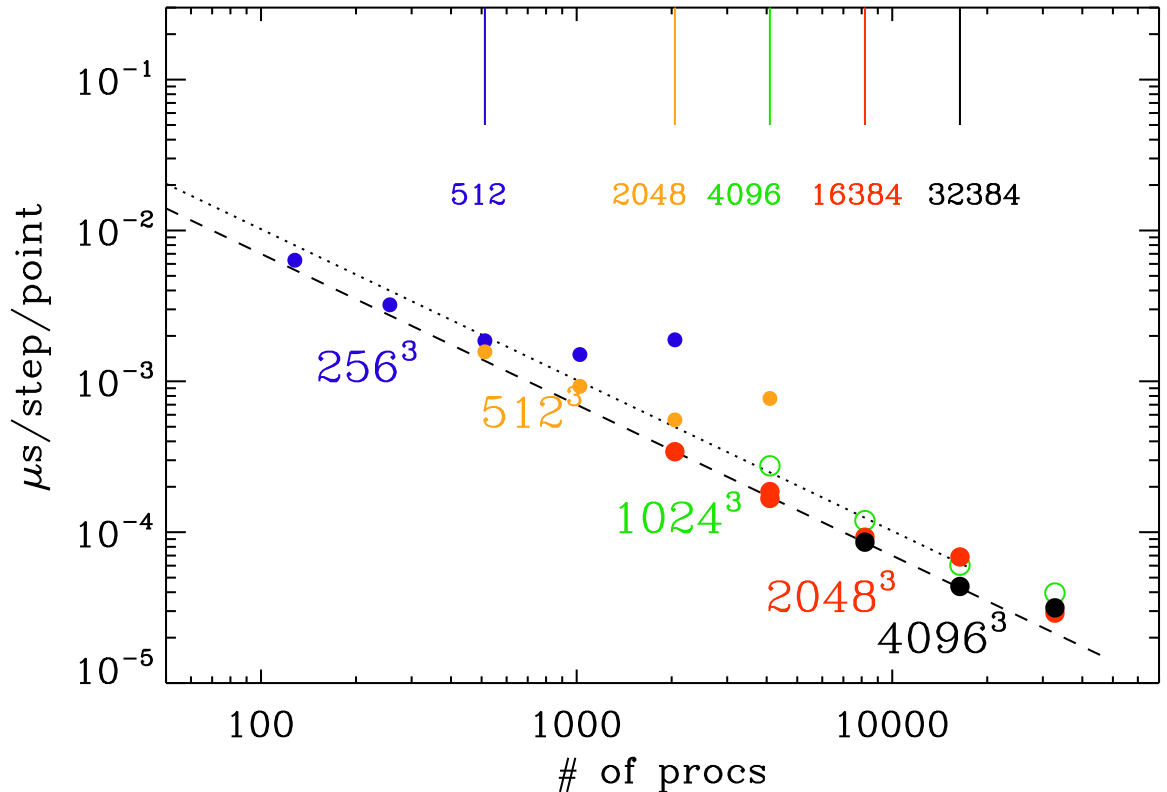

Figure 1: Strong scaling on Dardel. The dotted and dashed lines corresponds to 1.02μs/proc/step/point and 0.70μs/proc/step/point, respectively.

Performancewise, Cray with O2 optimization is equivalent to gnu with O3. While gnu-O3 is able to handle memory or whatever compiler problems much better, it is otherwise not better than Cray-O2, and often some 10–20% slows, but this is within the measurement accuracy; see https://github.com/pencil-code/website/blob/master/News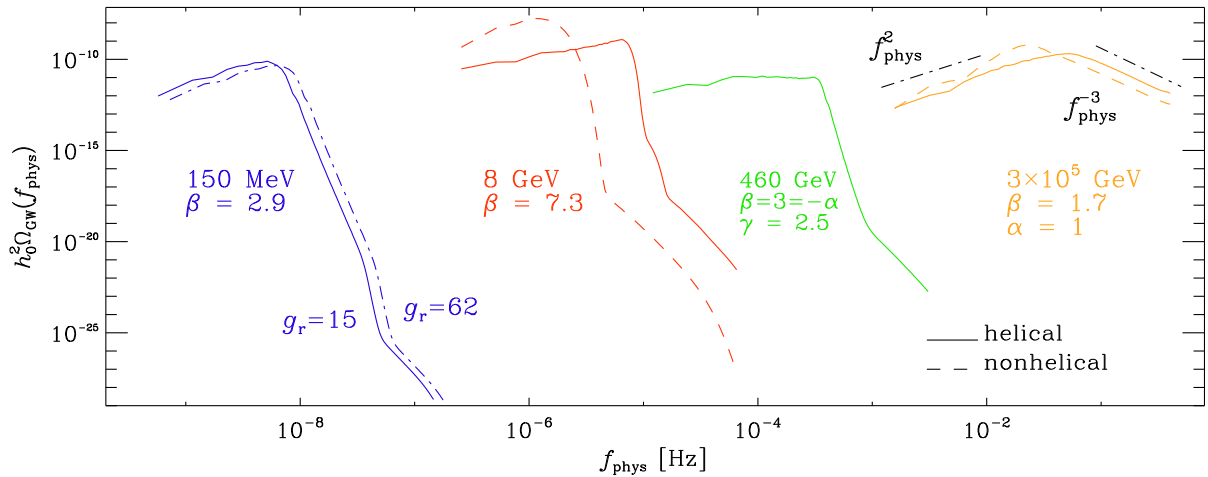Letters/2021/newsletter3.pdf. More details can be found on https://github.com/pencil-code/pencil-code/tree/master/doc/timings.

