

# Detailed project description: Turbulence and Dynamo in Accretion Flows

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## Abstract

Continued support for large-scale computing is being requested. In connection with the recently awarded ERC grant a number of new people have arrived that have all started working on various projects. We use the PENCIL CODE (<http://pencil-code.googlecode.com>), which is a sixth order finite difference code with a third order time stepping scheme. The code uses MPI and is running on a range of different platforms around the world and is designed to work on large Linux clusters. The scientific progress of the last 18 months is summarized in <http://www.nordita.org/~brandenb/AstroDyn/progress/report1/> and has resulted in 24 publications. Since 2009, Swedish computing resources were acknowledged in 20 refereed papers; see <http://www.nordita.org/~brandenb/AstroDyn/progress/computing/report10.pdf>.

## 1 Background

The group at Nordita working on dynamo theory has now expanded and is still expanding significantly owing to the ERC grant that we have secured. The people who are now members of the group are listed below

Mr Simon Candelaresi (PhD student)  
Mr Fabio Del Sordo (PhD student)  
Mr Koen Kemel (PhD student)  
Mr Jörn Warnecke (PhD student)  
Dr Niccolo Bucciantini (Nordita fellow)  
Dr Piyali Chatterjee (Post-doc)  
Dr Gustavo Guerrero (Post-doc)  
Dr Alexander Hubbard (Nordita fellow)  
Dr Chi-Kwan Chan (Nordita fellow)  
Dr Dhruvadya Mitra (assistant professor )  
Dr Matthias Rheinhardt (visiting professor)  
Mr Andreas Svedin (visitor)

## 2 Scientific content

The overall goal of this project is to understand the origin of the Sun's magnetic field, i.e. the solar dynamo, its location within the Sun, its 22 year period, and the origin of the equatorward migration of the sunspot belts. Current thinking places the dynamo in the tachocline, i.e. the

bottom of the convection zone where the internal angular velocity turns from nearly uniform in the interior to non-uniform in the convection zone. The idea is that the field strength there exceeds the equipartition value by a factor of 100, but such a result has not yet been obtained with dynamo theory and it would be below current helioseismological detection limits. One also assumes a turbulent magnetic Prandtl number of 100, instead of 1, which is predicted by theory and simulations. Such modifications of the theory are the result of trying to make the models reproduce the observations. However, such models ignore some important findings regarding the nonlinear behavior of the mean field dynamo effect (e.g., the so-called alpha and Babcock–Leighton effects) in the case of large magnetic Reynolds number. Recent research has provided new detailed insights that we feel should be followed up using more realistic settings such as spherical shell geometry.

Our research program proceeds in two parallel strands; one is connected with the development and exploitation of the spherical extension of the PENCIL CODE, and the other one is connected with important and unresolved problems that are to be addressed with the PENCIL CODE in its usual Cartesian configuration.

The prime objective of the PENCIL CODE is to be efficient on massively parallel machines. The code uses the message passing interface and is made cache efficient by assembling the right hand side for all equations along one-dimensional pencils first. It has been run on up to 1024 cores without loss of scaling. Partial differential equations are being solved to third order in time and to sixth order in space. The code is most efficient in 3-D, but for test purposes it runs also well in 2-D, 1-D, and 0-D (corresponding to solving ordinary differential equations). The user can code up easily new equations, but the equations currently supplied are those of compressible magnetohydrodynamics, including the effects of radiation, self-gravity, dust particles with inertia and coagulation, chemistry, variable ionization, cosmic rays. For turbulence and dynamo studies it has been critical to be able to solve with the correct diffusion operators. Alternatively, however, shock diffusion and subgrid scale modeling can be included. The PENCIL CODE is now hosted by Google Code through subversion (svn). It comes with an infrastructure where the code’s integrity is tested each night on several machines on currently 31 test problems. Therefore everybody uses normally always the latest version, which is made public every morning. The number of people having downloaded the code has risen to over 800 since its initial development in 2001.

In the following we list detailed steps of our research program. Background and technical details of each of the steps in this synopsis are explained in Section 2 of this proposal.

1. *Catastrophic quenching in a spherical shell*: Reproduce the catastrophic quenching behavior in a closed sphere or spherical shell sector using perfectly conducting boundary conditions and forced turbulence. Some work in this direction has already been done [1], but the results are not yet well understood nor entirely conclusive.
2. *Dynamo effect from the MRI*: Calculate the nonlinear  $\alpha$  effect and the turbulent diffusivity for turbulence driven by the magneto-rotational instability (MRI). Some work in this direction has already been done [2], but only a few representative test cases at relatively low resolution were done, nor was nonlinearity in the test-field method. This work is primarily relevant to accretion discs. However, understanding this case may also teach us general aspects of magnetically driven dynamos that may in some form also work in the Sun.
3. *Test-field method in spherical geometry*: Adapt the test-field method to spherical coordinates. Originally the test-field method was developed in connection with full spheres,

and then the test-fields consisted of field components of constant value or constant slope. However, only afterwards it became clear that the scale (or wavenumber) of the field components must be the same for one set of all tensor components, and so it is necessary to work with spherical harmonic functions as test fields. In other words, constant and linearly varying field components are problematic.

