The state of Anelastic Module in the Pencil Code

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The method

Continuity: $$\boldsymbol{\nabla}.(\rho\boldsymbol{u}) = 0$$

EOS: $$\rho = \rho(p, s)$$

NS: $$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}.\boldsymbol{\nabla})\boldsymbol{u} - \frac{\boldsymbol{\nabla} p}{\rho} + \boldsymbol{R}^v$$

Poisson Eq: $$\nabla^2 p = \nabla_i[\rho \boldsymbol{R}_i^v - \nabla_j(\rho u_i u_j)] = g(\rho, \boldsymbol{u})$$

Entropy: $$\frac{\partial s}{\partial t} = -(\boldsymbol{u}.\boldsymbol{\nabla})s - \frac{\boldsymbol{R}^s}{\rho T}$$

The method

- Linear treatment: $\rho=\rho_b+\rho'$; $s=s_b+s'$; $p=p_b+p'$ along with the relations,

$$\frac{\rho'}{\rho_b} = \frac{p'}{p_b} - \frac{T'}{T_b}$$
$$\frac{s'}{c_p} = \frac{T'}{T_b} - \frac{\gamma - 1}{\gamma}\frac{p'}{p_b}$$

Continuity equation now becomes: $\nabla \cdot (\rho_b \mathbf{u}) = 0$

The Poisson equation for the linearized set can also be solved in presence of gravity so that the z-direction is non-periodic and x, y-directions are periodic. The subroutine inverse_laplacian_z in anelastic.f90 uses tridag to do this.

Changes in Makefile.local

Isothermal	General
DENSITY: experimental/anelastic	-do-
Nonlinear:	
ENTROPY: noentropy	entropy (but will not work)
Linearized:	
ENTROPY: noentropy	experimental/entropy_anelastic

Additionally we need to set
FOURIER = fourier_fftpack
POISSON = poisson

Adiabatic case not coded yet.

Future plan: To merge entropy.f90 and entropy_anelastic.f90

List of f-array variables

- Velocity and entropy are registered variables as usual.
- Non-linear: Pressure (ipp), RHS of NS equation (irhs - irhs+2), ρ(irho) are communicated auxiliary variables.
- Linearized: Pressure, RHS of NS equation, ρ_b (irho_b), s_b (iss_b) are communicated auxiliary variables. The f-array with index iss now contains s'.
- Selection made by using logical flags *lanelastic_lin* and *lanelastic_full*.

Changes in the f-array and storage of density

Logical flag *lanelastic* = T in anelastic.f90

Facility to toggle internal flags *lanelastic_lin* and *lanelastic_full* defined in eos_idealgas.f90 via eos_init_pars namelist

Non-linear treatment uses f-index, irho as the auxiliary communicated density variable instead of ilnrho. Note, this is NOT similar to setting ldensity_nolog=T for fully compressible runs.

Linear treatment stores the density base state ρ_b in f-index, irho_b and the fractional change ρ'/ρ_b in the pencil p%rhop. The entropy base state is similarly stored with f-index iss_b.

Pressure is stored as an auxiliary communicated variable with f-index ipp.

Samples and set-ups

- sample/2d-tests/anelastic_decay solves the nonlinear anelastic set of equations for an isothermal ideal gas in 2D with periodic boundaries. The initial condition is a vortex in xz plane which decays with time.
- Rest of the set-ups are on Nordita's CVS server norlx51.

  f90/pencil-piyali/anelastic/2d_isothermal

  f90/pencil-piyali/anelastic/2d_entropy

  f90/pencil-piyali/anelastic/conv-slab-anelastic

Note about the Poisson equation

- Non linear case. For periodic boundaries, the Poisson equation will give a pressure p, whose average over the domain is zero.
  - Necessary to add an average pressure $\propto$ Mass for isothermal case (implemented)
  - This average pressure will have a nonlinear dependence on Mass such as M^{γ} (not implemented)
- For linearized equations,
  - In presence of gravity, the buoyancy term $\rho' g/\rho_b$, has to be expressed as $\rho'/\rho_b = p'/\gamma p_b - s'/c_p$ and pressure term taken to the LHS of the Poisson equation. (implemented)

What works and what doesn't

- Tested the Poisson solver for non periodic case and it works correctly.
- Decay problems seem to work fine. Though *dt* and decay rates for nonlinear and linear formulations in isothermal runs are different! (needs to be checked)
- The set up for a 3D polytropic slab with gravity in z direction doesn't show gravity waves. Compared this with a similar set-up for the full 3D compressible case which definitely shows the *Brunt-Väisälä* oscillations.
- Even though the anelastic solver doesn't give the correct answer, it still is *faster* than the fully compressible run by a factor of 6 for a 32^3 setup.