Figure 2: $h_0^2\Omega_{\mathrm{GW}}(f_{\mathrm{phys}})$ for runs with reheating temperatures T_{r} ranging from $150\,\mathrm{MeV}$ to $3 \times 10^5\,\mathrm{GeV}$. Dashed lines denote nonhelical runs and dashed-dotted show the result for $g_{\mathrm{r}} = 62$.

3 Scientific challenges

Impact of primordial magnetic fields on the formation of first luminous sources. The 21-cm signal, which is produced by the spin-flip transition of the ground state of neutral hydrogen, is a tracer of the matter in the intergalactic medium during the time when the first luminous sources of our Universe formed. This signal is affected by the population, distribution, and properties of these luminous sources (Giri, S. K., & Schneider, A., 2022). Therefore we can learn not only about the distribution of matter but also about the formation of first sources by observing this 21-cm signal. Current radio experiments, such as the Low-Frequency Array (Mertens et al., 2020), the Murchison Widefield Array (Trott et al., 2020), and the Hydrogen Epoch of Reionization Array have provided useful upper limits on this signal. In the near future, the much more powerful Square Kilometre Array will come online, which will be sensitive enough to not only detect the signal but also produce images of the distribution of neutral hydrogen in the sky (Giri, et al., 2018).

The power spectrum can be used to constrain not only the properties of the first luminous sources but also the dark matter models that affect structure formation in our Universe. We plan to use this 21-cm signal to study any cosmological model that affects the formation of sources in the early Universe. In this project, which will also use the PENCIL CODE, we aim to develop models to study the impact of primordial magnetic fields on the formation of first luminous sources and derive their signature on the 21-cm signal.

Inflationary magnetogenesis and gravitational wave production. In two recent papers with Ramkishor Sharma, a new Nordita postdoctoral researcher, we have performed new simulations of inflationary magnetogenesis and gravitational wave production. In those models, we solved the equations for electromagnetic waves during the post-inflationary phase (the reheating phase), and switched to magnetohydrodynamics at the beginning of the radiation-dominated phase; see Figure 2. In future models, we will include equations describing the evolution of scalar potentials in cosmology. This will lead to more realistic models. All models are three-dimensional and typical resolutions are of the order of 1024^3 to 2048^3 mesh points.

Gravitational wave polarization. We study the influence of helical magnetic fields on the production of gravitational waves. Gravitational waves provide an as yet unexplored window into the earliest moments of the Big Bang, not obscured by the last scattering surface given by the hitherto studied cosmic microwave background. The production of gravitational radiation from cosmological turbulence was calculated analytically by Kosowsky et al. (2002) and Gogoberidze et al. (2007). Helical magnetic fields

produce non-vanishing cross-polarization in the gravitational wave spectrum (Kahniashvili et al., 2005; Caprini & Durrer, 2006), which would be observable with LISA. Hindmarsh et al. (2017) have recently presented detailed numerical models of gravitational waves from phase transition nucleation bubbles produced during the electroweak phase transition. Our new work involves the calculation of gravitational waves using the PENCIL CODE, where a gravitational wave solver has already been successfully implemented.