4. *Dynamo in open shells with and without shear*: Calculate the saturation of the magnetic field and the underlying dynamo effects with open boundary conditions in a spherical shell sector with and without shear. One expects low saturation amplitude with magnetic energy of the mean field being inversely proportional to the magnetic Reynolds number in the absence of shear, but of order unity in the presence of shear. The shear is here critical, because it is responsible for the local driving of small scale magnetic helicity fluxes [3, 4, 5].
5. *Magnetic flux concentrations near the surface*: Test the scenario that the emergence of active regions and sunspots can be explained as the result of flux concentrations from local dynamo action via negative turbulent magnetic pressure effects [6] or turbulent flux collapse [7].
6. *CME-like features above the surface*: Analyze the nature of the expelled magnetic field in simulations that couple to a simplified version of the lower solar wind. It is possible that the magnetic field above the surface might resemble coronal mass ejections (CMEs), in which case more detailed comparisons with actual coronal mass ejections would be beneficial.
7. *Buoyancy-driven dynamo*: The turbulence in accretion discs is believed to be driven by the magnetorotational instability. It was one of the first examples showing cyclic dynamo action somewhat reminiscent of the solar dynamo [8, 9]. It was believed to be a prototype of magnetically driven dynamos [10, 11, 12, 13, 14]. In the mean time, another example of a magnetically driven dynamo has emerged, where magnetic buoyancy works in the presence of shear and stratification alone [15, 16, 17, 18]. This phenomenon is superficially similar to a magnetically dominated version of the shear–current effect [19, 20]. We are now in a good position to identify the governing mechanism by using the recently developed test-field method.
8. *Subgrid Model Construction from Direct Numerical Simulation*: Large eddy simulations are very important numerical tools to study turbulence flows. In this approach, turbulence is only resolved down to a cutoff scale. The unresolved subgrid physics are then described by extra terms in the equations. Although many subgrid models exist, they are mostly derived by assuming homogeneity and isotropy. However, magnetohydrodynamic turbulence can not be locally isotropic. Inverse cascade of magnetic helicity creates large-scale fields, which also breaks homogeneity. Because our group has experiences in measuring transport coefficients, we will construct more realistic subgrid models using local properties of the resolved flow. We will perform high resolution direct numerical simulations. By introducing a cutoff scale, we can decompose the high resolution simulations into large-scale fields and fluctuations. We will then measure subgrid models directly by correlating the fluctuating stresses with the large-scale fields.

### 3 Requested resources

Almost all the problems described above will principally use the PENCIL CODE<sup>1</sup>, which is hosted by Google-Code since 2008<sup>2</sup>. This is an open-source code developed by myself and my coworkers some of whom are part of this project. The performance of this code has been discussed at several international conferences; see, e.g., <http://www.nordita.org/~brandenb/talks/misc/PencilCode09.ppt>. The code has been optimized over the years and is still being improved in terms of performance and new features are also being added. Recently we have adapted and optimized this code for spherical polar coordinate system [21]. This addition to the code is going to be used in several of the problems listed in the previous section.

We have done exploratory runs for several of the problems described in the previous section in the PDC computers Neolith and Ferlin. At present it is almost impossible to run even medium-scale jobs, ones which require 32 processor cores or more, for significant time. It has been possible to do such medium-scale jobs in Neolith but for a very short while. In Neolith our previous allocation has been pitifully small. Our monthly allocation sustains for typically for about a week. This has principally two reasons. First is that we are a big group and working on several problems simultaneously. Secondly note that our group has four PhD students who must work on their separate problems in an independent fashion. This time we are applying for resources in the four machines Lindgren, Neolith, Akka, and Ferlin. Ferlin is ideal for many small-scale jobs, typically exploratory runs and to test new features of the code. On Neolith, Akka, and Lindgren we plan to use for our larger production runs.

Each of our production runs tend to have  $512^3$  meshpoints and require at least 64 but preferentially more processors. A typical run requires at least 250,000 time steps. In Brandenburg & Subramanian (2005b) we used three such runs in the paper, although, of course, several more have been computed that are not reported. With  $0.075\mu s$  per meshpoint and per timestep, this means 30 days of wall clock time (using multiple restarts) for just one run. This corresponds to a total of about 50,000 CPU hours. In order to address properly the critical question of the dependence on the magnetic Reynolds number we now have to move to runs with  $1024^3$  and  $2048^3$  meshpoints, which would require 400,000 CPU and 1,600,000 CPU hours, respectively. Note that the estimate above applies to most of the projects given in item 1 to 7 above. All the projects will of course not require same computational resources. This gives a time of more than 10,000,000 CPU hours. Divided over 12 months we would like to have about 800,000 CPU hours per month. For constructing subgrid models from direct numerical simulations, item 8 in the previous section, we will run an ensemble of eight  $1024^3$  simulations. Each of them requires about 100,000 time steps, which results again 50,000 CPU hours, where we assume that it takes 256 processors one second to take one time step. Out of the total demand, we need to cover about 300,000 CPU hours per month on Lindgren, 150,000 CPU hours on Neolith and Akka, and 50,000 CPU hours on Ferlin.

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<sup>1</sup> <http://www.nordita.org/software/pencil-code>

<sup>2</sup> <http://pencil-code.googlecode.com>

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