Chiral MHD. The chiral magnetic effect leads to a current along a magnetic field if the number of left- and right-handed Fermions is unequal. This effect has received significant attention in just the last few years. We are now able for the first time to perform a comprehensive study of the chiral magnetic effect in real turbulence. Earlier theoretical studies applied to neutron stars and the early Universe did not result in realistic estimates for the turbulence. Thus, the use of simulations is absolutely critical to making significant progress. Our recent work on the early Universe has brought us a significant step forward. We will now focus on neutron stars, which may have several important advantages. First, only one sign of chirality will be produced. Second, the timescales are short, giving us ample time for the subsequent inverse cascade to yield large length scales. Together with the helicity produced from rotation and stratification, the end result may produce a realistic model of observed pulsars.

Inverse cascading in non-helical MHD. Today, high resolution numerical turbulence simulations are feasible. They allow for fairly large turbulent inertial ranges, but not all the resources must be spent on a large inertial range. It is also important to allow for enough scale separation toward very long length scales to avoid artifacts from a finite domain size. To help optimizing the computational resources, various groups have employed numerical tools to approach the physically relevant regime: (i) reduce the viscous and diffusive subranges by adopting hyperviscosity and magnetic hyperdiffusivity, and (ii) to make the viscous and magnetic diffusion coefficients time-dependent, or do use, for example, slope-limited diffusion to prevent the code from crashing at early times when the turbulence is extremely vigorous and to allow for the physical viscosity to be as small as possible at late times. All these tools imply small artifacts that we need to examine very carefully. The slope-limited diffusion technique has recently been used in some of our simulations of chiral MHD. To assess the role of artifacts in simulations, we now need to perform more detailed studies by monitoring the evolutionary tracks in a *pq* diagram for different combinations of such numerical tools. We will therefore perform divers surveys to extract physically meaningful results from artifacts.

Entropy rain convection. Convection is a highly nonlocal phenomenon where low entropy blobs can descent over large depths through the convection zone. This causes an extra contribution to the enthalpy flux that is not described by a local entropy gradient, but by the value of the mean squared entropy fluctuation relative to the background. This always transports energy outward. The resulting stratification may end up being slightly subadiabatic instead of being slightly superadiabatic and therefore host gravity modes that may be observationally detectable. Corresponding simulations were produced by Brandenburg (2016) and Käpylä et al. (2017), but the effect of rotation is not well understood. This will be studied using upcoming simulations.

Solar/stellar dynamo simulations. Local simulations will be used to develop what we call smart boundary conditions for application in the global simulations. The purpose of such boundary conditions is to compactify the small-scale physics of the surface-driving layer in order to control the global simulations, which cannot resolve these scales, in a physically realistic way. Here we assume that (i) stellar turbulence is essentially driven by cooling in the surface-driving layer and (ii) large-scale structures like giant cells or a global dynamo field would not markedly affect the overall properties of the convection. Then, local Cartesian boxes, which extend vertically just deep enough so that the (non-physical) boundary conditions to be applied at their bottom have no significant effect on the near-surface convection (say, 30 Mm deep) and which are horizontally just wide enough to capture the essential topology and dynamics of the granulation, will be employed to solve the full convection problem with the necessary high grid resolution (say, 100 Mm horizontal extent) and with physically meaningful boundary conditions at their top. Time

series of the simulated physical quantities on a horizontal plane placed at the estimated bottom of the surface-driving layer inside the computational box will be employed to define the boundary conditions at the top of a global simulation model which extends from the bottom of the convection zone (say 200 Mm depth) with physically meaningful boundary conditions to the bottom of the surface-driving layer. A simple way of doing this consists in directly employing the quantities from the local-box simulations as Dirichlet boundary conditions of the global model. Due to its coarser resolution, the data have to be properly restricted. As the simulated model time interval of the global model will in general be much longer than the one of the local model, the problem arises of how the boundary values should be repeatedly used without introducing a strict periodicity. This approach will allow incorporating the NSSL in the global simulations, without needing to resolve the surface layer in one and the same model.

4   Research group and management

The work in the astrophysics group at Nordita covers a broad range of topics from kinetic simulations over gravitational wave physics and the early universe to solar physics and meteorology. Our research group consists currently of the following people:

Kyrylo Bondarenko (Nordita fellow)
Dr Sambit Giri (Nordita fellow)
Mr Yutong He (Nordita PhD student and Stockholm U)
Dr Oksana Iarygina (Marie Curie fellow)
Dr Lars Mattsson (guest researcher)
Dr Dhrubaditya Mitra (assistant professor)
Dr Ramkishor Sharma (Researcher)
Mr Patrik Tengnér (Master student)

The monthly usage within the group is monitored and discussed during our weekly group meetings.

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